Fractal Geometry Gets the Measure of Life's Scales

Living organisms come in a vast range of sizes—from microbes to whales, they span at least 21 orders of magnitude. Biologists have long been intrigued by this startling array of bodily dimensions, and for more than a century they have been trying to figure out how body size and physiology are related. What they have come up with so far is a big conundrum. Metabolic rate, for example, varies in proportion to the 3/4 power of an organism's mass—the bigger the creature, the slower its metabolism—and similar relationships have been found for variables such as life-span, age at first reproduction, and duration of embryonic development (1/4, 3/4, and –1/4 powers of mass). The common factor in all these relationships—the 1/4 power, which seems to hold in almost all organisms from microbes to higher plants and animals—has biologists stumped: 1/3 powers would be much more logical if metabolic rate reflected only geometric constraints of body size.

Now, on page 122, a team of researchers reports a new mathematical model for living organisms that may finally help solve the puzzle. Their model—a unique combination of the dynamics of energy transport and the mathematics of fractal geometry—is still very general, but it has produced results that conform well with observations of living systems, including the enigmatic 1/4-power scaling. Other biologists are keen to test their approach. "They have really come up with something quite unexpected," says ecologist William Calder of the University of Arizona.

Ecologists James Brown and Brian Enquist of the University of New Mexico in Albuquerque had been butting their heads against the 1/4-power law for a long time. "It's an intriguing relationship, and we just couldn't get anywhere until we began to wonder whether the transportation of materials around the body might be a key rate-limiting process," says Brown. The two quickly found that they needed more expertise in mathematics and physics to explore the complex mechanics of shunting supplies around bodies of different sizes. So, with the help of the Sante Fe Institute, which specializes in interdisciplinary theoretical research, they teamed up with physicist Geoffrey West of the Los Alamos National Laboratory.

The team analyzed organisms in terms of the geometry and physics of a network of linear tubes required to transport resources and wastes through the body. Such a system, they reasoned, must have three key attributes: The network must reach all parts of a three-dimensional body; a minimum amount of energy should be required to transport the materials in a fluid medium; and the terminal branches of the networks (for example, the capillaries in the circulatory system) should all be the same size, as cells in most species are roughly similar sizes. A key insight came when the team realized that such a network could best be described using a space-filling, fractal-like branching system. Fractal geometry, pioneered by physicist Benoit Mandelbrot, has been used to model many seemingly complex natural structures, from snowflakes to the branching patterns of streams, by repeatedly applying a relatively simple mathematical formula.

Branching out. Computer model of a vertebrate vascular system created using fractal geometry.

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"Researchers previously have tended to focus on individual parts of transport systems, such as major vessels or capillary beds, and have not focused on the whole network," says Brown. "By looking at a whole transport system, it is possible to see the fractal branching," he adds. "Combining energetics with fractal design is a fascinating approach," says comparative physiologist Ewald Weibel of the University of Berne in Switzerland.

Using this approach, the team has developed a general model for the design of distribution networks that incorporates both fractal geometry and hydrodynamics. The researchers believe that their model includes the most fundamental features of a real network. They found, for example, that when they initially tried to ignore some features, such as the elastic and pulsatile nature of blood vessels, "the model gave us all the wrong answers," says Brown. The researchers then added more detail to the model, "to incorporate special features of blood vessels and other features such as the multiple small, parallel vessels of plant vascular systems," Brown adds. And when this was done, the model predicted values for the scaling of structural and functional variables that were in close agreement with measured values. "It's stronger than any model that has come along before," says Calder. And to the team's delight, the model predicts 1/4-power scaling. "It's the fractal approach that gives us the 1/4-power scaling," says Brown.

The model makes several as yet untested predictions. For example, because the metabolic rate of single-celled organisms, like those of higher animals, also scales as the 3/4 power of mass, the model predicts that the distribution of materials within single cells will show similar geometric and physical properties to those found in multicellular organisms. It also predicts the degree of branching in a circulatory system: A whale is 10^7 heavier than a mouse, but the new model suggests that a whale needs only 70% more branches in its circulatory system to supply its body. Brown even suggests that a fractal circulatory system, by providing an optimum means of provisioning different body sizes, may be a major factor in the evolution of such a vast array of shapes and sizes in the living world. "If fractal distribution didn't happen, would you find 21 orders of magnitude among organisms?" he asks.

West, Brown, and Enquist are now planning to test some of the model's predictions; they are also looking at whether the model can help explain other aspects of ecology and evolution, such as why life-span, age at first reproduction, and embryonic development time also fail to scale to the 1/3 power of mass. "Since the rates of resource use, times of life histories, abundance of individuals, and numbers of species all vary predictably with body size, scaling may be one of the most fundamental features of biological diversity," says Brown.

Although the model is highly generalized, the team believes it may provide the starting point for other, more detailed, models. Says Weibel, "I'm not sure their model is the final one, but it has good predictive power, and study of the deviations from it is going to be very interesting."

–Nigel Williams