dispel the notion that *H. erectus* in general, and Eastern *H. erectus* in particular, were relatively slow to react to challenges posed by the environment," because they not only navigated deep-water straits but adapted to life on an island, where the environment is thought to have been far different from the forest habitat of the mainland.

The new findings also fit well with other work showing that Asian *H. erectus* has been underrated. Controversial new dates from sites in Java suggest that *H. erectus* persisted there from as early as 1.8 million years ago until as recently as 30,000 years ago, implying that they were able to adapt to varied terrain and climate. Other new studies suggest that *H. erectus* left behind sophisticated hand axes in southern China (see sidebar on page 1636). For those who have worked on Flores and long believed in *H. erectus*'s presence there, the new results are vindication. Says Sondaar: "I am happy that the finds of Verhoeven are finally recognized."

–Ann Gibbons

MATHEMATICS

Polyhedra Can Bend But Not Breathe

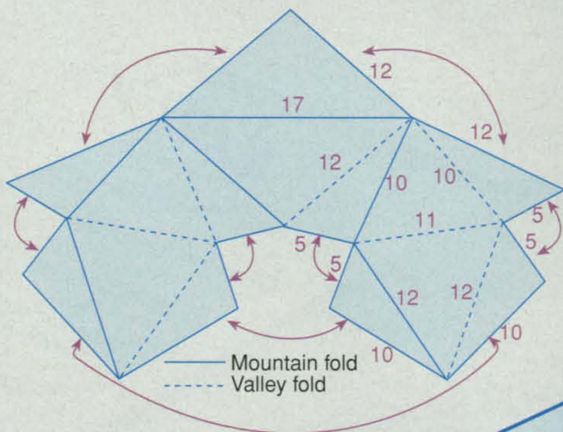
Anyone who has made an origami crane knows the delight and wonderment of conjuring a moving creature from the static geometry of lines and triangles. Although the flexibility of the paper is what allows the crane's wings to flap, mathematicians showed 20 years ago that a geometric equivalent could be constructed: a closed, three-dimensional figure made of rigid triangles, which can be squeezed or stretched into a new shape without distorting the faces. The finding upset what had been an article of faith for geometers and engineers—that a structure whose surfaces are made of triangles must be rigid. But a new proof shows that flexible polyhedra still face constraints: They have to keep their volume constant as they move. As Robert Connelly of Cornell University puts it, "You cannot build a mathematical bellows."

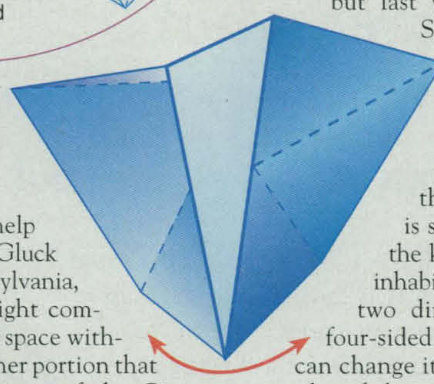
The discovery of flexible polyhedra, with their infinitely changeable angles, had blown a hole in the long-standing belief that a given set of edge lengths can yield only a finite number of shapes. But the new result, published jointly in the German journal *Beiträge zur Algebra und Geometrie* by Connelly, Idzhad Sabitov of Moscow State University, and Connelly's student Anke Walz, implies that edge lengths do narrow down one of the most important aspects of shape—the volume—to a finite number of possibilities.

In 1813, the French mathematician Augustin Louis Cauchy had proved that convex polyhedra—structures with flat faces, straight edges, and most important, no indentations—are always rigid. But that left open the question of whether polyhedra with indentations could flex. Around the turn of the century, a French engineer named Raoul Bricard found they could, if the faces were allowed to pass through each other. However, in the strictest sense Bricard's example, a flexible surface with eight faces, was not a polyhedron. For example, such a shape cannot be made into a physical object.

However, Bricard turned out to be on the right track. In the 1970s, Connelly managed to build a true flexible surface by elaborately altering Bricard's example, eliminating certain faces and allowing certain edges to detour around others. Later, Klaus Steffen of the University of Düsseldorf discovered a flexible polyhedron with only nine vertices and 14 triangular faces, which is believed to be the simplest one possible (see illustration).

As soon as these first models were built, mathematicians began playing around with them. "You could not help noticing," says Herman Gluck of the University of Pennsylvania, "that … though they might compress some portion of the space within, there was always another portion that expanded." Dennis Sullivan of the City University of New York blew smoke into a model and observed that none came out when the model was moved back and forth—suggesting again that it was not acting as a bellows.





Flexible figure. This crease pattern, with the dimensions shown, yields what may be the simplest flexible polyhedron *(right)*.

SOURCE: PETER CROMWELL, *POLYHEDRA*

The key to proving what Connelly called the Bellows Conjecture was a vast generalization of a formula discovered by an ancient Greek mathematician, Heron of Alexandria. Heron's formula says that the area, x, of a triangle with side lengths (a, b, c) must solve the following polynomial: $16x^2 + a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 - 2b^2c^2 = 0$. The volume of a tetrahedron has to satisfy a similar—but more complicated—polynomial. Connelly and, independently, Sabitov came up with the idea that the volume of any polyhedron might also solve some version of Heron's polynomial. If so, then the volume of a polyhedron with fixed side lengths could only change by jumping from one solution of the polynomial to another. But if the motion of the polyhedron is gradual, the volume cannot change suddenly. "It has no choice but to remain constant," says Gluck.

To prove that such polynomials exist for more complex polyhedra, Connelly and Sabitov found a way to divide these figures into tetrahedra, eliminate the edges of component tetrahedra that aren't actually edges of the final figure, and merge the known polynomials for the tetrahedra into a single polynomial for the entire shape. Even for a simple figure like an octahedron, the resulting polynomial involves 16th powers of the volume. Sabitov, in 1996, was the first to produce an algorithm that yields a polynomial for a general polyhedron, but last year's joint paper by Sabitov, Connelly, and Walz greatly simplifies the proof.

The proof still leaves plenty of mysteries. For one thing, the Bellows Conjecture is surprisingly sensitive to the kind of space the figure inhabits. It does not hold in two dimensions: A flexible four-sided figure, for example, can change its area without changing the side lengths. Connelly and Walz believe they can prove the Bellows Conjecture in four dimensions, but in higher dimensions, Connelly admits, "we're stuck."

Sabitov remains optimistic that in three dimensions, at least, mathematicians will soon have a complete understanding of how edge lengths determine the shape of polyhedra—not just their volume but also whether they can flex, and by how much. In the future, he says, "there will be a chapter of geometry titled 'The solution of polyhedra' as we now have 'The solution of triangles.' "

–Dana Mackenzie

Dana Mackenzie is a mathematics and science writer in Santa Cruz, California.