Duality in Perspective

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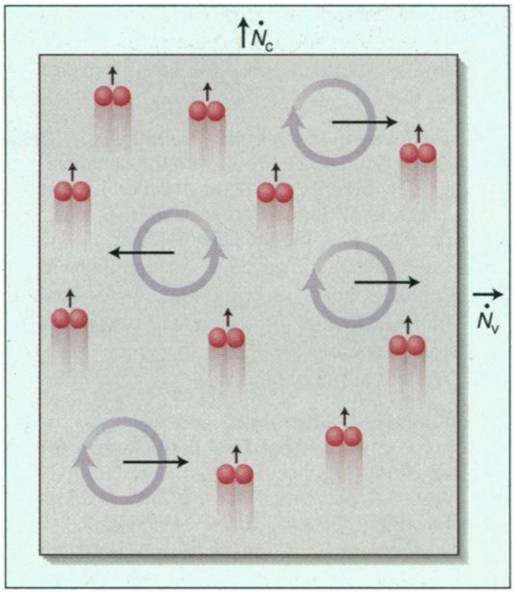
Duality transformations are mathematical methods that relate the physical properties of one system to those of another, possibly very different, physical system. If such a transformation exists, the systems are said to be dual to each other. Until recently, duality transformations were simply one of the items in theoreticians' bags of tricks used to conjure up exact solutions, or partial solutions, of statistical mechanics models of phase transitions, and of quantum field theories. However, as described in this issue (*1*), experimentalists have now gotten into the act, presenting tantalizing evidence of some potentially important physical consequences of duality symmetry in condensed matter systems.

The first use of duality in statistical mechanics was made by Kramers and Wannier in 1941 in a study of the two-dimensional (2D) Ising model, which describes a system of interacting magnetic spins. They found an exact transformation between the Ising model at some temperature and a dual model at a different temperature T_{dual}. Typically, the degrees of freedom in the dual model are different or have different interactions from those of the original model, or both. Nevertheless, all observable quantities are identical for the two models. Dual models thus often provide a new physical picture and way of thinking about the physics of a given system. This situation is reminiscent of (and in some special cases, actually related to) the wave-particle duality of quantum mechanics.

Remarkably, the Ising model has the special feature of being "self-dual": the dual model is identical to the original model (except that its temperature may be different). It follows that the phase transition between the magnetically ordered and disordered phases must occur at the unique temperature obeying $T^* = T^*_{dual}$. This observation allowed Kramers and Wannier to compute the critical temperature exactly. Further progress had to await the tour de force in which Onsager finally managed to fully solve the model.

It was recognized long ago that in electromagnetism, a duality transformation interchanges the role of **E** and **B** fields, and electric charges and magnetic monopoles. Seiberg and Witten (*2, 3*) have pointed out an analogous duality transformation in certain supersymmetric field theories. The purported duality in the quantum Hall effect (QHE) system is analogous to electric-magnetic duality for a 2D system.

Consider a somewhat simpler system, a 2D superconducting film (or more precisely an array of Josephson junctions). This is a canonical example in condensed matter physics of a system that exhibits approximate self-duality. We can crudely view a 2D superconductor at zero temperature as a Bose condensate of Cooper pairs of electrons. An important excitation of the system is a vortex, which is a topological point defect in the condensate wavefunction. If a boson moves around the vortex, the quantum phase of the system winds by $\pm 2\pi$. Associated with this gradient in the phase is a circulating current, much like a whirlpool in an ordinary fluid.



Dual view. Superconductor containing Cooper pairs (red circles) and magnetic vortices (circulating arrows). The two are related by dual symmetry.

Because vortices in 2D are point defects, it is possible to perform a duality transformation that rather directly interchanges the roles of particles and vortices. One is then viewing the vortices as the fundamental degrees of freedom and considering a wavefunction for the quantum system that depends on the positions of the vortices rather than the particles. Just as a vortex was a place where the phase of the particle wavefunction wound by $\pm 2\pi$, now a particle is a place where the phase of the vortex wavefunction winds by $\pm 2\pi$. It's all relative.

