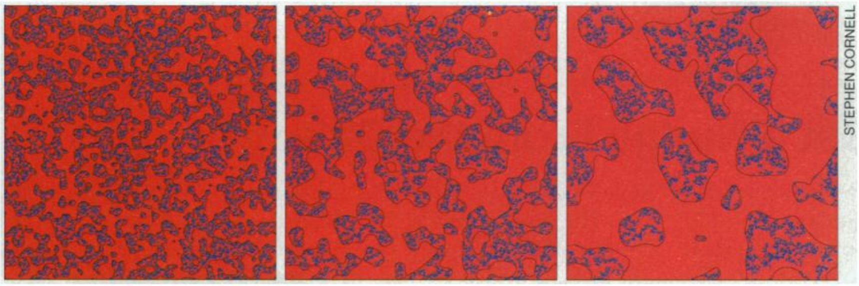
STATISTICAL MECHANICS

Persistence Pays Off in Defining History of Diffusion

Everyone knows that the smell of a cooked breakfast is sure to reach someone sleeping in a distant room of the house. That enticing information is carried mostly courtesy of air currents, but even if the air were dead still, the news would still arrive—in days or weeks rather than minutes. Billions of collisions with air molecules would ultimately convey the aroma of the long-cold breakfast to the far corners of the house. This process of diffusion is one of the basic modes of transport by which nature shifts material and heat from one place to another, and physicists thought it held few surprises.


STEPHEN CORNELL

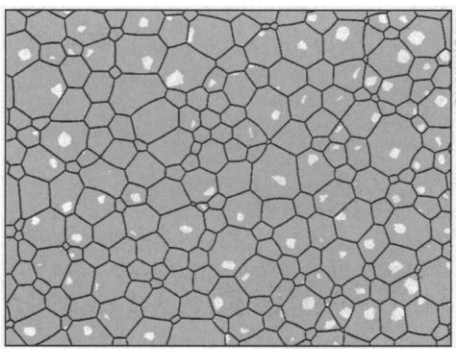
Red tide. Areas of high and low concentration *(red and blue)* coarsen in this diffusion simulation. A new analysis describes how the blue areas shrink over time.

But two separate groups of researchers, one French and the other a U.S.-Anglo-French collaboration, have found that there is more than meets the eye in the decades-old mathematical description of diffusion. In a series of recent papers, they have shown that the diffusion equation can do more than simply provide concentrations at specific points and times. They have identified a new property of the diffusion equation—a new exponent containing information about how the process progresses. "We are asking a question which depends on the whole history of the process, this is the key point," says one of the researchers, Alan Bray of Britain's University of Manchester.

This new work allows researchers to calculate just how much of a diffusing system is left unchanged at each moment—how much of the house the aroma of coffee and bacon still has to reach, for instance. And because the diffusion equation describes phenomena as diverse as the growth of bacterial colonies, the rate of chemical reactions, and the behavior of the stock market, the new analysis could have a broad impact. "These authors have opened a new field of research," says computational physicist Dietrich Stauffer of Cologne University in Germany. As Oxford University theorist John Cardy puts it, "One thought one knew everything that was to be known about simple equations like the diffusion equation, so mathematically it is a surprise."

The classical diffusion equation describes how the rate of change of the concentration of diffusing particles depends on differences in concentration. Using it, physicists can calculate how long it takes for a gas to diffuse across a room and reach a certain concentration on the far side, and chemists can figure out how long it takes for chemical species to migrate across a container.

But these types of calculations yield only final average values, and ignore all the fluctuations along the way. Two research teams—one consisting of Satya Majumdar of Yale University, Clément Sire of the Paul Sabatier University in Toulouse, France, and Bray and Stephen Cornell of Manchester University in the United Kingdom, and the other of Bernard Derrida, Vincent Hakim, and Reuven Zeitak of the Ecole Normale Supérieure in Paris—have independently found a way of bringing the fluctuations into the picture. They focused on a phenomenon they dub persistence, a measure of the fluctuations that, though random, always remain either above or below some overall average.


TAM WING YIM

Foam movie. Coalescing bubbles in foam simulations show "persistence."

