MATHEMATICS

Taking 'Hard' Problems to the Limit

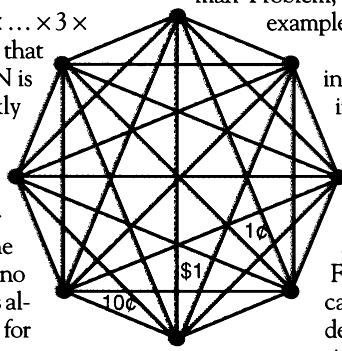
SAN DIEGO—If you suffered through college calculus, you may remember your professor harping on the notion of "limits." Simultaneously simple and subtle, the limit concept is central to the study of analysis. Loosely speaking, a limit is the residue of an infinite process; for example, it's the exact value on which an infinite sequence of approximations "converges." Because limits are meaningful only when infinity is involved, they haven't played much of a role in theories of purely finite processes such as computing, which takes place in finite-sized machines. But if Michael Freedman is correct, that's going to change in a big way.

Freedman, a mathematician at the University of California, San Diego, thinks that the notion of limits could be key to solving a famous question in computer science: whether certain problems, such as the Traveling Salesman Problem, are "hard," meaning they cannot be solved efficiently by any computer algorithm. Such problems underlie nearly all cryptography and computer security codes, and proving that these applications are based on something more than blissful ignorance has been a major goal of theorists in computational complexity for more than 2 decades.

By applying the notion of limits, Freedman explained here at the joint meetings of the American Mathematical Society and the Mathematical Association of America in January, he thinks he may be able to convert this messy question into a clear-cut proposition: whether or not single, infinitely large examples of these problems have solutions. Experts in computational complexity who know the outlines of Freedman's proposal—few know its details—are intrigued, partly because of Freedman's reputation. (He won a Fields Medal, mathematics' version of the Nobel Prize, in 1986, for work in topology.) "What he's doing sounds interesting," says Michael Sipser of the Massachusetts Institute of Technology, who has explored similar ideas about infinite limits.

Even though computers are strictly finite, the concept of infinity lurks in theories about what kinds of problems they can tackle: The complexity of an algorithm, say for solving the Traveling Salesman Problem, refers to how quickly the amount of work grows as the size of the problem (e.g., the number of cities the salesperson is to visit) "goes to infinity." Computer scientists are happiest with what they call polynomial-time algorithms, in which the amount of work required grows at a less-than-exponential pace as the problem gets larger. Such algorithms are considered "efficient." For the Traveling Salesman Problem, in contrast, a straightforward comparison of all possible itineraries to find the easiest one requires upward of $N! = N \times (N-1) \times (N-2) \times \ldots \times 3 \times 2 \times 1$ calculations—a number that is hideously large even when N is in the teens, and that quickly becomes astronomical.

But even though computer scientists suspect there is no efficient algorithm for solving such problems, no one has been able to prove that no such algorithm exists. "This is always an attractive situation for the pure mathematician, when there's a very clear, well-defined problem blocking understanding," Freedman says. "It's kind of like waving a red flag at a bull!"



Infinite salesperson. In this example, the journey's cost drops as the number of destinations goes to infinity, but there is no "cheapest" infinite itinerary.

Freedman's idea, which he notes is still highly speculative, is to extend the concept of computation to include infinitely large problems, each one a limit of increasingly large, finite examples. The trick is to define the extension so that a particular criterion holds: Whenever the original computational problem has a polynomial-time algorithm, each infinite example has a solution. The hope, then, is that for problems like the Traveling Salesman Problem, there will be infinitely large examples without solutions.

It's relatively easy to construct infinite examples of the Traveling Salesman Problem for which there is no easiest path through all the cities. The question is whether such examples can be viewed as limits of finite-size problems, according to Freedman's criterion. He is still casting about for the right way to define his limits. "I have a 'design criterion,' " he says, "and I'm trying to find a definition that will fit it."

The effort could come to naught, he admits, but he thinks the notion of limits is sure to pay off in other arenas—in unsolved problems in geometry, for example. It's too soon to tell just where, he says: "You could call it a fantasy or a research program. Research programs begin as fantasies."

–Barry Cipra

CHEMICAL ELEMENTS

The Weighty Matter of Names

Although the heaviest 19 elements don't exist in nature and have no known use, physicists have fought fiercely for the right to name them. Now, several pitched battles reflecting Cold War rivalries may have drawn to a close. Late last month, an international commission presented a revised list of names for elements 104 to 109 to chemistry's high court, the International Union of Pure and Applied Chemistry (IUPAC), which will rule on the names this August.

Created inside particle accelerators by fusing two lighter elements, these superheavy elements last at most a few seconds before disintegrating. Conflicts arise in part because it takes months to determine that a new element was formed and months more for an independent lab to confirm the feat. In the meantime, each lab's pride takes over, says Australian National University chemist Alan Sargeson, chair of the naming commission. "They promptly give a name to the element," he says, "and of course, they never agree."

For example, Lawrence Berkeley National Laboratory (LBNL) in California and the Joint Institute for Nuclear Research in Dubna, Russia, each claimed to be the first to have made element 104 in 1969 and element 105 in 1970. For 104, the commission adopted Berkeley's name: rutherfordium (Rf), which honors Oxford physicist Ernest Rutherford, who discovered the atom in 1911. Element 105 will be dubbed dubnium (Db), after the Russian lab.

Citing LBNL's undisputed claim to element 106, the commission accepted the name seaborgium (Sg), for Glenn Seaborg, whose LBNL team forged nine other weighty elements in the 1940s and '50s. A few years ago, the commission had rejected the name because Seaborg is still living. But the chemistry community persuaded the commission to reconsider.

Also undisputed, element 107 will be called bohrium (Bh), after Danish quantum-physics pioneer Niels Bohr. German physicist Otto Hahn, one of the first to study fission, has fallen out of the elemental pantheon, however. In 1994, IUPAC had proposed the name hahnium for element 108, but the commission instead opted for hassium (Hs), after the Latin name for Germany's Hesse province. That's the home of the Heavy-Ion Research Laboratory (GSI) in Darmstadt, which created elements 107 to 112. For 109, the commission has settled on the name meitnerium (Mt)—after Lise Meitner, who helped Hahn discover and explain nuclear fission.

The names must still be approved by delegates from 40 IUPAC countries, who will meet in Geneva in August. So, don't start memorizing the list just yet, says a weary Sargeson: "It may be revisited. It may have to be revised. I hope not."

–Erik Stokstad