Cold Dark Matter

Scott Dodelson, Evalyn I. Gates, Michael S. Turner

Motivated by inflation, the theory of big-bang nucleosynthesis, and the quest for a deeper understanding of fundamental forces and particles, a paradigm for the development of structure in the universe has evolved. It holds that most of the matter exists in the form of slowly moving elementary particles left over from the earliest moments—cold dark matter—and that the small density inhomogeneities that seed structure formation arose from quantum fluctuations around 10^{-34} seconds after the big bang. A flood of observations, from determinations of the Hubble constant to measurements of the anisotropy of cosmic background radiation, are now testing the cold dark matter paradigm.

According to the hot big-bang model, the universe began as a hot, nearly homogeneous soup of fundamental particles (*1*). Today, the most conspicuous feature of the universe is the abundance of structure—stars, galaxies, clusters of galaxies, superclusters, voids, and very large, sheet-like structures composed of galaxies (great walls) (*2*). The uniformity of the temperature of the cosmic background radiation (CBR) indicates that this structure must have arisen from small inhomogeneities in the distribution of matter. It is believed that this occurred through the attractive action of gravity operating over the past 15 gigayears (Gyr), amplifying the primeval inhomogeneities by a factor of about 10^5 (*3*).

This general picture was confirmed in 1992 when the differential microwave radiometer (DMR) on NASA's Cosmic Background Explorer (COBE) satellite detected differences in the CBR temperature in directions separated on the sky by around 10° at the level of 30 μK; the average temperature is 2.728 ± 0.002 K, so this corresponds to a relative temperature difference $\delta T/T \approx 10^{-5}$ (*4*). Because matter inhomogeneities give rise to temperature differences of comparable size, the DMR detection provided direct evidence for the existence of matter inhomogeneity, $\delta\rho/\rho \sim \delta T/T \sim 10^{-5}$, at the level needed to seed the formation of structure. Since then, CBR anisotropy of a similar size has been detected by more than 10 experiments, on angular scales from 0.5° to tens of degrees (Fig. 1) (*5*). The challenge now is to put together a detailed and coherent picture of structure formation. In doing so, many cosmologists believe that much will be revealed about the earliest moments of the universe and perhaps even the nature of the fundamental forces.

S. Dodelson is with the NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510–0500, USA. E. I. Gates is with the NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510–0500, USA, and the Department of Astronomy and Astrophysics, Enrico Fermi Institute, University of Chicago, Chicago, IL 60637–1433, USA. M. S. Turner is with the NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510–0500, USA; the Department of Astronomy and Astrophysics, Enrico Fermi Institute, University of Chicago, Chicago, IL 60637–1433, USA; and the Department of Physics, Enrico Fermi Institute, University of Chicago, Chicago, IL 60637–1433, USA.

In recent years, several approaches to structure formation have been pursued (*6*, *7*). Here we review the status of the cold dark matter (CDM) theory, which derives from two basic tenets. (i) The universe is spatially flat, which means that on average, the total energy density is equal to the critical energy density. Ordinary matter (baryons) contributes about 5% of the critical density, and slowly moving elementary particles left over from the earliest moments (CDM particles) contribute much (but perhaps not all) of the rest. (For example, a portion of the critical density could exist in the form of a cosmological constant.) (ii) The primeval density perturbations are nearly scale-invariant and arose from quantum-mechanical fluctuations during the earliest moments. Scale-invariant refers to the fact that fluctuations in the gravitational potential are independent of length scale. More precisely, the Fourier components of the primeval density field, δ_k, are drawn from a Gaussian distribution with variance given by a power spectrum $P(k) \equiv \langle|\delta_k|^2\rangle \sim Ak^n$, where k is the wave number (*8*). Perturbations of wavelength $\lambda = 2\pi/k \sim 1$ Mpc give rise to galaxies, of ~10 Mpc to clusters, and of ~100 Mpc to the largest structures observed (1 Mpc = 3.09×10^{24} cm).

The CDM theory draws from three important ideas: inflation, big-bang nucleosynthesis, and the quest to better understand the fundamental forces and particles. Inflation holds that the universe underwent an early (time $t \sim 10^{-34}$ s) rapid period of expansion during which it grew in size by a factor greater than 10^{25}. This rapid expansion is driven by vacuum energy, a form of energy predicted to exist by some unified field theories (*9*). The enormous growth in size leads to a universe that appears to be flat on the length scales that we can probe (up to the current horizon of 15 billion light years) and thus has critical density. Further, the growth allows quantum-mechanical fluctuations excited on microscopic scales ($\ll 10^{-16}$ cm) to be stretched in length to become variations in the energy density on astrophysical scales. The continual creation of quantum fluctuations and stretching of their wavelengths leads to fluctuations on all scales. The conversion of these fluctuations to energy density fluctuations occurs when vacuum energy decays into radiation at the end of inflation.

Big-bang nucleosynthesis refers to how the light isotopes D, ^{3}He, ^{4}He, and ^{7}Li were produced by nuclear reactions during the first few seconds. The agreement between the predicted and measured light-element abundances is an important confirmation of the hot big-bang model (*10*) and provides the best determination of the density of ordinary matter. The baryon density inferred from nucleosynthesis is between 1.5×10^{-31} g cm^{-3} and 4.5×10^{-31} g cm^{-3} and corresponds to a fraction of critical density that depends on the value of the Hubble constant (H_0) $\Omega_B = 0.008h^{-2}$

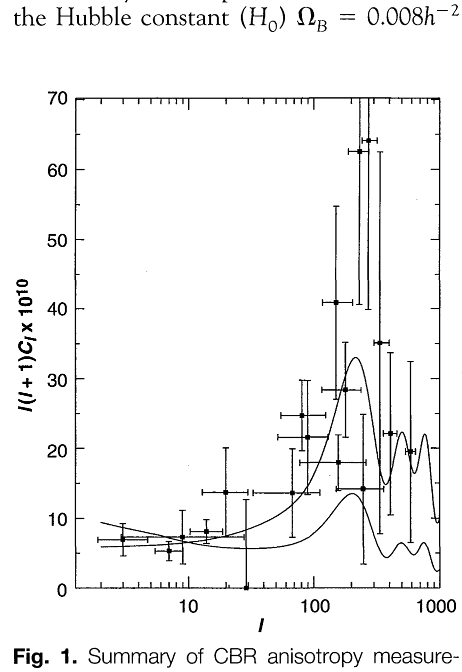

Fig. 1. Summary of CBR anisotropy measurements [adapted from (*5*)] and predictions for two CDM models. Plotted are the squares of the measured multipole amplitudes ($C_l = \langle|a_{lm}|^2\rangle$) versus multipole number l. The relative temperature difference on angular scale θ is given roughly by $\sqrt{l(l+1)C_l/2\pi}$ with $l \sim 200°/\theta$. The theoretical curves are standard CDM (upper curve) and CDM with $n = 0.7$ and $h = 0.5$ (lower curve).