The 2D system can find itself in one of two thermodynamic phases, insulating or superconducting. It turns out that the duality transformation maps one phase into the other. The figure shows Cooper pairs and vortices moving in the system. The electric current along the sample is given by $I = (2e)\dot{N}_c$, where $\dot{N}_c$ is the flux of Cooper pairs moving upward as shown in the diagram. The Josephson relation tells us that the voltage drop is given by the rate of slip of the order parameter phase between the two ends of the sample $V = (h/2e)\dot{N}_v$, where $\dot{N}_v$ is the flux of vortices moving transverse to the current as shown in the diagram.

In the superconducting phase the bosons are strongly condensed. This causes vortices and anti-vortices to strongly attract each other and become "confined" much like the quarks of elementary particle physics. The vortex flux and hence the voltage will vanish, meaning that current flows without dissipation, that is, we have a superconductor.

Consider now the opposite limit in which the metallic grains on which the Cooper pairs live are very tiny and well separated, so that it is hard for the pairs to tunnel between particles. It turns out that in this limit it is easy for vortices to move around rapidly. This causes charge excitations to become "confined," and we then have an insulator. Under the duality transformation we can view the insulator as a Bose condensate of vortices. The original particles now look like strongly confined vortices. This is not as absurd as it seems. In fact the insulator is, like the superconductor, completely dissipationless (that is, if we apply a voltage, no current flows, so the dissipated power $P = IV = 0$).

The dissipationless flow of vortices in the insulating phase is thus the dual of ordinary superconductivity. Notice further that when charges and vortices trade places, voltage and current are interchanged. A moving charge produces a current. A moving vortex induces a voltage. The nonlinear current-voltage characteristics of Josephson junction arrays exhibit a symmetry, under interchange of current and voltage as one crosses the phase boundary from superconductor to insulator (*4*), that is strikingly similar to that seen by Shahar *et al.* (*1*) in the quantum Hall effect.

At the critical point where the system is switching from superconductor to insulator, both charges and vortices are simultaneously mobile. This means that the system has finite dissipation. If the applied voltage induces a flux $\dot{N}_c$ of Cooper pairs, then the resistance at the critical point is $R^* = h/(2e)^2(\dot{N}_v/\dot{N}_c)$. It turns out that this dimensionless ratio is a universal number characteristic of the phase transition (*5*).

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If the superconductor were, like the Ising model, self-dual, then vortices and charges would necessarily behave in exactly the same way at the critical point. We would then have $\dot{N}_v = \dot{N}_c$, and the universal resistance would simply be the quantum of resistance for charge $2e$ Cooper pairs: $R^* = h/(2e)^2$. It turns out, however, that realistic models are not self-dual, and R^* differs from this value by a factor of order unity (5).

Quantum Hall systems seem to bear no relation to superconducting condensates. In reality, however, one can view the ordered state of a QHE system as a Bose condensate of composite objects consisting of electrons that have "swallowed" the vortices (6). Because of the peculiarities of quantum statistics in two dimensions, there is a second view, in which the duality of the QHE system appears as particle-hole symmetry (*1*) of composite fermion objects (*7*) consisting of electrons that have swallowed some, but not all, of the vortices.

There is tantalizing evidence both from experiment and theory that the quantum Hall localization transition may exhibit self-duality. Much more work is needed, however, to explore this fully.

References

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Animal Origins

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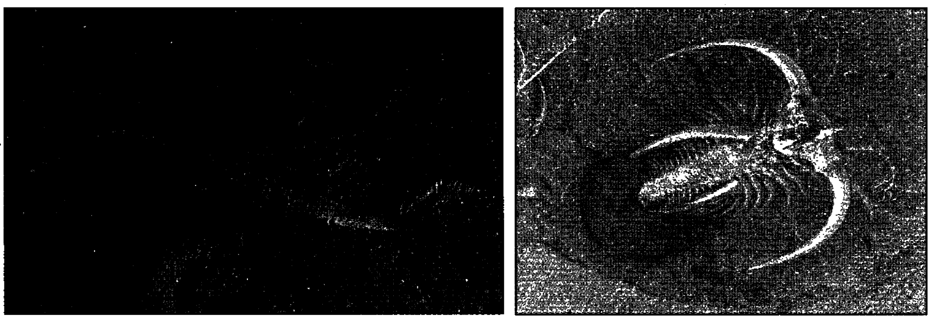
When did animals first evolve and proliferate? If we could answer this question, we would know more about an extraordinary episode of evolution.

