

# Direct Measurement of Angles Between Bond Vectors in High-Resolution NMR

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Angles between two interatomic vectors are measured for structure elucidation in solution nuclear magnetic resonance (NMR). The angles can be determined directly by using the effects of dipole-dipole cross-correlated relaxation of double-quantum and zero-quantum coherences. The measured rates can be directly related to the angular geometry without need for calibration of a Karplus-type curve, as is the case for scalar coupling measurements, and depend only on the rotational correlation time of the molecule as an empirical parameter. This makes the determination of torsional angles independent from the measurement of coupling constants. The two interatomic vectors can in principle be arbitrarily far apart. The method was demonstrated on the measurement of the peptide backbone angle  $\psi$  in the protein rhodniin, which is difficult to determine in solution by NMR spectroscopy.

Structure determination of biomolecules by NMR spectroscopy in solution relies on the measurement of interatomic distances (1) that are accessible by the nuclear Overhauser effect (NOE) (2) induced by the dipolar coupling between proton nuclei and the determination of torsional angles that can be derived from scalar coupling constants once an empirical correlation, the Karplus relation, of the measured scalar couplings with determined torsional angles (3) has been established. The accessibility of this information has grown rapidly over the last years by the introduction of multidimensional NMR spectroscopy (4) and the availability of uniformly  $^{13}\text{C}$ - and  $^{15}\text{N}$ -labeled biomacromolecules (5). Nevertheless, the current limitations on structure determination are twofold: NOE intensities fall off rapidly as the distance between the protons involved increases, and coupling constant information relies on calibration of Karplus relations.

Here, we introduce a method to determine the angles between two vectors connecting two pairs of nuclei within a molecule by solution NMR (Fig. 1). No empirical calibration is required to determine the included angle. The method relies on relaxation via cross-correlated dipolar couplings between the four nuclei that form the two internuclear vectors under consideration. Cross-correlated relaxation has so far been considered to be a cumbersome disturbance to high resolution spectra. It has been used only rarely to derive structural information (6). In solid-state NMR spectroscopy, the use of dipolar couplings to determine intertensorial angles is widely used (7) and has been recently

extended to biomolecules in the solid state (8, 9).

Dipolar couplings which constitute the through space magnetic interaction between two nuclei are averaged out in solution due to the isotropic tumbling of biomolecules. However, dipolar couplings are an effective source of relaxation of spin coherences (10) due to the stochastic modulation of the local fields according to the angular diffusion of the whole molecule in solution with a time constant in the ps to ns range. The efficiency of the relaxation depends on the time correlation of the dipolar coupling with itself (autocorrelation) or with another dipolar coupling (cross-correlation) (11). Without loss of generality, we discuss in the following a carbon-proton and a nitrogen-proton spin pair. Double-quantum coherence involving the nitrogen and carbon transitions can be excited and indirectly detected, which yields a four-line spectrum whose line splittings are the scalar couplings between  $\text{H}^{\text{N}}$  and  $\text{N}$  as well as  $\text{H}^{\text{C}}$  and  $\text{C}$  (Fig. 2). The relaxation rates  $\Gamma_{\mu\nu}$  for the four transitions that are reflected in the line widths are given by the following equations (12)

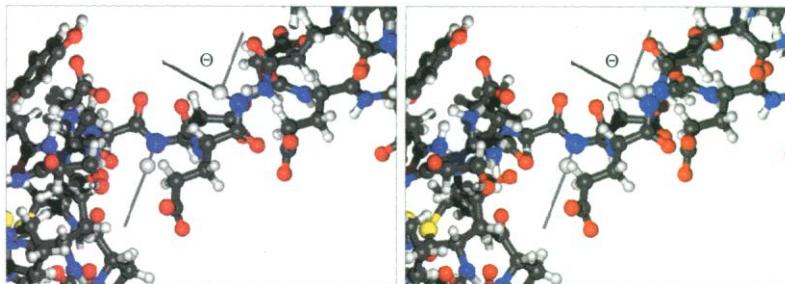
$$\begin{aligned}\Gamma_{\alpha\beta} &= \Gamma^a - \Gamma_i^{\text{CSA/DD}} + \Gamma_j^{\text{CSA/DD}} - \Gamma_{i,j}^c \\ \Gamma_{\alpha\alpha} &= \Gamma^a + \Gamma_i^{\text{CSA/DD}} + \Gamma_j^{\text{CSA/DD}} + \Gamma_{i,j}^c \\ \Gamma_{\beta\beta} &= \Gamma^a - \Gamma_i^{\text{CSA/DD}} - \Gamma_j^{\text{CSA/DD}} + \Gamma_{i,j}^c \\ \Gamma_{\beta\alpha} &= \Gamma^a + \Gamma_i^{\text{CSA/DD}} - \Gamma_j^{\text{CSA/DD}} - \Gamma_{i,j}^c\end{aligned}\quad (1)$$