$- 0.024h^{-2}$, where $H_0 = 100h$ km s^{-1} Mpc^{-1} (*11*). Allowing h to equal 0.4 to 0.8 (*12*) implies that ordinary matter can contribute at most 15% of the critical density. If the inflationary prediction is correct, then most of the matter in the universe must be nonbaryonic (Fig. 2).

This idea has received indirect support from particle physics. Attempts to further our understanding of the fundamental particles and forces have led to the prediction of new, stable or long-lived particles that interact feebly with ordinary matter. These particles, if they exist, should have been present in large numbers during the earliest moments and remain today in numbers sufficient to contribute most of the critical density (*13*). Two examples of particle dark-matter candidates that behave like CDM are a neutralino of mass 10 to 500 giga–electron volts, predicted in supersymmetric theories (*14*), and an axion of mass 10^{-6} to 10^{-4} eV, needed to solve a subtle problem of the standard model of particle physics (*15*).

A third possibility is that one (or more) of the three neutrino species has a mass between 2 and 30 eV; because they were once in thermal equilibrium, neutrinos this light move fast and are referred to as hot dark matter. The idea that most of the exotic matter exists in the form of fast-moving neutrinos was explored before the CDM theory and found to be inconsistent with observations (at least for inflationary density perturbations) (*16*). With hot dark matter, large structures form first and fragment into smaller structures. This conflicts with the fact that many galaxies with high red shifts are observed, which means that they are far away and old, whereas larger structures are only forming today (*17*). It is still possible that some of the exotic dark matter is light neutrinos.

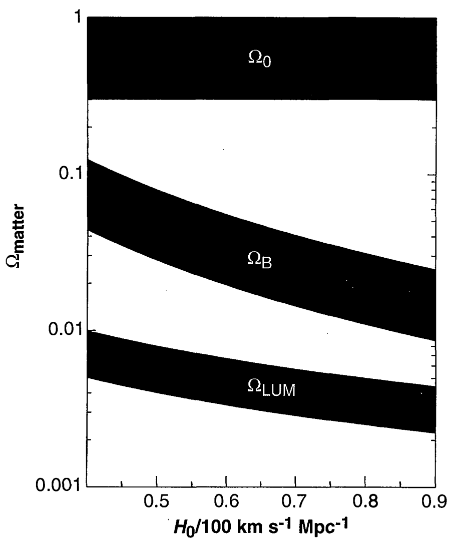

Fig. 2. Summary of knowledge of Ω. The lowest band is luminous matter (Ω_{LUM}), in the form of bright stars and associated material; the middle band is the big-bang nucleosynthesis determination of the density of baryons (Ω_B); and the upper region is the estimate of Ω_{matter} based on the peculiar velocities of galaxies (Ω_0). The gaps between the bands illustrate the two dark matter problems: Most of the ordinary matter is dark and most of the matter is nonbaryonic.

According to CDM theory, CDM particles provide the cosmic infrastructure; it is their gravitational attraction that forms and holds cosmic structures together. These structures form in a hierarchical manner, with galaxies forming first and successively larger objects forming thereafter (*18*). Quasars, the brightest and oldest galaxies, evolve from rare, very high density peaks and form at red shifts of up to five, whereas typical galaxies form from average-size density peaks at red shifts of one to three. Clusters of galaxies form at red shifts of around one, and today superclusters (objects made of several clusters of galaxies) are just becoming bound by the gravity of their CDM constituents. The formation of larger and larger objects continues. In the clustering process, regions of space are left devoid of matter. If the CDM theory is correct, CDM particles are the ubiquitous dark matter known to exist only by its gravitational effects, which accounts for most of the mass density in the universe and holds galaxies, clusters of galaxies, and even the universe itself together (*19*).

"Standard" Cold Dark Matter

When the CDM scenario emerged more than a decade ago, many referred to it as a no-parameter theory because it was so specific compared with previous models for the formation of structure. This was an overstatement, as there are cosmological quantities that must be specified. However, the data available did not require precise knowledge of these quantities to begin testing the model.

Broadly speaking, the parameters can be organized into two groups. First are the cosmological parameters: H_0; the density of ordinary matter, specified by $\Omega_B h^2$; and the power-law index n that describes the shape of the spectrum of density perturbations (*20*). The inflationary parameters fall into this category because there is no standard model of inflation; on the other hand, once determined, they can be used to discriminate among models of inflation. The second group specifies the composition of invisible matter in the universe: radiation, dark matter, and the cosmological constant. Radiation refers to relativistic particles: the photons in the CBR, three massless neutrino species (assuming none of the neutrino species has a mass), and possibly other undetected relativistic particles [some particle physics theories predict the existence of additional massless particle species (*21*)]. At present, relativistic particles contribute almost nothing to the energy density in the universe, $\Omega_{REL} \simeq 4.2 \times 10^{-5} h^{-2}$; early on, when the universe was less than about 10^{-5} of its present size, they dominated the energy content; the level of radiation today is important as it determines when the transition from radiation domination to matter domination took place. Dark matter could include other particle relics besides CDM. For example, each neutrino species has a number density of 113 cm^{-3}, and a neutrino species of mass $m_\nu = 5$ eV would account for about 20% of the critical density ($\Omega_\nu = m_\nu/90h^2$ eV). Predictions for neutrino masses range from 10^{-12} eV to several mega–electron volts, and there is some experimental evidence that at least one of the neutrino species has mass (*22*). Finally, there is the cosmological constant. Introduced and then abandoned by Einstein to prevent the expansion of the universe (*23*), and resurrected by Bondi, Gold, and Hoyle in 1948 to address an age crisis (*24*), it is still with us. In the modern context, it corresponds to an energy density associated with the quantum vacuum. At present, there is no reliable calculation of the value that the cosmological constant should have (*25*), and so its existence must be regarded as a logical possibility, with its value to be determined by observations.

The original no-parameter CDM model, often referred to as standard CDM (*18*), is characterized by simple choices for the cosmological and the invisible matter parameters: precisely scale-invariant density perturbations ($n = 1$) (*26*), $h = 0.5$, $\Omega_B = 0.05$, and $\Omega_{CDM} = 0.95$; no radiation beyond the photons and the three massless neutrinos; no dark matter beyond CDM; and a zero cosmological constant. Standard CDM focused attention on a specific scenario for the development of structure.

Although inflation models predict that the shape of the spectrum is approximately scale-invariant, the overall amplitude depends on parameters of the particular inflationary model. In standard CDM, the overall amplitude was fixed by comparing the predicted amount of inhomogeneity with that seen today in the distribution of bright galaxies. Galaxy-number fluctuations in spheres of radius $8h^{-1}$ Mpc are unity; adjusting the overall amplitude so that mass fluctuations in spheres of radius $8h^{-1}$ Mpc are unity, $\sigma_8 = 1$, corresponds to the assumption that light, in the form of bright galaxies, traces mass. Choosing σ_8 to be less than 1 means that light is more clustered than mass and is a biased tracer of mass. There is some evidence that bright galaxies are somewhat more clumped than mass with biasing factor $b \equiv 1/\sigma_8 \simeq 1 - 2$ (*27*).

