the high latitudes and parts of Eurasia (*13*) than the North Atlantic SST patterns (*12*).

The spatial structure of the breakdown of predictability can be assessed through a series of ensemble variance maps. Maps taken every other year for ensemble I's ensemble variance for North Atlantic dynamic topography (Fig. 4) show that ensemble variance is initially large in the region directly south of Greenland. Soon thereafter, large values are observed off the coast of east Greenland, which subsequently spread into the northern and northwestern part of the Atlantic. Analysis of convection reveals that the regions of early variance growth are also regions associated with the model's deep water formation. Convection acts as a downward pathway for atmospheric signals. The atmospheric signals were initialized differently for each member of the ensemble, and therefore it may be speculated that convection can enhance the rate of variance growth.

Our results suggest that the basis for long-term North Atlantic climate predictions rests on three physical properties of the North Atlantic Ocean. First, the ocean integrates the mostly "noisy" atmospheric fluxes, thus producing a red power spectrum for oceanic properties on time scales substantially longer than those of the synoptic atmosphere (*17*). This integrative property provides a long-term memory for the coupled ocean-atmosphere system and can be exploited for damped persistence predictions (*20*). Second, there is the special feature of the North Atlantic variability that involves the very active participation of thermohaline dynamics that can provide a significant oscillatory component to the multidecadal variability. These damped, roughly linear, oscillations in the oceanic circulation increase the amplitude of water mass changes at low frequencies over what can be expected from a purely red noise process. The signal from this oscillation can potentially be exploited for making useful multiyear to multidecadal oceanic predictions. Third, strong variations at high latitudes near Greenland are seen in SST, which can influence atmospheric variability extending in a predominantly downstream direction (eastward) (*13*). These variations can also be associated with extreme events in the North Atlantic variability, which are themselves quite predictable. In general, we conjecture that oceanic predictability of the North Atlantic and high-latitude multidecadal variability is greater (reaching up to 10 to 20 years) when the variability has a larger amplitude, including more extreme events.

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The Origin of Gravitational Lensing: A Postscript to Einstein's 1936 *Science* Paper

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Gravitational lensing, now taken as an important astrophysical consequence of the general theory of relativity, was found even before this theory was formulated but was discarded as a speculative idea without any chance of empirical confirmation. Reconstruction of some of Einstein's research notes dating back to 1912 reveals that he explored the possibility of gravitational lensing 3 years before completing his general theory of relativity. On the basis of preliminary insights into this theory, Einstein had already derived the basic features of the lensing effect. When he finally published the very same results 24 years later, it was only in response to prodding by an amateur scientist.

Sixty years ago, Einstein published a short note in *Science* entitled "Lens-Like Action of a Star by the Deviation of Light in the Gravitational Field" (*1*). The note is often considered as the pioneering study of gravitational lensing, although earlier contributions have been recognized [(*2*), chap. 1]. In 1920, Eddington discussed the possibility of seeing multiple images of a star if a massive object, acting as a gravitational lens, is suitably interposed between the star and an observer (*3*). A few years later, Chwolson pointed out that if a star, lens, and observer are in alignment, the observer will see a ring-shaped image of the star centered on the lens (*4*). Einstein's paper of 1936 deals with both effects in apparent ignorance (*5*) of these publications but is free of some of their shortcomings. However, it only gives the final formulas without any derivations. In view of this historical account and Chwolson's pioneering work, it has been suggested (*6*) that the ring-shaped images be renamed "Chwolson rings" rather than the current "Einstein rings."

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In the course of a research project on the genesis of general relativity at the Max Planck Institute for the History of Science, we have identified and reconstructed calculations by Einstein on gravitational lensing closely related to his 1936 paper in notes dated to the spring of 1912. These notes show that Einstein had developed the basic theory of gravitational lensing even before he completed the general theory of relativity in 1915.

Exploring consequences of a heuristic assumption about static gravitational fields, Einstein in 1911 published a paper on the deflection of light by the gravitational field of the sun (*7*). The prediction of light bending was confirmed in 1919 by the famous solar eclipse expedition led by Eddington.

