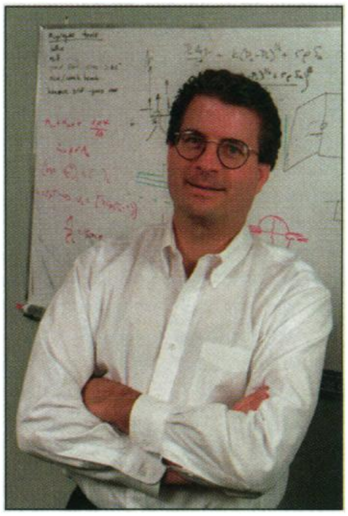
MATHEMATICS

Number Theorists Embark on A New Treasure Hunt



ANDY SCOTT

Numbers guy. Dallas banker Andrew Beal hopes to inspire young mathematicians with his prize.

What can mathematicians do for an encore now that the most famous problem in their subject has been solved? For over 3 centuries, Fermat's Last Theorem had tantalized both amateurs and professionals with all the ingredients of a real-life treasure hunt: a cryptic note left behind by French mathematician Pierre de Fermat; legends of well-known mathematicians who believed they had solved the puzzle, only to fall through hidden trap doors; a substantial reward offered by a German physician in his 1906 will. After Princeton University's Andrew Wiles proved the theorem in 1994, it seemed that the saga was finally over. But now, a Dallas banker has financed a sequel, offering a bounty of up to \$50,000 for a proof of a more general version of Fermat's Last Theorem.

"I've always loved brain teasers and logical puzzles," says Andrew Beal, the president of Dallas's largest locally owned bank. Beal, who never studied mathematics in college, first heard about Fermat's Last Theorem when Wiles announced his proof. After trying to solve the Fermat problem himself, Beal and mathematician Daniel Mauldin of the University of North Texas in Denton hatched the idea of a new prize for a new problem. "I was particularly thrilled to have someone … who's not a mathematician be interested in promoting mathematics," Mauldin says.

Fermat's original problem states that if two positive integers are raised to the nth power and then added, the sum can never be an nth power, provided n is greater than 2. But as he worked on the problem, hoping to find a simpler proof than Wiles's, Beal wondered whether a similar prohibition governs equations whose exponents vary. There are infinitely many examples in which the summands share a common factor: for instance, $17^4 + 34^4 = 17^5$. Much rarer are examples where the summands are "relatively prime"—that is, they have no prime factors in common. Exactly 10 are known (see box), and in all of them one of the exponents is 2. Beal is offering \$5000 to the first person who can prove that there are no relatively prime solutions in which all the exponents are 3 or greater. Like a lottery, the prize will rise by \$5000 each year, up to the maximum of \$50,000.

Until now, number theorists say, this conjecture has been in a sort of limbo, unpublished but not exactly unknown. Andrew Granville of the University of Georgia, Athens, has lectured for several years on the "Beal equation," $a^x + b^y = c^z$. He always writes the 10 known examples on the board. "Every time I give the lecture, someone in the audience asks, 'Have you noticed that, in all those examples, there is an exponent of 2?' " Granville says. But he has refrained from publishing this as a conjecture because he believes the evidence favors a slightly different version.

Granville's own claim is that the Beal equation has only a finite number of relatively prime solutions for which $1/x + 1/y + 1/z$ is less than 1. While Beal's conjecture limits x, y, and z to values greater than or equal to 3, Granville's allows the smallest exponent to be 2, provided the other two exponents are large enough. He and Henri Darmon of McGill University in Montreal have already proved that for any particular x, y, and z that satisfy the inequality, there are only a finite number of solutions, and he believes that the same will hold for all possible trios of exponents put together. "I'd be surprised but not incredibly surprised if [Beal's] conjecture is wrong," Granville says. "But I'd be absolutely devastated if there are infinitely many solutions" that meet his own condition.

Beal's prize could easily reach its \$50,000 limit before either version of the conjecture is proven, say mathematicians. Although Fermat's Last Theorem would be a special case of this one, says Ron Graham of AT&T Research, Wiles's method for proving the Fermat theorem probably won't suffice to prove Beal's conjecture. In that case, the problem may require a brand-new approach that would not only re-prove the Fermat theorem but a whole lot more.

In the long run, both Fermat's Last Theorem and the Beal conjecture may turn out to be incidental cases of a grander hypothesis called the "abc conjecture." Roughly, this says that whenever two numbers with many repeated factors (for example, nth powers) are added together, the result is a number that has relatively few repeated prime factors and is unlikely to be a power. But the formal version of the abc conjecture, with a spate of adjustable parameters and ad hoc functions, lacks the charisma to attract new treasure seekers—especially young ones.

Beal, who has also sponsored science fairs and Odyssey of the Mind competitions, is hoping to reach just those talents. "I'd like to do anything to stimulate young minds to look at mathematics," he says. One such mind belonged to a boy who went to the Milton Road Lending Library in Cambridge, England, in 1963 and checked out a book on Fermat's Last Theorem. That boy's name was Andrew Wiles.

–**Dana Mackenzie**

Dana Mackenzie is a free-lance science and mathematics writer in Santa Cruz, California.

A Limited Universe of Solutions?

Only 10 solutions are known for the equation $a^x + b^y = c^z$, where a, b, and c have no prime factors in common and $1/x + 1/y + 1/z$ is less than 1.

$1^n + 2^3 = 3^2$ (for any n)
$2^5 + 7^2 = 3^4$
$7^3 + 13^2 = 2^9$
$2^7 + 17^3 = 71^2$
$3^5 + 11^4 = 122^2$
$17^7 + 76271^3 = 21063928^2$
$1414^3 + 2213459^2 = 65^7$
$43^8 + 96222^3 = 30042907^2$
$33^8 + 1549034^2 = 15613^3$
$9262^3 + 15312283^2 = 113^7$

Mathematicians found the five large solutions in the last decade. Why the solutions fall into two groups is a mystery, says Andrew Granville of the University of Georgia, Athens. It's "absolutely staggering that there should be five 'big' solutions," he wrote in 1994. "Why on Earth is there such a big jump between the sizes of the two sets of solutions?" –D.M.