An important change occurred with the detection of CBR anisotropy by COBE in 1992 (*4*). The COBE measurement permitted a precise normalization of the amplitude of density perturbations on very large scales ($\lambda \sim 10^4 h^{-1}$ Mpc) without regard to biasing (*28*). For standard CDM, the COBE normalization leads to $\sigma_8 = 1.2 \pm 0.1$ or anti-bias, because $b = 1/\sigma_8 \simeq 0.8$ is less than 1. The pre-COBE normalization ($\sigma_8 = 0.5$) led to too little power on scales of $30h^{-1}$ Mpc to $300h^{-1}$ Mpc, as compared with what was indicated in red shift surveys, the angular correlations of galaxies on the sky, and the peculiar velocities of galaxies. The COBE normalization leads to about the right amount of power on these scales but appears to predict too much power on small scales ($\lesssim 8h^{-1}$ Mpc). Broadly speaking, standard CDM is consistent with many observations, but a consensus has developed that the conflict just mentioned is probably significant (*29*). This has led to a new look at the cosmological and invisible-matter parameters and to the realization that the problems of CDM theory reflect a poor choice for the standard parameters.

Cold Dark Matter Models

To determine a better choice of parameters, we first identify several robust tests of large-scale structure that any CDM model must satisfy; later we discuss other observations that are important but whose interpretation is more ambiguous. We focus on four "families" of CDM models, distinguished by their invisible matter content: standard invisible matter content (CDM), extra radiation (τCDM), small hot dark matter component (νCDM), and a cosmological constant (ΛCDM). There are of course other possibilities: τCDM + ΛCDM, and so on; for simplicity, we have not considered combinations.

For each family, we allow the cosmological parameters [h, $\Omega_B h^2$, n, and the level of gravity waves (*20*)] to vary and fix Ω_{CDM} so that the total energy density is equal to the critical density. For each model, we compute the expected CBR anisotropy and require that it be consistent with the 4-year COBE data set at the 2σ level (*30*). Having COBE-normalized, we compute the expected level of inhomogeneity in the universe today (*31*) and compare it with three observational constraints: the shape of the power spectrum, the power on cluster scales, and the early formation of objects. We choose the power spectrum to judge each model because it is the underlying foundation and is ultimately probed by a variety of cosmological observations. Our three constraints probe the power spectrum on a range of scales and at a time when those scales were still relatively smooth, enabling us to trust our linear calculations of the power spectrum (*32*).

The first constraint, the shape of the power spectrum on scales from a few megaparsecs to a few hundred megaparsecs (Fig. 3), comes from red shift surveys of the distribution of bright galaxies today (*33, 34*). In the absence of an understanding of the relation between the distributions of light (which is what these surveys determine) and mass, we leave the bias factor as a free parameter. We reject models whose power spectrum deviates from the measured power spectrum (*34*) by more than 2σ (a value of χ^2 whose likelihood is less than 5%).

The abundance of x-ray–emitting clusters is sensitive to the degree of inhomogeneity on scales around $8h^{-1}$ Mpc and thus provides a good means of inferring the value of σ_8. Following (*35*), we use $0.5 \leq \sigma_8 \leq 0.8$ for models with $\Omega_{matter} = 1$ and let this range scale with $\Omega_{matter}^{-0.56}$ for models with a cosmological constant ($\Omega_\Lambda = 1 - \Omega_{matter}$).

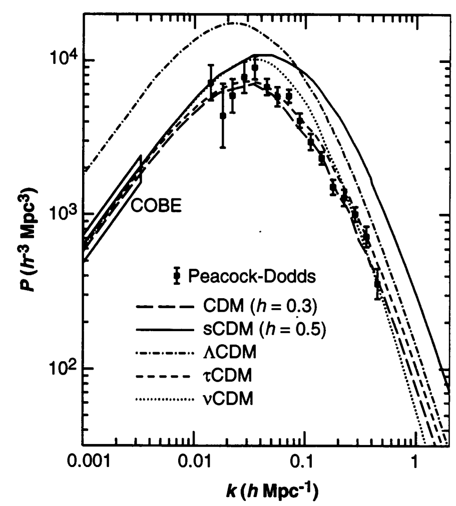

Fig. 3. Measurements of the power spectrum $P(k) = |\delta_k|^2$ and the predictions of different COBE-normalized CDM models. (COBE constrains the power spectrum at wavenumbers k around $2h \times 10^{-3}$ Mpc^{-1} as indicated by the rectangle.) The points are from several red shift surveys as compiled by (*34*); the models are: ΛCDM with $\Omega_\Lambda = 0.6$ and $h = 0.65$; standard CDM (sCDM), CDM with $h = 0.35$; τCDM (with the energy equivalent of 12 massless neutrino species), and νCDM with $\Omega_\nu = 0.2$ (unspecified parameters have their standard CDM values). The offset between a model and the points indicates the level of biasing. ΛCDM does not pass through the COBE rectangle because a cosmological constant alters the relation between the power spectrum and CBR anisotropy.

The formation of objects at high red shift (early structure formation) probes the power spectrum on small scales. At red shifts of two to four, hydrogen clouds, detected by their absorption features in the spectra of high red shift quasars (red shift $z \sim 4$ to 5), contribute a fraction of the critical density [$\Omega_{clouds} \simeq (0.001 \pm 0.0002)h^{-1}$ (*36*)]. Insisting that the predicted level of inhomogeneity is sufficient to account for this fraction at red shift four leads to a lower limit to the power on small scales ($\lambda \sim 0.2h^{-1}$ Mpc) (*37*).

Viable CDM models with standard invisible-matter content lie in a region that runs diagonally from a smaller H_0 and larger n to a larger H_0 and smaller n (Fig. 4). That is, higher values of H_0 require more tilt (tilt referring to deviation from scale invariance). As expected, standard CDM is outside of the allowed range. Current measurements of CBR anisotropy on the degree scale, as well as the COBE 4-year anisotropy data, preclude n less than about 0.7 (Fig. 1). This implies that the largest H_0 consistent with the simplest CDM models is slightly less than 60 km s^{-1} Mpc^{-1}. If the invisible-matter content is nonstandard, higher values of H_0 can be accommodated.

With tilt and hot dark matter, H_0 as large as 65 km s^{-1} Mpc^{-1} can be accommodated (Fig. 5). A neutrino content as low as 5% (corresponding to a neutrino mass of around

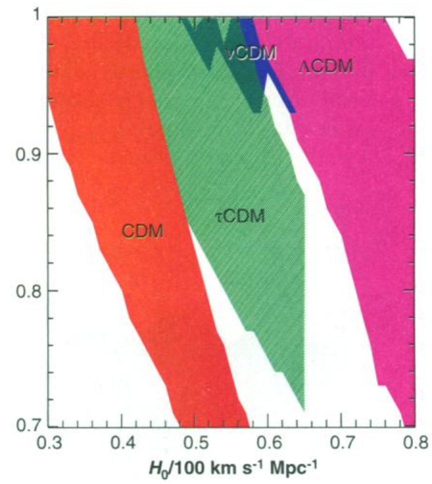

Fig. 4. Acceptable values of the cosmological parameters n and h for CDM models with standard invisible-matter content (CDM); with 20% hot dark matter (νCDM); with additional relativistic particles (the energy equivalent of 12 massless neutrino species, denoted τCDM); and with a cosmological constant that accounts for 60% of the critical density (ΛCDM). Here and in Figs. 5 through 7, a model is considered viable if it passes the three tests for any value of $\Omega_B h^2$ between 0.008 and 0.024 and any level of gravitational waves (*20*). The τCDM models have been truncated at a H_0 of 65 km s^{-1} Mpc^{-1} because a larger value would result in a universe that is younger than 10 Gyr.

