calibration images. And instead of being smeared into two spots along the direction of the Polar spacecraft's wobble, as Frank had claimed, closely spaced spots in UVI images fell in random directions from each other.

Parks has now repeated the same type of analysis on spots from Frank's visible-light imager and reports finding the same noiselike behavior. The spots are "internal to the camera," he said at the meeting. "There's no evidence anything is coming from the outside."

Frank responded that Parks is analyzing the wrong spots. All but the largest dark spots are instrument noise, Frank said, adding, "There's no reason to include these enormous amounts of noise. You have to do something else to the data." If the spots are indeed clouds of water from incoming small comets, he reasoned, a cloud's motion across the field of view should add a third spot to the two created by the wobble motion of the spacecraft. Five of the candidate atmospheric holes in Parks's UVI images, said Frank, have three spots in the triangular orientation predicted by Polar's wobble. "I don't see how you can miss that sort of thing," said Frank.

Frank also looked at whether the spots shrink and disappear when Polar's eccentric orbit carries it farther above the atmosphere, as they should if they are real. Eighty percent of the large dark spots that Frank thinks are real holes disappeared between altitudes of around 25,000 kilometers and 41,000 kilometers, he said. "There's nothing you can do about this," said Frank. "It's the ultimate test. The holes are a geophysical phenomenon."

Parks, among many others, remained unconvinced. He notes that if the holes are real, then the three spots each one produces should be connected by slightly darkened streaks where the image was smeared across the detectors, but the three spots in the example he saw Frank present actually had slightly brighter spaces between them. And although Parks has not yet checked the altitude dependence of spots, he notes that Bruce Cragin of CES Network Services in Farmers Branch, Texas, and his colleagues did apply the altitude test in 1987 to spots in images from the Dynamics Explorer spacecraft, in which Frank first discovered atmospheric holes. When spacecraft altitude varied by a factor of 3, the abundance as well as the size of spots remained the same.

"I don't think the San Francisco exercise has changed many minds," says planetary scientist Thomas Donahue of the University of Michigan, Ann Arbor, who has leaned toward accepting the atmospheric holes as real. But some minds could change after other researchers get their shot at the data. NASA is expected to fund more analyses this year, and Frank plans to release all his Polar data on CD-ROMs this month, so that other space scientists can see—or fail to see—the spots for themselves.

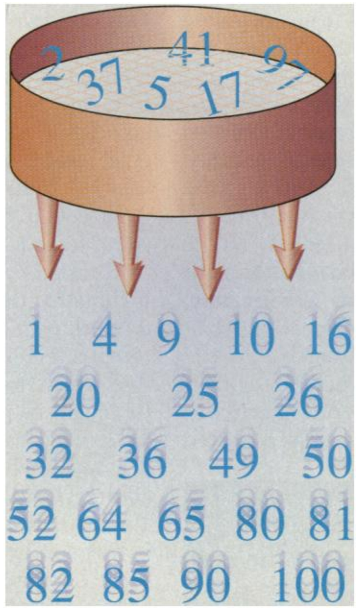
–**Richard A. Kerr**

MATHEMATICS

Sieving Prime Numbers From Thin Ore

Mathematicians have known since Euclid that there is an infinite number of prime numbers. For the last 100 years, they have even had a good way to determine, approximately, how many primes there are up to any given number. But the finer points of how primes are distributed remain, by and large, mysterious. In particular, mathematicians are almost always unable to take an infinite but sparsely distributed set of integers, such as the values of $n^2 + 1$, and tell how rich in primes it is.

That barrier is beginning to yield. In what number theorists are calling a major breakthrough, two mathematicians have developed powerful new techniques for assaying such "thin" subsets of integers for primes. As John Friedlander of the University of Toronto and Henryk Iwaniec of Rutgers University in New Brunswick, New Jersey, report in a paper to appear in the *Annals of Mathematics*, they have refined a tool known as the asymptotic sieve, developed in the 1970s by Enrico Bombieri of the Institute for Advanced Study in Princeton, New Jersey. Their first conquest: a remarkably thin sequence consisting of numbers of the form $a^2 + b^4$. Friedlander and Iwaniec's new sieve shows that even though most such numbers are composite—products of prime factors—the sequence includes an infinite number of primes.



Prime cut. The sieve captures the primes in the sequence of numbers of the form $a^2 + b^4$, up to 100.

ILLUSTRATION: L. CARROLL

"This is totally new," says Bombieri. The conclusion, he adds, "is what you would expect from heuristic arguments, but to prove things is another matter!" In his opinion, Friedlander and Iwaniec have written "one of the most important papers in analytic number theory of the century." It "will find a lot of applications" in exploring the distribution of primes.

Roughly speaking, a mathematical sieve determines the abundance of primes in a long list of numbers by estimating how many numbers remain when multiples of small primes are removed—a procedure that sifts out most composite numbers. For example, of the numbers between 169 and 289 (the squares of the primes 13 and 17), roughly half remain when you delete the even numbers, two-thirds of those remain after the multiples of 3 are removed, etc. Sieving the 120 numbers in the sequence yields an estimate of $120 \times (1/2) \times (2/3) \times (4/5) \times (6/7) \times (10/11) \times (12/13) = 23$ primes. That's close to the exact count, 22. Similar sieves can be designed for other number sequences. The real work comes in analyzing the errors in such estimates to get rigorous results.

The errors are easiest to estimate for "dense" sequences such as 1, 5, 9, 13, etc.—a progression that contains roughly one-fourth of all numbers up to a given size. But Friedlander and Iwaniec's sequence contains a vanishingly small fraction of the integers. That thinness makes it impossible to estimate the errors in the usual way, putting the sequence out of the reach of previous sieves. "Nobody dreamed you could analyze such sequences," says Bombieri.

The new techniques rely on special algebraic properties of the formula $a^2 + b^4$. In a number system known as the Gaussian integers, which enlarges the set of ordinary integers by including i, the square root of -1, such numbers can always be factored as $(a + b^2i)(a - b^2i)$. By putting their numbers into this form, Friedlander and Iwaniec were able to exploit the well-developed theory of algebraic numbers to get a handle on the errors when they applied their sieve.

The combination of algebraic number theory and sieve techniques is what Peter Sarnak of Princeton University finds most impressive. "These are two different worlds, algebra and the sieve," he notes. But because the technique relies on properties found in only a small fraction of sequences, many of sieve theory's dearest problems still look hopeless. In particular, numbers of the form $n^2 + 1$—1, 2, 5, 10, 17, 26, etc.—almost certainly include an infinite number of primes; indeed, number theorists have even conjectured an estimate for how many there are up to any given integer in the sequence. But no proof appears to be forthcoming. Number theorists are also convinced that each interval between consecutive squares contains at least one prime, but have no idea how to prove it.

"The answer to most of these questions is, we don't know," says Andrew Granville, a number theorist at the University of Georgia in Athens. "It's frightening how pathetic our knowledge is!"

–**Barry Cipra**