MATHEMATICS

Windows into Mathematical Minds

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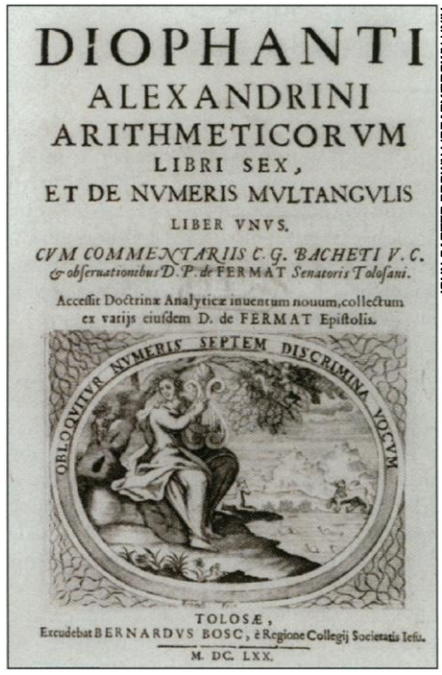
Fermat's Enigma. The Epic Quest to Solve the World's Greatest Mathematical Problem. SIMON SINGH. Walker, New York, 1997. xx, 315 pp., illus. $23. ISBN 0-8027-1331-9.

"I have discovered a truly marvelous proof which this margin is too narrow to contain."

These words might have been scribbled by a great detective, stopping to make notes while in pursuit of an elusive criminal. But in fact, they represented the greatest challenge to mathematicians for more than 300 years. Sometime around 1637, a French lawyer named Pierre de Fermat scrawled these words next to a mathematical statement that had remained unproven for over 1000 years. It took an additional 33 years for Fermat's notes to be published, and over three centuries more for Fermat's last theorem to be completely proven. During this time, many of the world's greatest mathematicians attempted, and failed, to prove the theorem. When Andrew Wiles of Princeton University announced that he had succeeded in 1993, the story ran on the front pages of newspapers around the world.

The saga of Fermat's enigma began in ancient Greece with the Pythagorean school of mathematicians. Pythagoras had proved that the equation $x^2 + y^2 = z^2$ had an infinite set of whole-number solutions (relating, of course, to the lengths of the sides of a right-angled triangle). What Pythagoras did not know was how many solutions existed if the exponent in his equation were a number greater than 2. Fermat's claim was that for any exponent greater than 2, there were no solutions at all. Fermat may have been the greatest mathematician never to publish his proofs—indeed, upon his death, many of his "theorems" were not proven at all. Over time, mathematicians were able to verify all of his claims—with the exception of the one in question. Fermat had provided a complete proof for the case in which the exponent is 4, and the great Swiss mathematician Leonhard Euler later proved that there are no solutions when the exponent is 3. Unfortunately, an infinite number of cases remained and the case-by-case method was doomed to fail.

Fermat's Enigma is constructed as a mystery novel. Although we know what will happen in the end, how we arrive at that conclusion proves most engaging. In solving their cases, great detectives must follow seemingly disparate leads and make connections that at first seem farfetched, but which ultimately prove to be incredibly insightful. In the case of Fermat's enigma, the process of following these leads introduces the reader to many great mathematicians and reveals connections between seemingly distinct areas of mathematics. Good novels contain more than plot, however; they construct an entire world in which their characters live. Simon Singh has done an admirable job of this, leading the reader through the professional world in which mathematicians lived in the past, as well as the world they inhabit today. His book is full of anecdotes that reveal how humanistic realities have sometimes intruded upon the abstract world of mathematics.



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Marginal notes. The frontispiece of C.-S. Fermat's edition of Diophantus's *Arithmetica*, published in 1670.

For example, one of the first mathematicians to make progress on more than a single case of the theorem was Sophie Germain. Germain's mathematical contributions are explained, but so are the circumstances of her life. Although some readers may know that she was forced to assume the identity of a man in order to gain respect as a mathematician, most will be surprised to hear of her role in securing the safety of the mathematician Carl Gauss during Napoleon's invasion of Germany. Vignettes such as these draw the reader into the mathematicians' world and are one of the strengths of the book.

A most intriguing detective, Andrew Wiles, dominates the latter chapters of the book. Contrary to the norms for a modern mathematician, Wiles worked for years in isolation attempting to solve this conundrum. His frustrations during this period and his elation on the proof's announcement are conveyed clearly. The anguish that he felt during the year in which his proof was considered incomplete is palpable, and the reader will share his sense of fulfillment at the end.

Writing a book about mathematical research for a general audience is a daunting task. Topics central to this particular proof, such as elliptical curves and modular forms, can intimidate graduate students, but here they are explained in just enough detail to convey their importance to the greater mission of proving the theorem. Indeed, another strength of the book is that the author, as well as the mathematicians whom he quotes, is able to explain complex mathematics in an understandable manner. Ten appendixes present some surprisingly accessible mathematics, including proofs by Euclid and Pythagoras as well as answers to old mathematical puzzles. These may be skipped by the reader without loss or read as endnotes to increase the rigor of the text.

Fermat's Enigma is an outgrowth of a 1-hour documentary coproduced by the BBC's *Horizon* and PBS's *NOVA* series. Singh directed the film and undoubtedly was frustrated at the amount of material that was left out because of time constraints. In this instance, the old cliché is true: The book *is* better than the movie. It represents a more thorough, deeper, and more satisfying treatment of the subject. The world of the mathematicians is more lucidly described, and seemingly disparate ideas are integrated more efficiently.

The dust jacket describes *Fermat's Enigma* as "the story behind NOVA's 'The Proof.'" This is probably the biggest error of the book. *Fermat's Enigma* is neither a mathematical proof nor a story about the making of a documentary. Readers expecting either will be disappointed. It is better described by its subtitle: *The Epic Quest to Solve the World's Greatest Mathematical Problem*. The book is a thematic tour through the history of mathematics, full of diversions made more interesting by their human element. Singh has said that the proof of Fermat's last theorem is of no practical value. This is not quite true. As this book shows, its story can provide insight into the detective-like world of mathematics and show us the human side of a discipline that is often thought to be entirely abstract.

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