1 eV or so) allows nearly scale-invariant perturbations ($n > 0.9$) with $H_0 = 50$ km s^{-1} Mpc^{-1}. The jagged shape of the allowed region in Fig. 4 arises because $\Omega_\nu = 0.2$ is almost precluded for any value of h and n, and higher content of hot dark matter is not viable at all (*38*). This is another indication that hot dark matter with inflationary perturbations is not viable.

The introduction of a cosmological constant permits H_0 as large as 80 km s^{-1} Mpc^{-1} (Figs. 4 and 6). Figure 7 shows the trade-off between H_0 and the radiation content: Lowering h or raising g_* has the same effect (see below). (Here $g_* = 2 + 0.454\,N_\nu$ measures the energy density in relativistic particles, where N_ν is the equivalent number of massless neutrino species.)

Changes in the different parameters from their standard CDM values alleviate the excess power on small scales in different ways. Tilt has the effect of reducing power on small scales when power on very large scales is fixed by COBE. A small admixture of hot dark matter works because fast-moving neutrinos suppress the growth of inhomogeneity on small scales by streaming from regions of higher density to regions of lower density.

A low value of H_0, additional radiation, or a cosmological constant all reduce power on small scales by lowering the ratio of matter to radiation. Because the critical density depends on the square of H_0 ($\rho_{\text{critical}} = 3H_0^2/8\pi G$), a smaller value corresponds to a lower density of matter as $\rho_{\text{matter}} = \rho_{\text{critical}}$ for a flat universe without a cosmological constant. Shifting some of the critical density to vacuum energy also reduces the matter density because $\Omega_{\text{matter}} = 1 - \Omega_\Lambda$. Lowering the ratio of matter to radiation reduces the power on small scales in a subtle way. Although the primeval fluctuations in the gravitational potential are nearly scale-invariant, density perturbations today are not, because the universe made a transition from an early radiation-dominated phase ($t \lesssim 1000$ years), in which the growth of density perturbations is impeded, to the matter-dominated phase, in which growth proceeds unimpeded. This introduces a feature in the power spectrum today (Fig. 3) whose location depends on the relative amounts of matter and radiation. Lowering the ratio of matter to radiation shifts the feature to larger scales, and with power on large scales fixed by COBE, this leads to less power on small scales.

Some of the viable models have been discussed previously as singular solutions—a cosmological constant (*13*, *39*), a very low H_0 (*40*), tilt (*41*), tilt + low H_0 (*42*), extra radiation (*21*), or an admixture of hot dark matter (*43*). We wish to emphasize that there is actually a continuum of viable models (Figs. 4 through 7), which arises because of imprecise knowledge of cosmological parameters and the invisible-matter sector. Other observations, especially the anisotropy of the CBR, will soon clarify the situation.

Other Considerations

Other observations test CDM theory, although their interpretations are more ambiguous or controversial. Presently, the best of these tend to distinguish the cosmological-constant family of models from the other three families (*44*). This is because models with standard invisible matter, extra radiation, or a hot dark matter component are all matter-dominated today and have the same kinematic properties: age for a given H_0 and distance to an object of given red shift. The introduction of a cosmological constant leads to an older universe and greater distance to an object at a fixed red shift.

Together, the age of the universe and determinations of H_0 have substantial leverage; so much so that some would say there is an age crisis in cosmology (*45*). This is because determinations of the ages of the oldest stars lie between 12 Gyr and 17 Gyr (*45*, *46*) and recent measurements of H_0 favor values between 60 km s^{-1} Mpc^{-1} and 80 km s^{-1} Mpc^{-1} (*47*), which, for $\Omega_{\text{matter}} = 1$, leads to a time back to the big bang of 11 Gyr or less (*48*). These age determinations receive additional support from estimates of the age of the galaxy that are based on the decay of long-lived radioactive isotopes and the cooling of white dwarf stars, and all methods taken together make a very strong case for an absolute minimum age of 10 Gyr (*49*). Within the uncertainties there is no inconsistency, though there is certainly tension, especially with $\Omega_{\text{matter}} = 1$ (Fig. 8).

Although large-scale structure considerations lessen the age problem, as they favor an older universe by virtue of a lower H_0 or cosmological constant, H_0 still has great leverage. If it is determined to be greater than about 60 km s^{-1} Mpc^{-1}, then only CDM models with nonstandard invisible-matter content can be consistent with large-scale structure. If H_0 is greater than 65 km s^{-1} Mpc^{-1}, ΛCDM is the lone possibility. The issue of H_0 is not settled yet, but

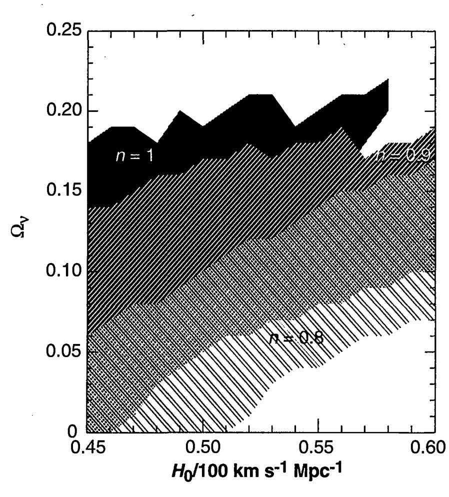

Fig. 5. νCDM models: Acceptable values of Ω_ν and h for $n = 0.8$, 0.9, and 1.0. Note that no model is viable with Ω_ν greater than about 0.2 and that even a small admixture of neutrinos has important consequences.

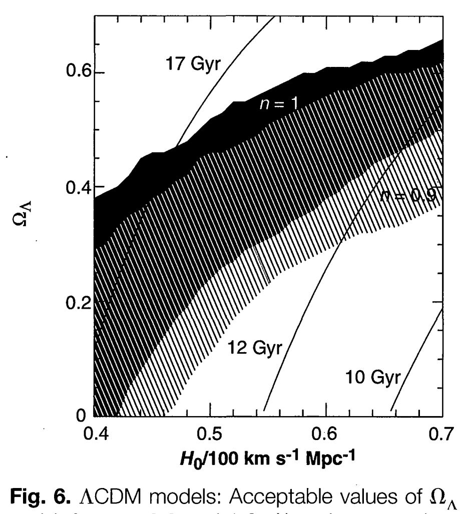

Fig. 6. ΛCDM models: Acceptable values of Ω_Λ and h for $n = 0.9$ and 1.0. Also shown are isochrones corresponding to 17, 12, and 10 Gyr. Note that large-scale structure considerations generally favor an older universe—smaller h or larger Ω_Λ.

