OPTICAL PHYSICS

Solitons Made Simple

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Solitons, or solitary waves, are localized wave entities that propagate with little change of form. They occur under special circumstances from wave propagation in nonlinear dispersive media. On his discovery of solitons in 1834, J. Scott Russell (*1*) wrote that while riding along the Edinburgh-Glasgow Canal, he had observed rolling forward "a round and well-defined heap of water which continued its course along the channel apparently without change of form or diminution of speed" (see figure). Although solitons exist ubiquitously in many branches of science, theoretical attempts to understand them have been almost exclusively mathematical. As Haus and Wong (*2*) stated in a recent article on the subject, "The soliton concept is a sophisticated mathematical construct based on the integrability of a class of nonlinear differential equations." Indeed, ever since Korteveg and de Vries (KdV) (*3*) first derived in 1895 their famous equation for water waves, study of solitons has always been an active area of research in mathematical physics. There are perhaps 100 known integrable nonlinear differential equations with soliton solutions, including the KdV equation, the nonlinear Schrödinger equation, and the sine-Gordon equation. Although such investigations do reveal some general structure and behavior of solitons, much physical insight is lost in the complexity of mathematics. It is particularly difficult for nonexperts to appreciate the results. Now, as Snyder and Mitchell report on page 1538 of this issue (*4*), solitons and their characteristic behavior can be described with elementary mathematics related to simple physical pictures.

Snyder and Mitchell consider spatial optical solitons in their model. Spatial and temporal solitons refer to solitary waves that retain, respectively, their spatial and temporal profiles while traveling. The latter has been intensively studied as a possible candidate for carriers in future optics communications (*2*). Theories for the two types of solitons are, however, basically identical. Spatial solitons can be seen as the result of a delicate balance between optical diffraction and self-focusing in a medium whose refractive index n depends on the light intensity I. The important contribution of Snyder and Mitchell is their recognition that the mathematical complexity of soliton solutions arises only because of the dependence of n on spatially varying intensity $I(\mathbf{r})$; it is not fundamental to solitons. The complexity can be eradicated if n would depend not on $I(\mathbf{r})$ but on the total beam power $P = I(\mathbf{r})d\mathbf{r}$ and $\mathbf{r}$. This can be achieved in their heuristic model by assuming that the medium has a nonlocal response with a correlation length much larger than the beam diameter. The nonlinear wave equation then becomes linear and readily solvable, but the solution for solitons still contains the essential characteristic features. Physically, the model transforms the problem into a simple case of linear propagation of thin beams in a wave guide. With the assumption that the beams always stay close to the axis, the refractive index $n(P, \mathbf{r})$ makes the wave equation identical to the time-dependent Schrödinger equation for a linear harmonic oscillator, the solution of which is well-known to all physicists. The physics of solitons can then be readily appreciated.

Thus, Snyder and Mitchell's work has liberated us from dealing with complicated mathematics such as the inverse scattering technique (*5*) to understand solitons. It provides an easy access to more complex soliton phenomena, namely, higher order solitons and soliton-soliton interaction. The latter, for example, would require a highly sophisticated mind in mathematics to extract a good physical understanding from the mathematical solutions, but Snyder and Mitchell's approach allows a quick penetration of the mist to attain a direct visualization of the physical results. A critic might question whether their model can be realized physically: It is indeed difficult to find a medium whose optical response can satisfy the requirement of a correlation length much larger than a beam diameter. Liquid crystals and photorefractive materials are known to exhibit nonlocal response, but of the materials reported to date, all have a correlation length not more than a few micrometers. Possibly Snyder and Mitchell's predictions will serve as an incentive for experimentalists to devise ways of extending correlation lengths. However, in my opinion, physical realizability of the model is not important. Solitons of a specific system are not the same as those of other systems anyway. It is "soliton physics made easy" that renders the model invaluable.

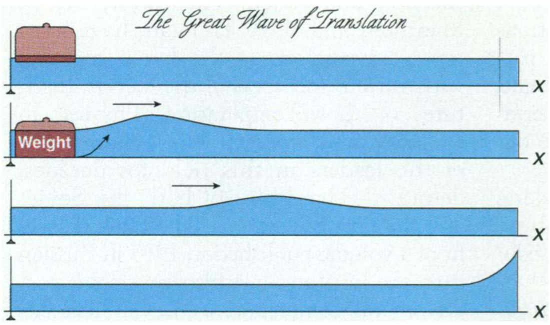

Illustration adapted from a copperplate etching by J. Scott Russell (*6*) depicting the 30-foot tank he built in his back garden in 1834. [Courtesy Chris Eilbeck, Heriot-Watt University, Edinburgh]

Snyder and Mitchell's accessible solitons have circumvented the usual difficulties in our search for a better understanding of soliton physics. This achievement is a giant step forward. With their model, not only can old soliton phenomena be easily understood, but new soliton phenomena can be readily predicted. Their approach is not limited to optical solitons; extension to solitons in other disciplines should be feasible. Such theoretical advances will undoubtedly encourage more experimental research on solitons. Thus, Snyder and Mitchell's work could be the stimulant for a new surge of soliton activities in the near future.

References and Notes

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4. A. W. Snyder and D. J. Mitchell, *Science* **276**, 1538 (1997).
5. V. E. Zakharov and A. B. Shabat, *Zh. Eksp. Teor. Fiz.* **61**, 118 (1991) [*Sov. Phys. JETP* **34**, 62 (1972)].
6. J. S. Russell, plate XLVII, in (*1*).
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