The indices  $i$  and  $j$  denote in this case the  $\text{N}-\text{H}_i^{\text{N}}$  and the  $\text{C}-\text{H}_j^{\text{C}}$  vector, respectively. The term  $\Gamma^a$  defines the contributions due to autocorrelated relaxation for the individual spectral lines.  $\Gamma_{i,j}^{\text{CSA/DD}}$  describe the sum of all interactions due to dipole-CSA cross-correlated relaxation for the  $\text{N}-\text{H}_i^{\text{N}}$  and the  $\text{C}-\text{H}_j^{\text{C}}$  vector, respectively. CSA is chemical shift anisotropy and refers to the orientation dependence of chemical shifts.  $\Gamma_{i,j}^c$  is the desired contribution resulting from dipole-dipole cross-correlated relaxation.

These relaxation rates can be determined from the intensities of the corresponding resonance lines in a constant-time NMR experiment, in which the intensity  $I_{\mu\nu}$  of a resonance line has decayed by  $\exp(-\Gamma_{\mu\nu}T)$  after a given time  $T$ . Except for the overall relaxation  $\Gamma^a$  which does not yield any structural information all other relaxation rates can be determined according to the equations

$$\begin{aligned}\Gamma_{i,j}^c &= \frac{1}{4T} * \ln\left(\frac{I(\alpha\beta)*I(\beta\alpha)}{I(\alpha\alpha)*I(\beta\beta)}\right) \\ \Gamma_i^{\text{CSA/DD}} &= \frac{1}{4T} * \ln\left(\frac{I(\alpha\beta)*I(\beta\beta)}{I(\alpha\alpha)*I(\beta\alpha)}\right) \\ \Gamma_j^{\text{CSA/DD}} &= \frac{1}{4T} * \ln\left(\frac{I(\beta\beta)*I(\beta\alpha)}{I(\alpha\alpha)*I(\alpha\beta)}\right)\end{aligned}\quad (2)$$

where  $T$  is the delay in the experiment during which double-quantum coherence evolves.  $I(\alpha\alpha)$ ,  $I(\alpha\beta)$ ,  $I(\beta\alpha)$ , and  $I(\beta\beta)$  are the intensities of the corresponding resonance lines in the spectrum. For a macromolecule that tumbles slowly in solution, the cross-correlated dipolar relaxation depends on the angle  $\theta$  between the  $\text{C}-\text{H}_j^{\text{C}}$  and  $\text{N}-\text{H}_i^{\text{N}}$  bonds (11, 13) according to



**Fig. 1.** Stereoview showing a close-up of the structure of rhodniin. The two gray lines indicate the bond vectors of two spin pairs. The angle between the two interatomic vectors is indicated by  $\theta$ . Atoms are depicted by the following colors: hydrogen, white; carbon, black; sulfur, yellow; oxygen, red; and nitrogen, blue.