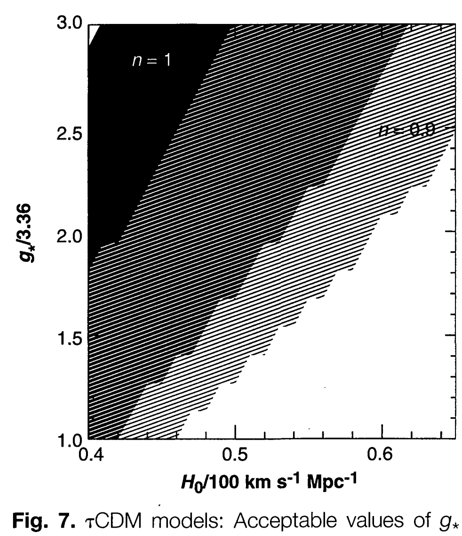

Fig. 7. τCDM models: Acceptable values of g_* and h for $n = 0.9$ and 1.0. The quantity g_* counts the total number of relativistic species. Photons and three massless neutrino species correspond to $g_* = 3.36$; with the energy equivalent of N_ν massless neutrino species $g_* = 2 + 0.454N_\nu$.

the Hubble Space Telescope (HST) is making progress by finding Cepheid variable stars (a reliable distance indicator) in galaxies in our local neighborhood and using them to calibrate bright standard candles, including Type Ia and Type II supernovas, which can be seen at great enough distance to determine H_0 accurately.

If CDM is correct, baryons make up a small fraction of matter in the universe. Most of the baryons in galaxy clusters are in the hot, x-ray–emitting intracluster gas and not the luminous galaxies. The measured x-ray flux fixes the mass in baryons, whereas the measured x-ray temperature fixes the total mass (through the virial theorem). The ratio of baryon mass to total mass has been determined from x-ray measurements for more than 10 clusters and is found to be $\simeq (0.04 - 0.1)h^{-3/2}$ (*50*). Because of their size, clusters should represent a fair sample of the cosmos and thus the baryon-to-total mass ratio should reflect its universal value, $\Omega_B/\Omega_{\text{matter}} \simeq (0.008 - 0.024)h^{-2}/\Omega_{\text{matter}}$. These two ratios are consistent for models with $\Omega_{\text{matter}} \sim 0.3$; that is, those with a cosmological constant. However, important assumptions are made in this analysis: that the hot gas is unclumped and in virial equilibrium and that magnetic fields do not provide significant pressure support for the gas. If any one of these is not valid, the actual baryon fraction would be smaller (*51*), allowing for consistency with a larger value of H_0 without recourse to a cosmological constant.

The halos of individual spiral galaxies like our own are not large enough to be certain to provide a fair sample of matter in the universe; for example, some of the baryonic matter has undergone dissipation and condensed into the disk of the galaxy. Nonetheless, halos are expected to be primarily CDM particles. Indeed, visible stars, hot gas, cold gas, and dust account for only a tiny fraction of the mass of the halo of our own galaxy (*52*). Dark stars (or MACHOs), which have been detected by their microlensing of stars in the Large Magellanic Cloud, may well contribute a significant fraction of the halo (30% or perhaps more) (*53*). A large MACHO fraction of the halo is easier to understand in ΛCDM, where the ratio of baryons to CDM is higher. However, unless the MACHO fraction of the halo exceeds 50%, it is not inconsistent with any of the CDM models (*54*).

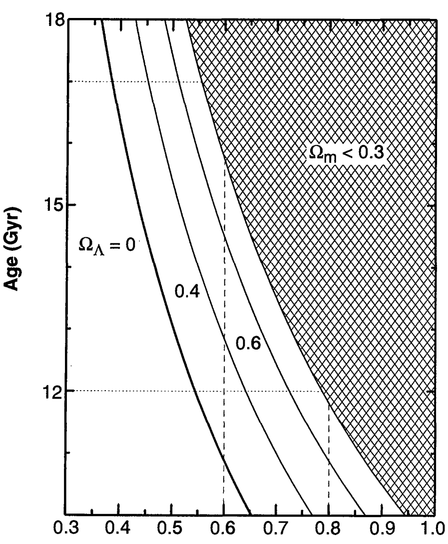

Fig. 8. The relation between age and H_0 for flat-universe models with $\Omega_{\text{matter}} = 1 - \Omega_\Lambda$. The cross-hatched region is ruled out because $\Omega_{\text{matter}} < 0.3$. The dotted lines indicate the favored range for H_0 and for the age of the universe (based on the ages of the oldest stars).

The mean mass density of the universe is an important test of inflation and CDM, but a definitive determination is still lacking. The measurement that averages over the largest volume and thus is potentially most useful uses the peculiar velocities of galaxies. Peculiar velocities arise because of the inhomogeneous distribution of matter, and the mean matter density can be determined by relating peculiar velocities to the observed distribution of galaxies. When this technique is applied to the peculiar velocity of our own galaxy, Ω_{matter} of at least 0.3 is inferred (*55*); an analysis of the peculiar velocities of thousands of other galaxies indicates a value close to unity (*56*). If a high value for Ω_{matter} is shown to be correct, it would lend support to the inflationary prediction and rule out ΛCDM.

A different approach to the mean density is through the deceleration parameter q_0, which quantifies the slowing of the expansion due to the gravitational attraction of matter in the universe. Its value is given by $q_0 = \frac{1}{2}\Omega_{\text{matter}} - \Omega_\Lambda$ (vacuum energy actually leads to accelerated expansion) and can be determined by relating the distances and red shifts of distant objects. In all but ΛCDM, $q_0 = 0.5$; for ΛCDM, $q_0 \sim -0.5$. Two groups (*57*) are trying to measure q_0 by using high–red shift ($z \sim 0.4 - 0.7$) Type Ia supernovas as standard candles; the preliminary results of one group suggest that q_0 is positive (*58*). Together, these two groups discovered almost 40 Type Ia supernovas earlier this year, and both groups may soon be able to determine q_0 with a precision of ± 0.2.

Gravitational lensing of distant quasistellar objects (QSOs) by intervening galaxies provides another path to q_0. The distance to a QSO of given red shift is larger for smaller q_0, and thus the probability for its being lensed by an intervening galaxy is greater. Present analyses suggest that $\Omega_\Lambda <$ 0.66 (*59*), which at present is not a problem for ΛCDM (Fig. 6).

These above considerations, with the possible exception of the preliminary determination of q_0, favor ΛCDM (*44*). Most troubling for the other models are the recent measurements of H_0 (*47*) and the ratio of gas to total mass in clusters (*50*). However, we hesitate to give this conclusion too much weight yet. First, the introduction of a cosmological constant is a big step, one that twice before proved to be a misstep. Second, a cosmological constant raises the fundamental question of the origin of the implied vacuum energy, about 10^{-8} eV^4 (*25*). Finally, the two key observations that could rule out the simplest cold dark matter models—the value of H_0 and the gas ratio in clusters—are not completely settled.

