Because higher energy electrons penetrate deeper into the electron mirror before they are reflected, the field has more time to bend their paths, introducing a pattern of aberrations that exactly cancels those introduced at the lens.

"This is the answer to a problem that has plagued electron microscopy for the past half-century," says University of Wisconsin physicist Brian Tonner. While it could ultimately benefit the two most common kinds of electron microscopes—scanning and transmission—its most immediate application will be for so-called photoelectron emission microscopes (PEEMs), says O. Hayes Griffith, a physical chemist at the University of Oregon, who collaborated with Rempfer.

A PEEM bombards surfaces with intense visible or ultraviolet light to spur the emission of electrons, then focuses the electrons into an image. The emitted electrons come in a wide range of energies, however, resulting in strong chromatic aberration. One solution has been to restrict the wavelength of light, eliciting electrons at just a single energy—but also producing a dim image. By eliminating the aberrations with the mirror, "you could make a bright, quick picture without losing resolution," says Martin Kordesch, a physicist and electron microscopist at Ohio University. The mirror could also increase the ultimate resolution of these microscopes from 70 angstroms to 10 or fewer, roughly the scale of atoms, says Griffith.

Surface scientists, who study the composition and chemical behavior of surfaces, would be the most immediate beneficiaries. But the corrected optics could also aid biologists. For example, researchers examining receptors on a cell surface now have to label them with fluorescent markers. "That's like seeing the headlights of a car far away," says Griffith. With a corrected PEEM, "you could see membranes right down to the cellular proteins, like looking at a landscape in daylight."

That's still in the future, though. The Oregon group has not yet attached their mirror to a working electron microscope. Meanwhile, Tonner's research group at the Wisconsin Synchrotron Radiation Center is trying to build a prototype corrected PEEM, and in Europe, a consortium of German groups has mounted a multimillion-dollar effort to build a corrected microscope. But in the race for higher resolution, says Tonner, Rempfer and her electron mirror have "made a quantum leap."

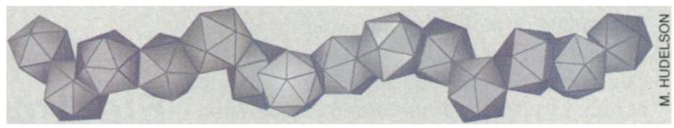
**–Erik Stokstad**

MATHEMATICS

# How to Play Platonic Billiards

SAN DIEGO—Billiards is a game of geometry. Expert players delight in setting up incredible "trick" shots based on careful calculation of angles and distance. Now, Matthew Hudelson has added a new dimension to this pastime, as he reported here last month at the joint meetings of the American Mathematical Society and the Mathematical Association of America. With a little help from a computer, the Washington State University mathematician has demonstrated some amazing three-dimensional shots for a cue ball bouncing around inside three equal-sided, or Platonic, solids—the eight-sided octahedron, 12-sided dodecahedron, and the 20-sided icosahedron. Each of Hudelson's shots hits each side of the pertinent solid and returns to its exact starting point and direction of travel.

That's no trick on a 2D billiard table, provided it has a regular shape. Just start the ball at the midpoint of one side and aim for the midpoint of an adjacent side. But add a dimension and the problem gets far more interesting. When Hudelson took up the challenge, mathematicians had worked out the required trajectories for only the two simplest Platonic solids. Hugo Steinhaus gave the answer for the cube in the 1950s, and John H. Conway and Roger Hayward independently solved the tetrahedron in the early 1960s. Both cases were described in a 1963 *Scientific American* column by Martin Gardner, who says he is "quite impressed" with Hudelson's extension. Conway, now at Princeton University, agrees: "It's a very nice result."



**Plumber's nightmare.** Part of a chain of icosahedra.

In principle, solving the problem is just a matter of algebraic bookkeeping—tracking the equations of the straight lines the ball follows as it bounces around. A standard theoretical approach is to glue together a sequence of "reflected" copies of the shape or solid at the sides in the order in which you expect they will be hit. If there is indeed a trajectory that hits the sides in the prescribed order, then there will be a straight line that stays inside the glued-together construction, because each time the ball hits a wall, the mirror image of the ricochet is a straight line that continues the incoming shot.

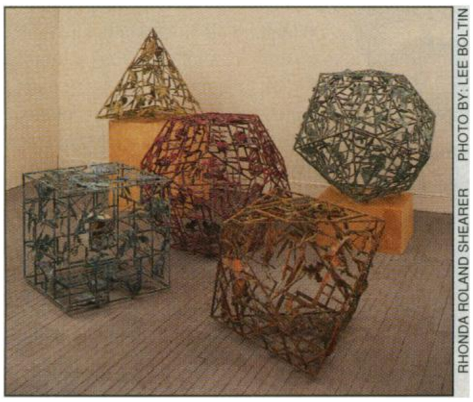
What makes things difficult is the sheer number of possibilities that have to be investigated, particularly for the dodecahedron and icosahedron. Even for the octahedron, there are hundreds of different ways to glue together eight copies of the shape, each corresponding to a different order in which the ball hits the walls. That's a lot of algebra.

Hudelson took up the problem last summer after hearing it mentioned in a geometry seminar at Washington State. He started with the octahedron. First, he fashioned a likely arrangement of stuck-together octahedra made with cardboard and tape. The resulting "plumber's nightmare," as Hudelson calls the tube, told him that there would be trajectories that hit the eight walls in the order he had guessed. It was then a relatively simple exercise in computer algebra to identify one path that returned the ball to its starting point and its original direction of travel.

For the dodecahedron and icosahedron, Hudelson had the computer do all the work. To get started, he wrote a program that generated random initial trajectories and followed them for the first 12 or 20 bounces. Running the program 100,000 times for each solid, he got about 50 trajectories that hit all 12 sides in the dodecahedron and about five that hit all 20 sides in the icosahedron. Each of the successful trajectories hit the walls in the same order, which suggested that there is essentially only one solution to the problem for each solid. He then went on to identify the one trajectory that took the ball back to its starting point and direction.

Hudelson doesn't see immediate applications for these virtuoso shots: "It just seemed like a hole that needed filling." However, he notes that theoretical physicists, who use billiards on odd-shaped tables as a model of the behavior of atoms jumping chaotically between energy states, may turn out to be avid players (*Science*, 20 December 1996, p. 2014). If and when physicists make the leap from two to three dimensions, Hudelson's Platonic shots will be ready to show them a trick or two.

**–Barry Cipra**



**Anyone for a game?** Mathematicians have mastered return shots within five Platonic solids.