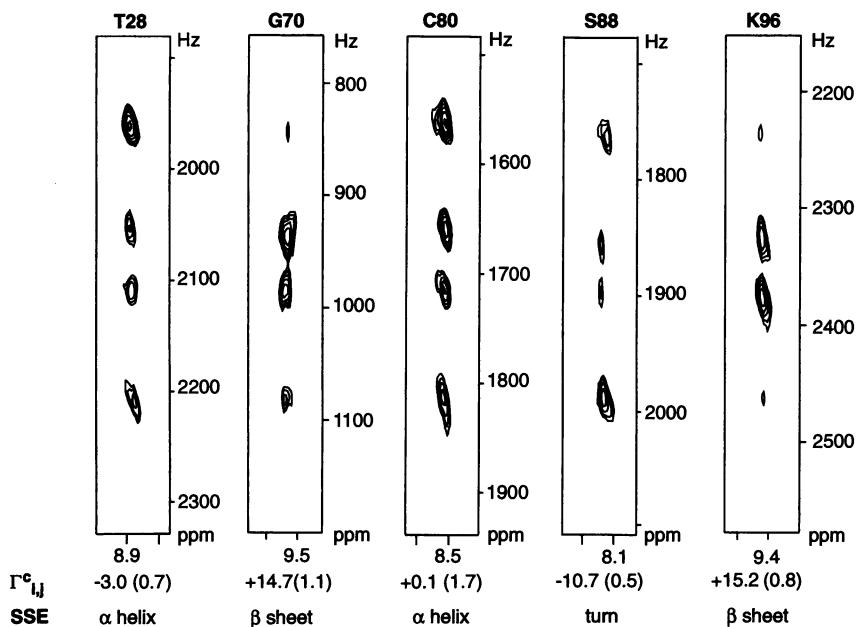
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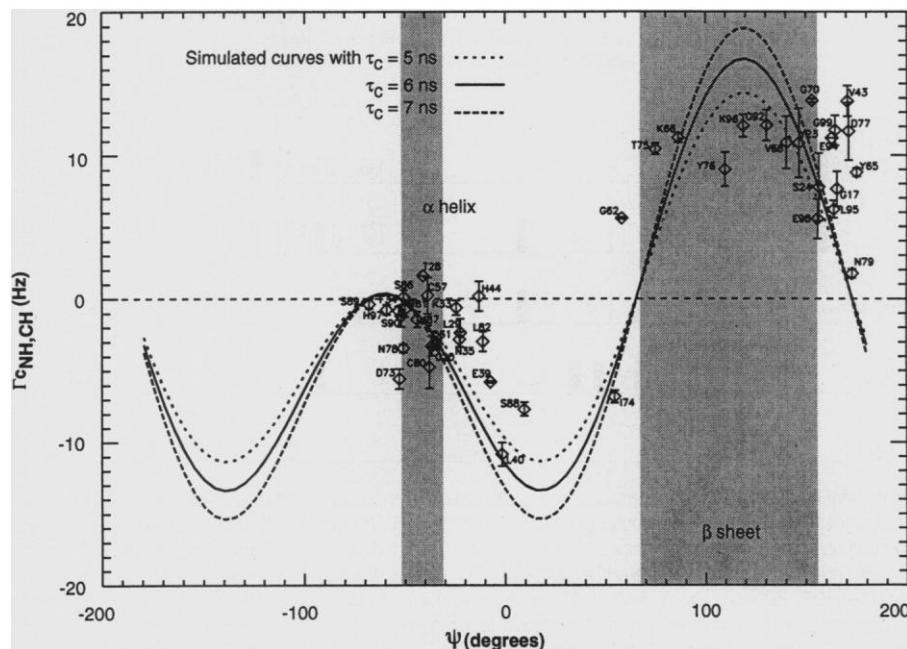


tracted from the two-dimensional experiment. The extracted relaxation rates  $\Gamma_{ij}^c$  with their standard deviation are given on

the bottom of the plot for the selected peaks. Furthermore, the secondary structure element (SSE), which is found in the NMR



**Fig. 5.** Experimentally obtained peak shapes for selected residues in rhodniin. The  $H^N$  chemical shift (600 MHz) of residue  $k+1$  (indicated on top of planes) is given on the horizontal axis. Double-quantum coherence which evolves between the nuclei  $C_k^{\alpha}$  and  $N_{k+1}$  is represented on the vertical axis. Deviations from the intensity ratio (1:1:1) that would be found without cross-correlated relaxation can clearly be seen. Two residues within an  $\alpha$ -helical (T28 and C80), as well as two residues within a  $\beta$  sheet (G70 and K96), together with one residue from a turn motive (S88) are shown. The mean relaxation rates in hertz as extracted according to Eq. 2 and their standard deviations are given below each residue. Single-letter abbreviations for the amino acid residues are as follows: C, Cys; G, Gly; K, Lys; S, Ser; and T, Thr.



**Fig. 6.** Experimental and simulated relaxation rates  $\Gamma_{ij}^c$  in Hz due to cross-correlated relaxation as a function of the backbone angle  $\psi$  and three overall correlation times  $\tau_c = 5.0, 6.0,$  and  $7.0$  ns. The  $\psi$  values for the selected residues are derived from the NMR structure of rhodniin that has been calculated with NOE data and torsional angle restraints. The range of  $\psi$  values found in  $\alpha$  helices and  $\beta$  sheets are shaded in gray.

structure for the respective residue, is cited. The experimental (from the three-dimensional experiment) and simulated (20) relaxation rates (for three correlation times  $\tau_c = 5.0, 6.0,$  and  $7.0$  ns) are drawn in Fig. 6 as a function of the backbone angle  $\psi$ . The extracted cross-correlated relaxation times agree very well with the  $\psi$  angles found in the NMR structure.