But in 1912, Einstein was still trying to persuade astronomers to test various astrophysical consequences of his new ideas on gravitation. Erwin Freundlich was one of the few astronomers who actively engaged in such work. In spring 1912, Einstein, then a professor in Prague, visited Berlin where he met Freundlich, who was working at the Königliche Sternwarte, the Royal Observatory. It is quite possible that the issue was discussed at a meeting with Freundlich. At any rate, Einstein did the gravitational lensing calculations during his Berlin visit as evidenced by notes found in a small notebook dated to the period 1910 to 1914 (8) (Fig. 1).

These calculations appear interspersed between various notes referring to Berlin appointments and addresses during his visit of 15 to 22 April 1912 (9). Notes on the gravitational lensing effect are contained on eight pages of the notebook and primarily deal with (i) the possibility of a double image of the source as a result of gravitational light bending and (ii) the magnification of the intensity of these images. Einstein started by sketching the geometrical constellation of gravitational lensing with a light-emitting source and a lensing star separated by a distance R (Fig. 1). An observer is located at distance R' from the lens along the axis formed by the light source and lens and at a small distance r off of this axis. Einstein then wrote down the basic lensing equation

$$r = \rho \frac{R + R'}{R} - \frac{R'\alpha}{\rho} \qquad (1)$$

which expresses the condition that a light ray passing the lens at a distance ρ will reach the observer. According to Einstein's 1911 paper (7), the light ray is bent by an angle α/ρ, where α depends on the mass of the lens and differs by a factor of 2 from the value following from general relativity. Introducing dimensionless units, Einstein arrived at the quadratic equation for ρ labeled (1) in his notes (Fig. 1). The two solutions to this quadratic equation correspond to the fact that a light ray can reach the observer after passing either side of the lens.

Considering the apparent brightness of the two images, Einstein also correctly calculated the magnification factor for the intensity of the deflected light [labeled (3) in Fig. 1]. On the following pages in the notebook, Einstein further discussed this magnification factor, transforming the bracketed expression of the equation labeled (3) in Fig. 1 into the form