Conclusion

The testing of CDM and inflation continues. The 10-m Keck telescope and HST are providing the deepest images of the universe and revealing details of the formation and evolution of galaxies and clusters of galaxies. In ΛCDM, structure forms earlier than in τCDM. The Keck has detected deuterium in high red shift hydrogen clouds (*60*). This is a confirmation of big-bang nucleosynthesis and has the potential of pinning down the density of ordinary matter to a precision of 10%. In searching for viable models, we allowed the baryon density to vary from 1.5×10^{-31} g cm^{-3} to 4.5×10^{-31} g cm^{-3}; a 10% determination would trim the number of acceptable models.

The level of inhomogeneity in the universe today is determined mainly from red shift surveys, the largest of which contains about 30,000 galaxies. Two larger surveys are underway. The Sloan Digital Sky Survey will cover 25% of the sky and obtain positions for 200 million galaxies and red shifts for a million galaxies (*61*). The Anglo-Australian Two-Degree Field is targeting hundreds of 2° patches on the sky and will obtain 250,000 red shifts (*61*). These two projects will determine the power spectrum much more precisely and on scales large enough ($500h^{-1}$ Mpc) to connect with measurements from CBR anisotropy on angular scales of up to 5°, allowing bias to be probed.

The most fundamental element of CDM—the existence of the CDM particles themselves—is being tested. The interaction of CDM particles with ordinary matter occurs through feeble forces, which makes their existence difficult to test. Experiments with sufficient sensitivity to detect the CDM particles that hold our own galaxy together if they are in the form of axions of mass 10^{-6} to 10^{-4} eV (*62*) or neutralinos of mass tens of giga–electron volts (*63*) are now under way. Evidence for the existence of the neutralino could also come from particle accelerators searching for other supersymmetric particles (*64*). In addition, a variety of experiments sensitive to neutrino masses are operating or are planned: accelerator-based neutrino oscillation experiments at Fermilab, CERN, and Los Alamos (*65*); solar-neutrino detectors in Japan, Canada, Germany, Russia, and Italy (*66*); and experiments at e^+e^- colliders (LEP at CERN, BES in Beijing, and CESR at Cornell) to the study of the tau neutrino (*67*).

CBR anisotropy probes the power spectrum most cleanly as it is related directly to the distribution of matter when density perturbations were small (*68*). Current measurements are beginning to test CDM and distinguish different CDM models (Fig. 1); for example, a spectral index $n < 0.7$ is ruled out. More than 10 groups are making measurements with instruments in space, on balloons, and at the South Pole, and within a few years the angular power spectrum will be much better known. Two new space missions have been approved: MAP, to be launched in 2001 by NASA, and COBRAS/SAMBA, to be launched by the European Space Agency in 2004. Each will map CBR anisotropy with more than 30 times the angular resolution of COBE. MAP should determine the angular power spectrum out to multipole number 500, and

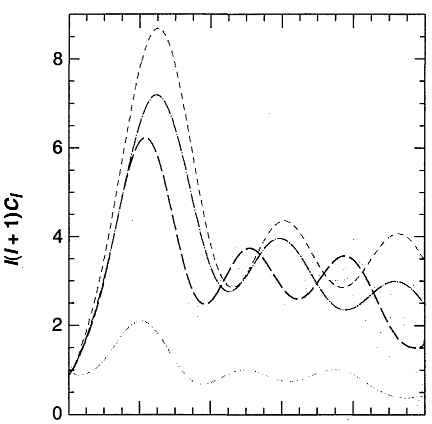

Fig. 9. Predicted angular power spectra of CBR anisotropy for several viable CDM models and the anticipated uncertainty (per multipole) from a CBR satellite experiment similar to MAP (angular resolution of 0.3° and noise level, inverse weight per solid angle, $w^{-1} = (15\ \mu K)^{-2}$ deg^{-2}). From top to bottom the CDM models are: CDM with $h = 0.35$, τCDM with the energy equivalent of 12 massless neutrino species, ΛCDM with $h = 0.65$ and $\Omega_\Lambda = 0.6$, νCDM with $\Omega_\nu = 0.2$, and CDM with $n = 0.7$ (unspecified parameters have their standard CDM values).

COBRAS/SAMBA out to multipole number 2000, each to a precision determined by sampling variance alone. The results should discriminate among the different CDM families (Fig. 9) as well as determining other cosmological parameters.

The location of the first peak in the angular power spectrum (Fig. 9) is an important test of inflation (*69*). All CDM models predict that it lies in roughly the same place. On the other hand, in an open universe (total energy density less than critical) the first peak occurs at a larger value of l (smaller angular scale). Theoretical studies (*70*) indicate that the results of MAP and COBRAS/SAMBA should be able to determine n to a precision of less than 1%, Ω_Λ to a few percent, $\Omega_{\rm total}$ to less than 1%, Ω_ν to enough precision to test νCDM (*71*), the baryon density to less than 10%, and even H_0 to 1%. In addition to testing CDM and inflation, measurements of CBR anisotropy can also be used to infer the value of the inflationary potential and its first two derivatives (*72*), which would provide information about the underlying inflationary model and possibly even insight about the unification of the forces and particles of nature.

The concepts of inflation and CDM are an attempt to extend our knowledge of the universe to within 10^{-32} s of the big bang. The number of observations that are testing the CDM theory is growing fast, and prospects for not only testing the theory but also discriminating among the different CDM models are excellent. If CDM is shown to be correct, an important aspect of the standard cosmology—the origin and evolution of structure—will have been resolved, and a window to the early moments of the universe and physics at very high energies will have been opened.

