excess emission in clusters, then the intracluster medium must be multiphase on a large scale, with a component of long-lived cold clouds.

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Generation of X-rays from Comet C/Hyakutake 1996 B2

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The instability resulting from the relative motion of newly picked-up cometary photoions and the solar wind generates lower hybrid waves that are capable of accelerating electrons to the keV range of energies. These electrons may be responsible for the x-rays from comet C/Hyakutake 1996 B2 seen by the Röntgen X-ray Satellite. The inferred x-ray photon power depends on the electron energy, with keV electrons providing values of photon power two orders of magnitude greater than 100 eV electrons. These observations and in particular, spectral resolution of the x-rays, should provide more insight into the composition of the comet.

Observations by the Röntgen X-ray Satellite (ROSAT) on 26 to 28 March 1996 of x-ray emission from comet C/Hyakutake 1996 B2 (*1*) appear to have been a surprise to most comet researchers. However, plasma data taken by the Vega spacecraft (*2*, *3*) from the encounter with comet P/Halley suggests that this observation can be explained by wave-particle interactions producing keV electrons, which are necessary for x-ray emission. Observations of strong plasma wave turbulence (*2*) together with energetic electrons (*3*) in the keV energy range observed in the sunward mantle region separating the solar wind and the interior cometary ionosphere of Halley corresponds to the same region where the x-rays were observed from Hyakutake. Halley's plasma wave turbulence is in the range 4 to 40 Hz corresponding to frequencies between the ion and electron gyro-frequencies and is close to the lower hybrid frequency which is the geometric mean of the electron and ion gyro-frequencies of the order of tens of hertz. Lower hybrid waves (*4*) are effective in energizing electrons, therefore it is not surprising that an energetic electron population up to several keV was also observed at Halley (*3*) in the same region as the lower hybrid wave activity (*2*). Hyakutake's gas production rate (*5*) (2×10^{29} molecules per second at 1 AU) is similar to Halley's (7×10^{29} molecules per second at 1 AU) (*6*) so it is possible that the electron population at Hyakutake might have the appropriate energies to produce the observed x-ray emission. A previous attempt to take an x-ray image of comet Bradfield 1979X (*7*) yielded an upper limit $\sim 10^{14}$ erg s^{-1} for the total x-ray power emitted at the comet. It was assumed that x-rays would result from the precipitation of auroral type keV electrons following the cometary analogy of a terrestrial substorm. This process implies that the emission, should it occur, is sporadic.

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Here we will demonstrate that the interaction of the cometary plasma and the solar wind produces waves in the lower hybrid frequency range that are responsible for the production of suprathermal electrons with energies in the range of 100 eV up to several keV, which are necessary for bremsstrahlung and cometary gas K-shell radiation of x-rays. Close to the sun ($\sim$1 AU) every cometary nucleus is surrounded by an expanding gas cloud that consists mostly of water with some carbon dioxide and other molecules that is ionized largely by photoionization. These ions, created in the solar wind, immediately see the $\mathbf{v}_{SW} \times \mathbf{B}_{SW}$ electric field which accelerates them to high energies. ($\mathbf{v}_{SW}$ and $\mathbf{B}_{SW}$ are the solar wind velocity and magnetic field vectors, respectively). These ions, called pick-up ions, form in the solar wind frame an ion beam gyrating in the solar wind magnetic field. The energy source for this process is the relative motion between the solar wind and newly created cometary ions that results in the modified two stream instability (*8*) of an ion beam. The excitation of waves in the lower hybrid frequency range by this instability and subsequent absorption of the wave energy by electrons is the main mechanism for converting energy from the solar wind flow into plasma electrons, and then accelerating them parallel to the magnetic field forming a suprathermal electron component.

We obtained simple approximate formula for the characteristic energy and density of the suprathermal electrons using the conservation equations between the flux of energy from the ions into wave turbulence and absorption of the turbulent wave energy by the electrons. The cometary ion flux F_i at a distance r from the surface of the comet can be estimated by equating the photoionized part of the cometary gas outflow to the flow of the cometary ions picked up by the solar wind such that

$$F_i = 4\pi r^2 n_{ci} u = Q_s - Q_s \exp(-r/v_g\tau) = \frac{Q_s r}{v_g \tau} \qquad (1)$$

where Q_s is the initial flux of gas molecules at the comet surface (*5*) (2×10^{29} molecules per second at 1 AU), τ^{-1} is the rate of photoionization ($\tau = 10^6$ s), v_g is the initial gas velocity at 1 AU ($v_g = 10^5$ cm s^{-1}), and n_{ci} is the cometary ion density (*9*). From Eq. 1 we can estimate the density of the cometary ions, n_{ci}, to be ≈ 10 cm^{-3} for $u = 3 \times 10^6$ cm s^{-1} corresponding to the downstream shocked solar wind velocity (*10*) at the distance $r = 50{,}000$ km which is close to the position where x-ray emission is being generated. These cometary photoions picked up by the solar wind excite the lower hybrid waves which in turn are absorbed by the suprathermal electrons through Cherenkov resonance with the waves.