A graphic way of visualizing persistence at work is to imagine a container full of a gas whose concentration varies from place to place, in which regions above average concentration are colored red and those below are average blue. The initially speckled red and blue pattern would gradually evolve, with the boundaries between regions constantly shifting; as smaller regions coalesce, larger and larger regions of both colors would form. To see persistence, introduce a new rule: Once a patch has become red, its color stays red, even though the patch's actual concentration may continue to fluctuate above and below the average. A blue colored patch, on the other hand, can turn into a red patch. Over time, red patches will gradually swamp the whole container and blue would slowly diminish, leaving only tiny isolated scraps. "The regions where the concentration has always been below the mean will decrease in time," says Bray.

The researchers propose that the area of this determinedly below-average region will diminish over time in proportion to the time raised to the power of a negative exponent they term theta. "This is the exponent we are calculating," says Bray. This new power law does not alter the diffusion equation; it simply gives extra information—it allows researchers to glean some information about the system as the diffusion progresses.

The two theoretical teams arrived at their persistence exponents via rather different routes. Majumdar and his colleagues tackled classical diffusion scenarios, such as two diffusing chemicals reacting to give a third inert and immobile product, whereas Derrida's group explored a model of magnetism based on the diffusion equation. Both groups, however, came up with the same values of theta, which depends on the dimensions of the problem. As they report in the 30 September *Physical Review Letters*, theta has the value 0.2358 for a three-dimensional situation, such as a gas in a box, and 0.1862 in a two-dimensional plane, for example a liquid film.

Even before the two groups achieved this result, a team led by Daniel Beysens of the Center of Nuclear Studies of France's Atomic Energy Commission in Grenoble had reported experimental evidence that supports the persistence exponents. "My experiments are concerned with the behavior of droplets of dew condensing on a surface," explains Beysens. As the water drops grow, they touch each other and fuse to make a bigger drop centered at the center of mass of the two previous drops, a coarsening effect similar to that exhibited by diffusing systems. Beysens' team studied how the area of surface left unwetted changes over time. Their data yielded a power law, with an exponent similar to that calculated by Majumdar and Derrida. One of Derrida's colleagues, Reuven Zeitak, sees industrial applications in Beysens' result. "The problem is, when spraying a surface with a disinfectant, how long until each spot of the surface has been touched by the disinfectant at least once?" he says.

Now, the mathematical results are stimulating new efforts to measure the persistence exponents in other systems. Following a talk by Majumdar at Lucent Technology's Bell Labs in New Jersey, two experimentalists, Bernie Yurke and Andrew Pargellis, reanalyzed videotaped results of a past experiment in which liquid crystal molecules held between two plates could twist either clockwise or anticlockwise, resulting in light and dark patches when viewed in polarized light. Light and dark regions coalesced and grew in time, but some fraction of the original area between the planes never flipped, yielding a value of theta. "It turns out to be 0.19, very close to the diffusion approximation," says Majumdar.

Zeitak, meanwhile, has been collaborating with Joel Stavans of the Weizmann Institute in Rehovot, Israel, and Wing-Yim Tam of Hong Kong University of Science and Technology on the collapse of foams. This effect, known to drinkers and brewers the world over, is the coalescence of small bubbles in a foam to make larger bubbles. Their results, currently being written up, indicate that the original small bubbles demonstrate persistence obeying a power law similar to that in diffusion models.

Now Majumdar's team is beginning to apply the concept of persistence to attack different types of problem. One example is so-called critical phenomena, in which systems undergo abrupt and dramatic changes in their properties—the sudden magnetization of a lump of heated iron once its temperature falls below a critical value, for example. In the 28 October issue of *Physical Review Letters*, Majumdar has shown that a persistence power law can give a probability that, at the critical temperature, the system still has not flipped its magnetic direction after a certain length of time—results backed up by Stauffer in the October issue of *International Journal of Modern Physics C*.

The real power of persistence, and its application in real physical situations such as industrial reactions and biological systems, will have to wait for experimenters to come up with definitive ways of measuring the new exponents. Yurke, meanwhile, wonders what other surprises are lurking in effects that physicists thought they understood. "It makes me wonder about what else we might be overlooking in these systems," he says.

–Andrew Watson

Andrew Watson is a writer in Norwich, U.K.

ASTRONOMY

Do Comets Get a Nudge From the Galaxy?