REFERENCES AND NOTES

1. For example, see P. J. E. Peebles, D. N. Schramm, E. Turner, R. Kron, *Nature* **352**, 769 (1991); M. S. Turner, *Science* **262**, 861 (1993).
2. For example, see M. J. Geller and J. P. Huchra, *Science* **246**, 897 (1989).
3. For example, see P. J. E. Peebles, *The Large-Scale Structure of the Universe* (Princeton Univ. Press, Princeton, NJ, 1980); G. Efstathiou, in *The Physics of the Early Universe*, J. A. Peacock, A. F. Heavens, A. T. Davies, Eds. (Adam-Higler, Bristol, U.K., 1990).
4. G. F. Smoot *et al.*, *Astrophys. J.* **396**, L1 (1992).
5. D. Scott, J. Silk, M. White, *Science* **268**, 829 (1995).
6. For example, see A. Vilenkin, *Phys. Rep.* **121**, 263 (1985); A. Albrecht and A. Stebbins, *Phys. Rev. Lett.* **69**, 2615 (1992); N. Turok, *ibid.* **63**, 2652 (1989); A. Gooding, D. Spergel, N. Turok, *Astrophys. J.* **372**, L5 (1991).
7. P. J. E. Peebles, *Nature* **327**, 210 (1987); *Astrophys. J.* **315**, L73 (1987); R. Cen, J. P. Ostriker, P. J. E. Peebles, *ibid.* **415**, 423 (1993).
8. The power spectrum $P(k) \equiv \langle|\delta_k|^2\rangle = Ak^n$. Scale invariance corresponds to $n = 1$. A is a numerical constant that depends on the model of inflation and its value sets the overall level of inhomogeneity.
9. For example, see E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990), chap. 8.
10. Although there is a general consensus about the basic correctness of primordial nucleosynthesis, there has been debate about the details; see, for example, N. Hata *et al.*, *Phys. Rev. Lett.* **75**, 3977 (1995) and C. Copi, D. N. Schramm, M. S. Turner, *ibid.*, p. 3981. The range for the baryon density we quote here encompasses the full range discussed in the literature.
11. For example, see C. J. Copi, D. N. Schramm, M. S. Turner, *Science* **267**, 192 (1995).
12. M. Fukugita, C. J. Hogan, P. J. E. Peebles, *Nature* **366**, 309 (1993); G. Jacoby *et al.*, *Proc. Astron. Soc. Proc.* **104**, 599 (1992).
13. M. S. Turner, *Physica Scripta* **T36**, 167 (1991).
14. For example, see G. Jungman, M. Kamionkowski, K. Griest, *Phys. Rep.* **267**, 195 (1996).
15. G. G. Raffelt, *ibid.* **198**, 1 (1990).
16. S. D. M. White, C. Frenk, M. Davis, *Astrophys. J.* **274**, L1 (1983).
17. For example, see G. Kauffmann, *Mon. Not. R. Astron. Soc.* **274**, 153 (1995).
18. An overview of CDM is given by G. Blumenthal *et al.*, *Nature* **311**, 517 (1984).
19. For example, see V. Trimble, *Annu. Rev. Astron. Astrophys.* **25**, 425 (1987); M. S. Turner, *Proc. Natl. Acad. Sci. U.S.A.* **90**, 4827 (1993).
20. In addition, inflation predicts a nearly scale-invariant spectrum of gravitational waves, which can lead to CBR anisotropy that is not simple to distinguish from that caused by density perturbations. If the normalization A is determined from CBR anisotropy measurements, as is now done, the level of gravitational radiation must be specified. In our calculations we have taken the possibility of gravity waves into account.
21. S. Dodelson, G. Gyuk, M. S. Turner, *Phys. Rev. Lett.* **72**, 3578 (1994); J. R. Bond and G. Efstathiou, *Phys. Lett. B* **265**, 245 (1991).
22. For example, see S. Parke, *Phys. Rev. Lett.* **74**, 839 (1995); C. Athanassopoulos *et al.*, *ibid.* **75**, 2650 (1995); J. E. Hill, *ibid.*, p. 2654; K. S. Hirata *et al.*, *Phys. Lett. B* **280**, 146 (1992); Y. Fukuda *et al.*, *ibid.* **335**, 237 (1994); R. Becker-Szendy *et al.*, *Phys. Rev. D* **46**, 3720 (1992).
23. A. Einstein, *Sitzungsber. Preuss. Akad. Wiss.* **1917**, 142 (1917). This article is translated into English in J. Bernstein and G. Feinberg, *Cosmological Constants* (Columbia Univ. Press, New York 1986).
24. F. Hoyle, *Mon. Not. R. Astron. Soc.* **108**, 372 (1948); H. Bondi and T. Gold, *ibid.*, p. 252.
25. S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
26. In spite of the fact that it is difficult to find inflationary models that predict precisely $n = 1$, this choice was taken for simplicity.
27. H. Lin *et al.*, *Astrophys. J.* **464**, 60 (1996).
28. CBR anisotropy on the angular scale θ arises primarily because of inhomogeneity on length scales $\lambda \sim 100h^{-1}$ Mpc (θ/deg).
29. J. P. Ostriker, *Annu. Rev. Astron. Astrophys.* **31**, 689 (1993); A. Liddle and D. Lyth, *Phys. Rep.* **231**, 1 (1993).
30. C. L. Bennett *et al.*, *Astrophys. J.* **464**, L1 (1996); K. M. Gorski *et al.*, *ibid.*, p. L11; M. White and E. F. Bunn, *ibid.* **450**, 477 (1995).
31. Computation of both the CBR anisotropy and the level of inhomogeneity today depends on the invisible-matter content and the cosmological parameters and requires that the distribution of matter and radiation be evolved numerically. We have carried out these calculations by integrating the coupled Einstein and Boltzmann equations. For details see, for example, P. J. E. Peebles and J. T. Yu, *Astrophys. J.* **162**, 815 (1970); M. L. Wilson and J. Silk, *ibid.* **243**, 14 (1981); J. R. Bond and G. Efstathiou, *ibid.* **285**, L45 (1984); J. A. Holtzman, *Astrophys. J. Suppl.* **71**, 1 (1989); A. van Dalen and R. K. Schaefer, *Astrophys. J.* **398**, 33 (1992); N. Sugiyama and N. Gouda, *Prog. Theor. Phys.* **88**, 803 (1992); S. Dodelson and J. M. Jubas, *Phys. Rev. Lett.* **70**, 2224 (1993).
32. There are now a variety of tests based on numerical simulations that confront models with data on scales that have become nonlinear (*73*). They represent an important step in testing models, but because the