In steady state, the energy flux lost by the pick-up ions is equal to the energy flux carried away by the suprathermal electrons. This leads to the energy flux balance equation

$$\alpha n_{ci} m_{ci} u^3 \simeq n_{Te} \varepsilon_e \left(\frac{\varepsilon_e}{m_e}\right)^{1/2} \qquad (2)$$

where α is the transfer efficiency from the cometary ions of mass m_{ci} and n_{Te} is the density of suprathermal electrons with energy ε_e and mass m_e. By balancing the growth rate γ_i of the lower hybrid waves due to the cometary pick-up ions with the Landau damping rate γ_e (*11*) by energetic electrons we obtain

$$\gamma_i + \gamma_e \sim \frac{\partial f_{ci}}{\partial v_\parallel} + \frac{m_{ci}}{m_e}\left(\frac{\partial f_e}{\partial v_\parallel}\right) = 0 \qquad (3)$$

where f_{ci} is the pick-up ion distribution function and f_e is the electron distribution function. From Eq. 3 we get

$$\frac{n_{Te}}{\varepsilon_e} \simeq \frac{n_{ci}}{m_{ci}u^2} \qquad (4)$$

From Eqs. 2 and 4 we find that the average energy of the suprathermal electrons is given by

$$\varepsilon_e \simeq \alpha^{2/5}\left(\frac{m_e}{m_{ci}}\right)^{1/5} m_{ci}u^2$$

For $\alpha = 10^{-1}$, and water ions, $m_{ci}u^2 \simeq 1$ keV then the average energy of the suprathermal electrons is $\varepsilon_e \sim 100$ eV and their density is given by

$$n_{Te} \simeq n_{ci}\left[\alpha^2\left(\frac{m_e}{m_{ci}}\right)\right]^{1/5} \sim 1 \text{ cm}^{-3}$$

This is a powerful source of suprathermal electrons with an energy flux of 10^{19} erg s^{-1} through the region of x-ray emission. The electron energy flux is about two orders of magnitude less than the total power available from the solar wind indicating a reasonably high efficiency of energy transformation. This region also corresponds closely to the region where intense lower hybrid waves were observed by the Vega satellite during its encounter with Halley (*2*).

The presence of intense lower hybrid waves can result in field aligned electrons accelerated up to energies in the keV range exceeding the average ion energy. The electron energy spectrum can be obtained by solving the quasi-linear velocity diffusion equation

$$v_\parallel\left(\frac{\partial f_e}{\partial x_\parallel}\right) = \left(\frac{e^2}{m_e^2}\right)\left(\frac{\partial}{\partial v_\parallel}\right)\left[\frac{\langle |E|^2\rangle}{\Delta k_\parallel v_\parallel}\cdot\left(\frac{\partial f_e}{\partial v_\parallel}\right)\right] \qquad (5)$$

where $v_\parallel$ is the electron velocity parallel to the solar wind magnetic field, $\Delta k_\parallel$ is the width of the wave spectrum in parallel wavenumbers $\Delta k_\parallel v_\parallel \simeq \omega$, $(\Delta k_\parallel \simeq k_\parallel)$ and $\langle E^2\rangle$ is the average of the square of the lower hybrid wave electric field. In writing Eq. 5 we used the standard expression for the parallel velocity diffusion coefficient [see for example (*12*)] and the dispersion equation $\omega = (\omega_{ce}/k_\parallel)k$, for lower hybrid waves. Note that for lower hybrid waves $k = (k_\perp^2 + k_\parallel^2)^{1/2} \simeq k_\perp$. From Eq. 5 we obtain the following for the maximum electron energy ε_{emax} given by

$$\varepsilon_{emax} \simeq \left[\left(\frac{e^2}{2\pi m_e^{1/2}}\right)\frac{l\langle E_f^2\rangle}{\omega}\right]^{2/3} \qquad (6)$$

where $l_\parallel$ is the interaction length $\sim 10^9$ cm. Using for the wave energy spectral density values obtained from observations at Halley by the Vega spacecraft $[\langle E_f^2\rangle \sim 1\ (\text{mV})^2/(\text{m}^2\ \text{Hz})]$ it is possible to estimate from (6) the maximum energy of accelerated electrons as