$$\sqrt{1 + \frac{1}{r_0^2\left(1 + \frac{1}{4} r_0^2\right)}} \qquad (2)$$

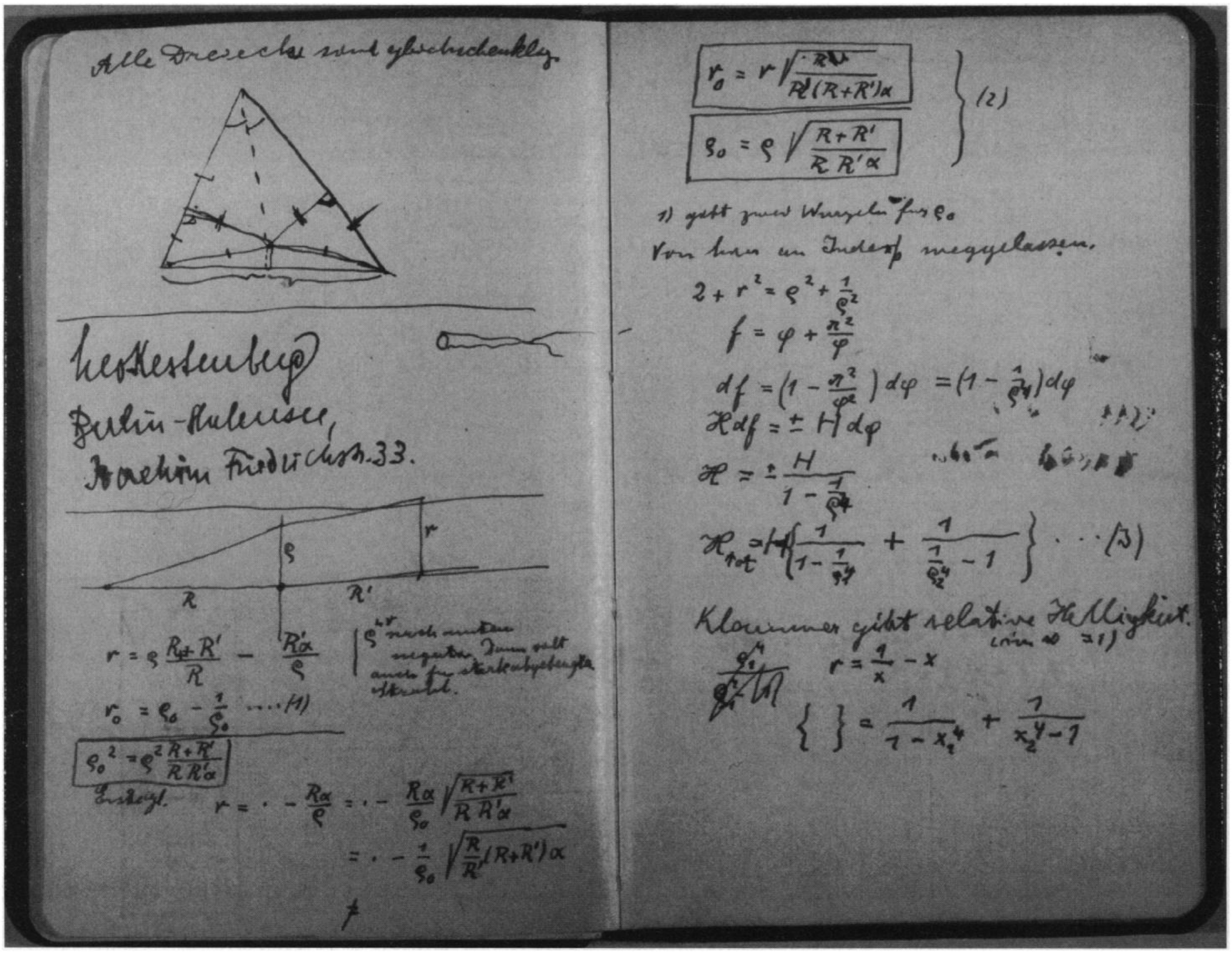

Fig. 1. Notes about gravitational lensing dated to 1912 on two pages of Einstein's scratch notebook (*12*). [Reproduced with permission of the Einstein Archives, Jewish National and University Library, Hebrew University of Jerusalem]

with r_0 given as

$$r_0 = r\sqrt{\frac{R}{R'(R + R')\alpha}} \tag{3}$$

[labeled (2) in Fig. 1]. He also estimated the orders of magnitude of the predicted effects.

Despite this work in 1912, Einstein did not publish his results until 1936. He must have already concluded, as he put it in his 1936 paper, that "there is no great chance of observing this phenomenon" [(1), p. 507] and probably for this reason did not pursue his idea any further. Even when he took up the subject again in 1936, it was only because of the initiative of an outsider to the field, a Czech electrical engineer and amateur scientist, Rudi W. Mandl. Mandl had approached Einstein and asked him to consider the possibility of a lensing effect caused by gravitational light bending. In his endeavors he was supported by "a small sum of money," granted to Mandl by the Science Service, to enable him to visit Einstein in Princeton and discuss his ideas (10). Einstein indeed (re)did the calculations (whether he recalled that he had done the same calculations in 1912 is unknown to us) and conceded that the results might warrant a brief publication. Figure 2 shows the relevant calculations for his 1936 paper in a draft headed "Letter to Mister Mandl," dated 2 June 1936.

Closer inspection shows that this draft contains the two relevant formulas published in his 1936 note: the angle of sight for an Einstein ring and the magnification factor. What is more surprising is that the lensing equation appearing as the first equation in Fig. 2

$$\frac{\varepsilon_0\Delta_0}{\Delta} = \frac{\Delta}{b} + \frac{\Delta - x}{a} \tag{4}$$

can be directly translated into the lensing equation that appears as the first equation in Fig. 1 by making the following identifications: $R \equiv b$, $R' \equiv a$, $\rho \equiv \Delta$, $r \equiv x$, $\alpha \equiv \varepsilon_0\Delta_0$.

All consequences follow from the general lensing equation, and indeed, the expressions for the magnification factor obtained in 1912 and 1936 are equivalent. The magnification obtained in 1936

$$\frac{\sqrt{A}}{\xi}\,\frac{1 + \frac{\xi^2}{2A}}{\sqrt{1 + \frac{\xi^2}{4A}}} \tag{5}$$

with $\xi \equiv xb/(a + b)$ and $A \equiv \varepsilon_0\Delta_0 ab/(a + b)$, as it appears in the calculation in Fig. 2, can in fact easily be transformed into Eq. 2 by noting that $r_0 \equiv \xi/\sqrt{A}$.