The methodology can be applied for all so-called "out and back" NMR experiments that are used for the assignment of proteins and oligonucleotides. The limitations with respect to size are the same as for these assignment experiments, because the proposed methodology relies exclusively on scalar couplings via one bond that are the biggest intramolecular interactions known for molecules in isotropic solutions. Long-range structural information is available if two nuclei can be correlated in an efficient way which can be achieved most efficiently for large deuterated proteins in which the  $H^N, H^N$ -NOE can be used for coherence transfer.

The proposed method yields different information from recently proposed measurement of residual dipolar couplings (21) and evaluation of  $^{15}N$ -relaxation rates in anisotropically tumbling molecules (22). The residual dipolar couplings provide projection information of bond vectors on anisotropy tensors of the magnetic susceptibility. Relaxation rates yield projection information of bond vectors on tensors that describe the anisotropy of the diffusive reorientation of the molecule. Thus, they provide indirect information about the relative orientation of interatomic vectors with respect to each other. By contrast, the proposed method yields direct information about bond vector projections provided double or zero quantum coherence can be excited between them. The cross-correlated relaxation rate is related to the interbond angle  $\theta$  in an ambiguous way because of the  $(3\cos^2\theta - 1)/2$  relationship (Eq. 3). However, this is also found for  $J$ -coupling constants, residual dipolar couplings, and projection on anisotropic reorientational tensors. The methodology can be extended to cross-correlation between dipolar couplings and CSA tensors of nitrogen (23, 24) and  $C_{\alpha}$  carbon (23, 25), which can potentially be used to resolve the ambiguities of the  $\Gamma_{ij}^c$  ( $\psi$ ) relation. The new spectroscopic parameter is uniquely powerful when a new set of NMR active nuclei is investigated in an organic molecule, for example, where neither the correlation time needs to be determined nor the interatomic distances need to be known, as  $\Gamma_{ij}^c$  changes sign at  $\pm 54^\circ$  and  $\pm 126^\circ$ . Fluctuations of  $\theta$  that are on a time scale longer than  $\tau_c$  lead to

averaging of the cross-correlated relaxation rate which may reveal slow motion up to 1 ms.

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## Large Molecular Third-Order Optical Nonlinearities in Polarized Carotenoids

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Garito and co-workers have suggested a mechanism to dramatically increase the second hyperpolarizability,  $\gamma$ , in linear  $\pi$ -electron-conjugated molecules. Polarization is introduced that leads to a difference between the dipole moments of the molecule's ground state and excited state. Here a series of carotenoids was examined that had increasing intramolecular charge transfer (ICT) from the polyenic chain to the acceptor moiety in the ground state, and  $\gamma$  was measured for these compounds as a function of wavelength by third-harmonic generation. The compound with the greatest ICT exhibited a 35-fold enhancement of  $\gamma_{\text{max}}$  (the  $\gamma$  measured at the peak of the three-photon resonance) relative to the symmetric molecule  $\beta$ -carotene, which itself has one of the largest third-order nonlinearities known. Stark spectroscopic measurements revealed the existence of a large difference dipole moment,  $\Delta\mu$ , between the ground and excited state. Quantum-chemical calculations underline the importance of interactions involving states with large  $\Delta\mu$ .

Molecules with large third-order optical nonlinearities (characterized by large second hyperpolarizabilities,  $\gamma$ ) are required for photonic applications including all-optical switching, data processing, and eye and sensor protection. In contrast to second-order nonlinear materials, for which the basic structure-property relations are both relatively well understood and well explored, for third-order materials a comparable level of understanding is only just emerging. Garito and co-workers have suggested a mechanism to dramatically in-

crease  $\gamma$  (1, 2), and they calculated that polarized polyenes could have very large  $\gamma$ , relative to unsubstituted polyenes of comparable length, because of symmetry breaking. A parallel situation exists in semiconductor quantum wells where asymmetrization has led to some of the largest second- and third-order nonlinearities ever observed in the mid- to far-infrared portion of the spectrum, both with and without the presence of permanent dc fields (3–5). The results of recent computational and experimental studies on small model systems agree with the predictions of Garito *et al.* (6–9). Nonetheless, molecules that exhibit extremely large  $\gamma$  according to the mechanism predicted by Garito *et al.* are still quite unusual (10–12). Indeed,  $\gamma$  values are still, perhaps with rare exceptions (13), not sufficiently large for realistic applications to ultrafast information-processing optical devices. Here we explored the hypothesis of Garito *et al.* further and found that large enhancements in  $\gamma$  can be achieved in a variety of acceptor-substituted carotenoids as determined by wavelength-dependent third-harmonic generation experiments (THG).

Several groups have shown from a sum-over-states analysis of  $\gamma$  derived from perturbation theory that three classes of terms

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