$$\varepsilon_{emax} \sim 5 \text{ keV} \qquad (7)$$

We now estimate the luminosity of the x-rays assuming that they are a combination of bremsstrahlung emission and inner shell radiation from collisionally excited oxygen atoms. For bremsstrahlung emission the power radiated by one electron in 1 cm^{-3} is given by

$$g = \int \hbar\omega n_0 v_e d\sigma(\omega) \qquad (8)$$

where $\hbar$ is Planck's constant divided by 2π, ω is the radiation frequency, $n_o = Q_s/2\pi r^2 v_g$ is the number density of water molecules at distance r, and $d\sigma(\omega)$ is the differential cross-section for bremsstrahlung given by

$$d\sigma(\omega) = \left(\frac{16}{3}\right)\left(\frac{Z^2e^2c}{\hbar v^2}\right) r_o^2 ln\left(\frac{b_{max}}{b_{min}}\right)\left(\frac{d\omega}{\omega}\right) \qquad (9)$$

$r_o = e^2/m_e c^2$ is the classical electron radius, Ze is the nuclear charge of the water molecule, and $b_{max}/b_{min} = mv^2/\hbar\omega$, note that $ln(mv^2/\hbar\omega)$ is of order 1.

The power radiated in 1 cm^3 is $\int_{v_{min}}^{v_{max}} gf_e dv_e$. If the distribution function is constant in velocity up to a maximum value v_{max} then $f_e = n_{Te}/v_{max}$ and the total luminosity (in ergs^{-1}) from a shell of thickness Δr surrounding the comet nucleus is given by

$$L = 2\pi \int gf_e dv_e r^2 \Delta r \approx$$

$$3 \times 10^{-25}\frac{Q_s}{v_g}\Delta r n_{Te} Z^2 \sqrt{\varepsilon_{max}(\text{eV})} \qquad (10)$$

For the parameters encountered at the comet we estimate the total luminosity of x-rays due to bremsstrahlung to be of the order of 2×10^{13} erg s^{-1} corresponding to an x-ray bremsstrahlung efficiency of $\sim 2 \times 10^{-6}$ in the keV photon energy range. In addition to bremsstrahlung radiation there is also radiation from partially ionized oxygen and other heavy cometary ions whose bound electrons are excited by collisions with the suprathermal electrons. The bound electrons quickly de-excite and radiate any energy they receive from this source.

Line radiation produced by excitation of bound electrons in inelastic collisions with keV plasma electrons is found to be more effective than bremsstrahlung. Detailed calculations of the intensity of this radiation, known in fusion research as impurity radiation (*13*), have shown that this radiation is a powerful source of energy loss in fusion reactors. The basic physics of this radiation can be explained using the simple picture proposed in (*14*) where it is assumed that the cross section of the process is determined by inelastic Coulomb electron-electron collisions with a small impact parameter thereby leading to relatively large angle scattering of plasma electrons. The Coulomb logarithm, which usually increases the cross section for electron-electron collisions in a plasma by a factor of 10 to 20 due to the input from distant collisions, for line radiation is a strong function of the Z factor and is of the order ≤ 1. The total luminosity due to recombination or line radiation from the cometary shell, described earlier, is given by (*13*)

$$L_I \approx 5 \times 10^{-18}\Lambda\, n_{Te}\frac{Q_s\Delta r}{v_g}\left[\frac{1}{\sqrt{\varepsilon_e(\text{eV})}}\right] \qquad (11)$$

The numerical factor Λ for oxygen atoms has been estimated in (*14*) as 0.1 which is close to the value obtained by detailed calculations (*13*).

From Eq. 11 we estimate the luminosity of line radiation for the parameters we have used above as $L_I \simeq 3 \times 10^{14}$ erg s^{-1} for photon energies in the keV energy range; this corresponds to an x-ray efficiency of 3×10^{-5}, which is comparable with the values obtained in terrestrial aurora (*15*). The keV photons predicted by our theory corresponds to the energy interval where the ROSAT high resolution imager (HRI) is also most sensitive. For an observed count rate, a keV photon spectrum would require an input sensitivity about 100 times or more less than would be needed for a 100 eV input photon spectrum. The variability of the x-ray flux can also be explained by the comet experiencing varying solar wind fluxes, resulting in a change to the suprathermal electron spectra and hence x-rays. If this hypothesis is correct, observation of the x-rays from comets could give remote information on solar wind conditions.

