mula for quantum chaos, with the zeros of the zeta function playing the role of energy levels, and prime numbers (or, more precisely, their logarithms) playing the role of the lengths of periodic orbits.

The first inklings of this connection came in the early 1970s. Hugh Montgomery, a number theorist at the University of Michigan, had discovered a formula that describes the statistics of the spacing between consecutive zeros of the zeta function. During a visit to the Institute for Advanced Study in Princeton, New Jersey, he was introduced to the physicist Freeman Dyson. "Dyson asked me what I was working on, and I told him this result and mentioned this function, and he recognized it as the function that arises in quantum mechanics," Montgomery recalls. To Dyson, the zeros of the zeta function seemed to be behaving precisely like the solutions to the complex mathematical models in statistical mechanics that physicists were then using to calculate energy levels in large nuclei. "It just so happened that he [Dyson] was one of the two or three physicists in the entire world who had worked all of these things out!" Montgomery says.

The next inkling came in the 1980s, when researchers began to grapple with quantum chaos. To theoretical physicists including Oriol Bohigas at the University of Paris, Montgomery's discovery suggested that the zeta function could be used as a stand-in for more complicated models in quantum chaos—and a means of generating truckloads of ersatz energy levels, which could be used to test predictions of the trace formula. What's nice about the zeta function is that its zeros are comparatively easy to compute. With the advent of computers, number theorists have generated zeros by the gigabyte.

Spectral zeros. Over the last decade, Andrew Odlyzko, a mathematician at AT&T Bell Laboratories (now AT&T Labs—Research), has computed hundreds of millions of zeros of the zeta function, often skipping to extremely high levels before beginning the tabulation. (In one series of computations, for example, he identified the 10^{20}th zero of the zeta function, and then calculated the next 500 million zeros.) Odlyzko's data on the spacing between zeros of the zeta function agree almost perfectly with what physicists expect of energy levels in quantum chaos. "It's the first phenomenological insight that the zeros are absolutely, undoubtedly 'spectral' in nature," as the trace formula hypothesis predicts, says Peter Sarnak, a mathematician at Princeton University. The computations are giving physicists access to statistical features they couldn't hope to see in other mathematical models, much less in laboratory experiments.

And if the physicists are right about the zeta function's connection to quantum mechanics, number theorists will be delighted, because that would imply that a conjecture about the location of the function's zeros, known as the Riemann Hypothesis, is also true. The zeros of the zeta function are complex numbers, which means they can be represented as points in the "complex plane"—each has a "real" x coordinate and an "imaginary" y coordinate. They are known to lie within a certain "critical strip," and in his original paper, Riemann made the additional remark that within that strip, the zeros all seem to lie on a straight line. This property, if true, has profound number-theoretic consequences. There is a logjam of results in the mathematics literature that go "If the Riemann Hypothesis is true, then. . . ." Number theorists have been looking for the last hundred years for a bit of dynamite.

The connection with quantum mechanics could provide it. That's because energy levels are necessarily positive numbers, and therefore always lie along a straight line in the complex plane. If researchers could show that the zeta function really does simulate a quantum system, they would be well on their way to proving the Riemann Hypothesis.

No one expects the zeros of the zeta function to pop up in the spectrum of an actual atom, as they are an idealized representation of quantum chaos, but theorists hope they may spot them in an abstract mathematical model of a quantum system. "I'm encouraged that what we're looking for exists," Sarnak concludes. "It doesn't mean we can find it, but what we're looking for is definitely there."

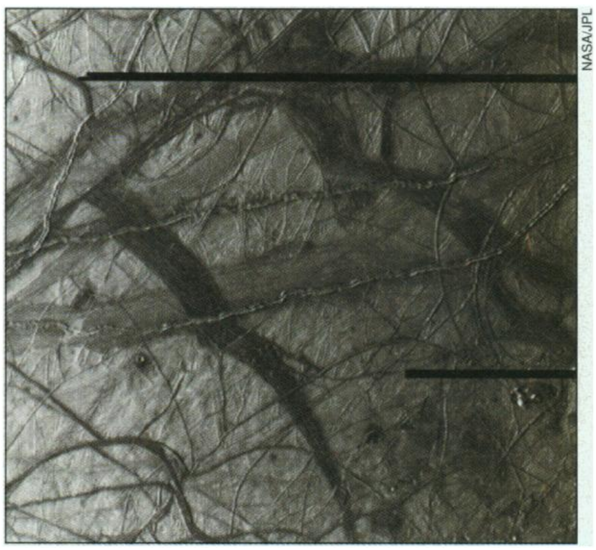
–**Barry Cipra**

PLANETARY SCIENCE

Does Europa's Ice Hide an Ocean?

An image sent down late last week from the Galileo spacecraft orbiting Jupiter offers the sharpest view yet of the Jovian moon Europa, adding new hints that there may be a water ocean sealed beneath the moon's icy surface—which raises the possibility, however faint, of life on this distant world.

To a geologist's eye, the intricate tangle of crisscrossing ridges seen in the image suggests icy volcanic eruptions. For example, two narrow, squiggly ridges running horizontally through the West Virginia–sized tract of crust look like long fractures where eruptions have spewed icy debris to either side, says Arizona State University planetary geologist Kelly Bender of the Galileo imaging team. Two dark bands angling across the image are reminiscent of Earth's volcanic midocean ridges, with a narrow central ridge bounded on both sides by sets of parallel ridges. That suggests that dirty, heat-softened ice rose to the surface at the central ridge and spread away as new icy crust, says Bender.



Europa's furrowed face. Long fractures suggest ice volcanism—and therefore heat or perhaps even liquid water—on this moon.

Where there's volcanism, there's heat. And if there's enough heat, perhaps generated by Jupiter's gentle gravitational massaging of the moon, then there's a good chance for an ocean of liquid water, and therefore life, below the ice-covered surface. And all this ice volcanism must be relatively young, given the dearth of meteorite impact craters; only one 3-kilometer crater (left center) is obvious in this image.

In the coming months, Galileo should return stunning images with 20 times better resolution, more than enough to brighten any geologist's holidays.

–**Richard A. Kerr**

More Galileo images are available at: http://www.jpl.nasa.gov/releases/ganyhq.html