The traditional view of the origins of multicellular animals, supported by a literal reading of the fossil record, is that animals appeared about 565 million years ago (Ma) during the Vendian period of the latest Neoproterozoic era. Within 20 million years, by the beginning of the Cambrian period of the Paleozoic era, all of the major groups (phyla) had become established—the first deep burrowers, grazers, predators, and skeleton-building animals. Early comparative work on globin proteins led Runnegar (*1*) to postulate a much earlier origin of animals, between 900 and 1000 Ma. On p. 568 of this issue, Wray *et al.* now offer support for an even earlier origin and diversification of the major phyla—between 1000 and 1200 Ma—on the basis of the nucleotide sequences of seven genes in living species from 16 phyla.

Arguing that the rate of substitution of nucleotides is roughly constant, or clock-like, the authors of the new work used the post-Cambrian fossil record of vertebrates to calibrate observed differences in gene sequences among species by noting the time of occurrence of the last common ancestor of these species. Acceptance of their proposed ancient origin of animals hinges on the accuracy of the calibration and on the assumption that the rate of nucleotide substitution is constant. Wray *et al.* show that vertebrates do not differ from other animal groups in average rates of molecular evolution, and therefore that the calibration is reasonable; but the assumption of constancy is less secure. It may be, for example, that rates of molecular substitution speed up during times of diversification, when many evolutionary branches are undergoing rapid morphological change. Such acceleration could have occurred during the Vendian-Cambrian interval, before the period over which molecular rates of evolution were calibrated, as well as during later episodes of evolutionary proliferation in the Mesozoic and Cenozoic eras. Moreover, if morphological evolution generally speeded up beginning in Neoproterozoic time, as the fossil record strongly implies (*2*), then the divergence times calculated by backward extrapolation from the vertebrate record would be too early. Wray *et al.* point out that the assumption of constancy would have to be violated on a grand scale in order to wring a Vendian origin of animals out of their data, but they acknowledge that the actual time of divergence of phyla remains poorly constrained. Greater precision in this and other analyses of molecular divergence (*3*) can come only when the possibility that molecular evolution accelerates in most or all groups during certain intervals of Earth's history is closely scrutinized.

An origin of animals somewhere between 700 and 900 Ma, during the Early or Middle Neoproterozoic, seems likely on geochemical grounds. This was a time when oxygen levels rose in the ocean and atmosphere (*4–6*), perhaps causally linked to changes in Earth's tectonic regime and to the formation of continents (*7*). There was likely a rise in primary productivity (rate of fixation of organic matter) (*5–6*), which together with more oxygen made possible collagen synthesis (*8*) and higher levels of energy use.

Why are animals missing from the pre-Vendian fossil record? The answer may reside in their minute size. Although Neoproterozoic oceans supported bottom-dwelling seaweeds and single-celled planktonic algae (*2, 5, 9*), there is no indication that these primary producers were being consumed by grazing or suspension-feeding animals (*10*), and the only known potential animal traces are tiny horizontal tracks on, or 1 mm beneath, the surface of the seafloor (*11*). Pre-Vendian animals may therefore have been minute creatures crawling and sliding between grains of mud or sand on the seabed (*12*). Unmineralized animals prominent in



Even older than old. The major animal phyla appear in the rock record about 540 million years ago, in the form of fossils such as *Pikaia* (left), a chordate, and *Marella* (right), an arthropod. New molecular evidence suggests that these phyla actually diverged closer to a billion years ago. [Photos by C. Clark]

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