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ties perpendicular to the magnetic field. The frequency of the lower hybrid waves is between the gyrofrequencies of the electrons (ω_{ce}) and the ions (ω_{ci}) which means that these waves can be in simultaneous Cherenkov resonance with the relatively slow but unmagnetized ions perpendicular to the magnetic field and fast magnetized (hence magnetic field aligned) electrons. Cherenkov resonance occurs when the phase velocity of the wave and the particle velocity are equal; under these conditions strong interaction between the waves and particles is possible and results in energy transfer from the wave to the particle or vice versa. The lower hybrid waves provide the intermediary step in transferring energy between the ions and electrons.

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An Economics Approach to Hard Computational Problems

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A general method for combining existing algorithms into new programs that are unequivocally preferable to any of the component algorithms is presented. This method, based on notions of risk in economics, offers a computational portfolio design procedure that can be used for a wide range of problems. Tested by solving a canonical NP-complete problem, the method can be used for problems ranging from the combinatorics of DNA sequencing to the completion of tasks in environments with resource contention, such as the World Wide Web.

Extremely hard computational problems are pervasive in fields ranging from molecular biology to physics and operations research. Examples include determining the most probable arrangement of cloned fragments of a DNA sequence (*1*), the global minima of complicated energy functions in physical and chemical systems (*2*), and the shortest path visiting a given set of cities (*3*), to name a few. Because of the combinatorics involved, their solution times grow exponentially with the size of the problem (a basic trait of the so-called NP-complete problems), making it impossible to solve very large instances in reasonable times (*4*).

In response to this difficulty, a number of efficient heuristic algorithms have been developed. These algorithms, although not always guaranteed to produce a good solution or to finish in a reasonable time, often provide satisfactory answers fairly quickly. In practice, their performance varies greatly from one problem instance to another. In many cases, the heuristics involve randomized algorithms (*5*), giving rise to performance variability even across repeated trials on a single problem instance.

In addition to combinatorial search problems, there are many other computational situations where performance varies from one trial to another. For example, programs operating in large distributed systems or interacting with the physical world can have unpredictable performance because of changes in their environment. A familiar example is the action of retrieving a particular page on the World Wide Web. In this case, the usual network congestion leads to a variability in the time required to retrieve the page, raising the dilemma of whether to restart the process or wait.

In all of these cases, the unpredictable variation in performance can be characterized by a distribution describing the probability of obtaining each possible performance value. The mean or expected values of these distributions are usually used as an overall measure of quality (*6–9*). We point out, however, that expected performance is not the only relevant measure of the quality of an algorithm. The variance of a performance distribution also affects the quality of an algorithm because it determines how likely it is that a particular run's performance will deviate from the expected one. This variance implies that there is an inherent risk associated with the use of such an algorithm, a risk that, in analogy with the economic literature, we will identify with the standard deviation of its performance distribution (*10*).

Risk is an important additional characteristic of algorithms because one may be willing to settle for a lower average performance in exchange for increased certainty in obtaining a reasonable answer. This situation is often encountered in economics when trying to maximize a utility that has an associated risk. It is usually dealt with by constructing mixed strategies that have desired risk and performance (*11*). In analogy with this approach, we here present a widely applicable method for constructing "portfolios" that combine different programs in such a way that a whole range of performance and risk characteristics become available. Significantly, some of these portfolios are unequivocally preferable to any of the individual component algorithms running alone. We verify these results experimentally on graph-coloring, a canonical NP-complete problem, and by constructing a restart strategy for access to pages on the Web.

To illustrate this method, consider a simple portfolio of two Las Vegas algorithms, which, by definition, always produce a correct solution to a problem but with a distribution of solution times (*5*). Let t_1 and t_2 denote the random variables, which have distributions of solution times $p_1(t)$ and $p_2(t)$. For simplicity, we focus on the case of discrete distributions, although our method applies to continuous distributions as well. The portfolio is constructed simply by letting both algorithms run concurrently but independently on a serial computer. Let f_1 denote the fraction of clock cycles allocated to algorithm 1 and $f_2 = 1 - f_1$ be the fraction allocated to the other. As soon as one of the algorithms finds a solution, the run terminates. Thus, the solution time t is a random variable related to those of the individual algorithms by

$$t = \min\left(\frac{t_1}{f_1}, \frac{t_2}{f_2}\right) \qquad (1)$$

The resulting portfolio algorithm is characterized by the probability distribution $p(t)$ that it finishes in a particular time t. This probability is given by the probability that both constituent algorithms finish in time $\geq t$ minus the probability that both algorithms finish in time $> t$

$$p(t) = \left[\sum_{t' \geq f_1 t} p_1(t')\right]\left[\sum_{t' \geq f_2 t} p_2(t')\right] - \left[\sum_{t' > f_1 t} p_1(t')\right]\left[\sum_{t' > f_2 t} p_2(t')\right] \qquad (2)$$

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