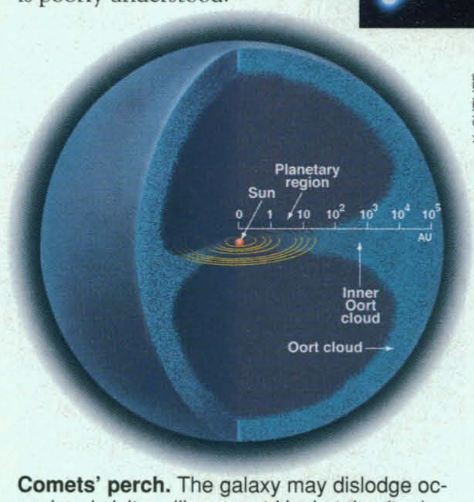
For decades, astronomers have known that most of the comets in the solar system are just rare visitors. Their true home is the Oort cloud, a vast spherical halo of icy objects that extends from beyond Pluto outward for more than a light-year. But what sends distant comets on their wanderings has been controversial. Now two astronomers at the University of Southwestern Louisiana, John Matese and Daniel Whitmire, offer a new candidate: the mass of the Milky Way as a whole, all the way to its distant core.

Matese and Whitmire, who present their idea in the 20 November *Astrophysical Journal*, say the galaxy's influence is unmistakable: Their calculations of how the Milky Way would stir the Oort cloud mesh neatly with an analysis of comet orbits. But the idea that the feeble gravitational pull of the distant galaxy could be influencing the solar system is hard for some other astronomers to swallow. "I don't mean to say that they're wrong," said Brian Marsden of the Harvard-Smithsonian Center for Astrophysics. "But I've played around with the [comet] data for years—and you can see some very funny things."

The roots of the puzzle go back to 1950, when the Dutch astronomer J. H. Oort calculated that the comets passing near Earth were shaken loose from a distant cloud. Oort's idea, says Matese, was that "a star came by and really shook them up" or some other "quasi-violent" event took place, kicking a few comets into elongated orbits that take them into the inner solar system every few hundred or thousand years.

But in the past decade, many astronomers, pointing to the lack of evidence for the sporadic "comet showers" that violent disruptions of the Oort cloud should unleash, have concluded that such events are responsible for only one-fifth or so of the comets we see. The most widely accepted explanation for the other four-fifths, says Matese, is that they are kneaded out of the Oort cloud by what he calls "gentle galactic tugging." But the mechanism of that tugging is poorly understood.

JOHN CHUMACK/PHOTO RESEARCHERS



K. SUTLIFF

Comets' perch. The galaxy may dislodge occasional visitors like comet Hyakutake *(top)*.

It has to come from some galactic tidal force that pulls on the orbiting comets, sapping their angular momentum so that they can plunge toward the sun. One possibility is the so-called *z*-tide, which comes from nearby stars and matter and pulls in a direction perpendicular to the plane of the galaxy. The other, 16 times weaker, is the so-called radial tide—the combined gravitational pull of the matter in the galactic interior, including all the stars and dark matter in its very core—that pulls toward the center of the galaxy. Since the gravitational attraction of nearby matter is so much larger, many astronomers think the *z*-tide is the main effect pulling comets from the Oort cloud.

Not necessarily, say Matese and Whitmire. The two astronomers claim that the weaker radial tide has a disproportionately strong effect on some comets—strong enough to account for fully one-third of all long-period comets. The reason: It can act on comets that are untouched by the *z*-tide. The *z*-tide's orientation perpendicular to the galaxy means it can only reduce the angular momentum of Oort cloud comets whose orbits are tilted relative to the galaxy. The radial tide, on the other hand, can affect the motion of comets moving in the plane of the galaxy. For those comets, Matese says, the radial tide "is the only game in town."

An analysis of comet tracks bears out the calculations, he and Whitmire say. Because the two galactic tides act in different directions, the paths of the comets they dislodge should be different. The analysis showed the influence of the radial tide in one-third of the tracks, as predicted. And with more than 80 comets in their database, says Whitmire, the statistical strength of their conclusion is "unheard of" in astronomy.

But Marsden thinks the comet data just aren't good enough to support Matese's and Whitmire's analysis. "I have no problem with [their] theoretical analysis," he says, "but it's a miserable, crummy set of data ... I say that having provided a lot of that data myself." With better data or a different analysis, Marsden says, the effect might disappear—and another way for the galaxy to pluck comets from the Oort cloud might emerge.

–Charles Seife

Charles Seife is a free-lance science writer in Scarsdale, New York.