As a result of the persistent urging of Mandl, Einstein published these results in 1936. As Einstein acknowledged, "Some time ago, R. W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request. This note complies with his wish" [(1), p. 506]. In a letter to the editor of *Science*, James Cattell, Einstein expressed himself more frankly: "Let me also thank you for your cooperation with the little publication, which Mister Mandl squeezed out of me. It is of little value, but it makes the poor guy happy" (11). In spite of Einstein's pessimism, gravitational lensing was observationally confirmed in 1979 and has since become an important area of astrophysical research (2).

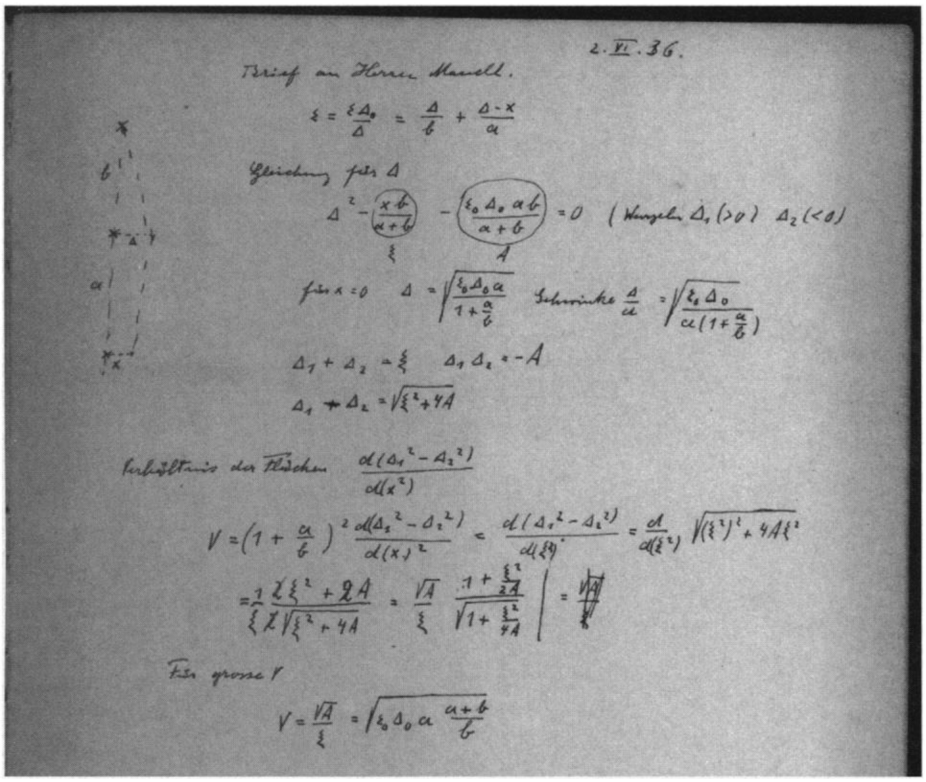

Fig. 2. Calculations on gravitational lensing made by Einstein in 1936 as background for a letter to amateur scientist R. W. Mandl. Einstein here derives the equations published in his note of the same year. The calculations are essentially equivalent to the ones of 1912 (Fig. 1). [Reproduced with permission of the Einstein Archives, Jewish National and University Library, Hebrew University of Jerusalem (call no. 3-011-55)]

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9. The most direct evidence is a list of appointments for his Berlin visit found on p. 36. One of the less direct hints is found in Fig. 1. Leo Kestenberg, probably an acquaintance of Elsa Löwenthal (later Einstein's second wife), was working as a piano teacher at the Klindworth-Scharwenka-Conservatory in Berlin, where Elsa Löwenthal gave a performance in February 1913.
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12. Figure 1 shows two facing pages of the notebook published in the facsimile reproduction of (8) as pages 43 and 46. Between these pages, a loose sheet containing further notes on the lensing effect was found, which was reproduced in (8) as pages 44 and 45.

13. We thank G. Castagnetti, J. Eisenstaedt, H. Goenner, and M. Janssen for helpful comments and Z. Rosenkranz and the Einstein Archives, Hebrew University of Jerusalem, for permission to reproduce the figures.

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