simulations are so time consuming, they are not very useful in spanning the large sets of parameters we are interested in here.
33. Because the distance d to a given galaxy is determined from its red shift z through Hubble's law, $d = zH_0$. Such surveys are called red shift surveys.
34. J. Peacock and S. Dodds, *Mon. Not. R. Astron. Soc.* **267**, 1020 (1994).
35. S. D. M. White, G. Efstathiou, C. S. Frenk, *ibid.* **262**, 1023 (1993).
36. K. Lanzetta, A. M. Wolfe, D. A. Turnshek, *Astrophys. J.* **440**, 435 (1995); L. J. Storrie-Lombardi, R. G. McMahon, M. J. Irwin, C. Hazard, *ibid.* **468**, 128 (1996); L. J. Storrie-Lombardi, R. G. McMahon, M. J. Irwin, *Mon. Not. R. Astron. Soc.*, in press.
37. Following others, we have used the Press-Schechter formalism to compute the number of clouds that form by a given red shift. See, for example, A. R. Liddle, D. H. Lyth, P. T. P. Viana, M. White, *Mon. Not. R. Astron. Soc.*, in press.
38. C.-P. Ma and E. Bertschinger, *Astrophys. J.* **434**, L5 (1994); A. Klypin, S. Borgani, J. Holtzman, J. Primack, *ibid.* **444**, 1 (1995); A. R. Liddle, D. H. Lyth, R. K. Schaefer, Q. Shafi, P. T. P. Viana, *Mon. Not. R. Astron. Soc.* **281**, 531 (1996).
39. M. S. Turner, G. Steigman, L. Krauss, *Phys. Rev. Lett.* **52**, 2090 (1984); P. J. E. Peebles, *Astrophys. J.* **284**, 439 (1984); G. Efstathiou *et al.*, *Nature* **348**, 705 (1990); L. Kofman and A. A. Starobinskii, *Sov. Astron. Lett.* **11**, 271 (1985).
40. J. G. Bartlett, A. Blanchard, J. Silk, M. S. Turner, *Science* **267**, 980 (1995).
41. R. Cen, N. Gnedin, L. Kofman, and J. P. Ostriker, *Astrophys. J.* **399**, L11 (1992); R. Davis *et al.*, *Phys. Rev. Lett.* **69**, 1856 (1992); F. Lucchin, S. Mattarese, S. Mollerach, *Astrophys. J.* **401**, L49 (1992); D. Salopek, *Phys. Rev. Lett.* **69**, 3602 (1992); A. Liddle and D. Lyth, *Phys. Lett. B* **291**, 391 (1992); J. E. Lidsey and P. Coles, *Mon. Not. R. Astron. Soc.* **258**, 57 (1992); T. Souradeep and V. Sahni, *Mod. Phys. Lett. A* **7**, 3541 (1992); F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, A. Olinto, *Phys. Rev. D* **47**, 426 (1993).
42. M. White, D. Scott, J. Silk, M. Davis, *Mon. Not. R. Astron. Soc.* **276**, L69 (1995).
43. Q. Shafi and F. Stecker, *Phys. Rev. Lett.* **53**, 1292 (1984); M. Davis, F. Summers, D. Schlegel, *Nature* **359**, 393 (1992); J. Primack *et al.*, *Phys. Rev. Lett.* **74**, 2160 (1995); D. Pogosyan and A. A. Starobinskii, *Astrophys. J.* **447**, 465 (1995).
44. L. Krauss and M. S. Turner, *Gen. Relativ. Gravit.* **27**, 1137 (1995); J. P. Ostriker and P. J. Steinhardt, *Nature* **377**, 600 (1995).
45. M. Bolte and C. J. Hogan, *Nature* **376**, 399 (1995).
46. B. Chaboyer, P. Demarque, P. J. Kernan, L. M. Krauss, *Science* **271**, 957 (1996).
47. A. Reiss, R. P. Krishner, W. Press, *Astrophys. J.* **438**, L17 (1995); M. Hamuy *et al.*, *Astron. J.* **109**, 1 (1995); W. Freedman *et al.*, *Nature* **371**, 757 (1994).
48. The time back to the big bang (t_{BB}) depends on H_0, Ω_{matter}, and Ω_Λ; for $\Omega_{matter} = 1$ and $\Omega_\Lambda = 0$, $t_{BB} = \frac{2}{3} H_0^{-1}$, or 13 Gyr for $h = 0.5$ and 10 Gyr for $h = 0.65$. For a flat universe with a cosmological constant, the numerical factor is larger than 2/3.
49. J. Cowan, F. Thieleman, J. Truran, *Annu. Rev. Astron. Astrophys.* **29**, 447 (1991).
50. S. D. M. White *et al.*, *Nature* **366**, 429 (1993); U. G. Briel *et al.*, *Astron. Astrophys.* **259**, L31 (1992); D. A. White and A. C. Fabian, *Mon. Not. R. Astron. Soc.* **273**, 72 (1995); A. E. Evrard, C. A. Metzler, J. F. Navarro, *Astrophys. J.*, in press.
51. A. Babul and N. Katz, *Astrophys. J.* **406**, L51 (1993).
52. J. Bahcall *et al.*, *ibid.* **435**, L51 (1994); C. Flynn, A. Gould, J. Bahcall, in preparation.
53. C. Alcock *et al.*, *Phys. Rev. Lett.* **74**, 2867 (1995); E. Gates, G. Gyuk, M. S. Turner, *ibid.*, p. 3724.
54. E. Gates and M. S. Turner, *Phys. Rev. Lett.* **72**, 2520 (1994).
55. M. Strauss and J. Willick, *Phys. Rep.* **261**, 271 (1995).
56. A. Dekel, *Annu. Rev. Astron. Astrophys.* **32**, 319 (1994).
57. S. Perlmutter *et al.*, *Astrophys. J.* **440**, L1 (1995); B. Leibundgut *et al.*, *ibid.*, in press.
58. S. Perlmutter *et al.*, *ibid.*, in press.
59. For example, see C. S. Kochanek, *Astrophys. J.*, in press.
60. A. Songaila *et al.*, *Nature* **368**, 599 (1994); R. F. Carswell *et al.*, *Mon. Not. R. Astron. Soc.* **268**, L1 (1994); M. Rugers and C. J. Hogan, *Astrophys. J.* **459**, L1 (1996); D. Tytler, X.-M. Fan, S. Burles, *Nature* **381**, 207 (1996); D. N. Schramm and M. S. Turner, *ibid.*, p. 193; S. Burles and D. Tytler, *Science*, in press.
61. For example, see J. E. Gunn and D. H. Weinberg, in *Wide-Field Spectroscopy and the Distant Universe: Proceedings of the 35th Herstmonceaux Workshop*, in press.
62. L. J. Rosenberg, *Particle World* **4**, 3 (1995).
63. D. O. Caldwell, in *Particle and Nuclear Astrophysics in the Next Millennium*, E. W. Kolb and R. D. Peccei, Eds. (World Scientific, Singapore, 1995), pp. 38–50.
64. J. E. Ellis, *Physics World*, July 1994, p. 31.
65. J.-P. Revol, in *Particle and Nuclear Astrophysics in the Next Millennium*, E. W. Kolb and R. D. Peccei, Eds. (World Scientific, Singapore, 1995), pp. 147–159.
66. See, for example, J. N. Bahcall, F. Calaprice, A. B. MacDonald, Y. Totsuka, *Physics Today* **49**, 30 (1996).
67. See, for example, *Proceedings of the Third Workshop on the Tau Lepton, Nucl. Phys. B* (*Proc. Suppl.*) **40** (1995).
68. W. Hu and N. Sugiyama, *Phys. Rev. D* **51**, 2599 (1995).
69. M. Kamionkowski, D. N. Spergel, N. Sugiyama, *Astrophys. J.* **426**, L57 (1994); G. Jungman, M. Kamionkowski, A. Kosowsky, D. Spergel, *Phys. Rev. Lett.* **76**, 1007 (1996); W. Hu and M. White, *ibid.*, in press.
70. L. Knox, *Phys. Rev. D* **52**, 4307 (1995); G. Jungman, M. Kamionkowski, A. Kosowsky, D. Spergel, *ibid.*, in press.
71. S. Dodelson, E. Gates, A. S. Stebbins, *Astrophys. J.*, in press.
72. M. S. Turner, *Phys. Rev. D* **48**, 3502 (1993); E. Copeland *et al.*, *Rev. Mod. Phys.*, in press.
73. For example, A. Klypin, J. R. Primack, and J. Holtzman, *Astrophys. J.* **466**, 13 (1996); S. Ghigna *et al.*, *Astrophys. J.* **437**, L71 (1994); R. Cen, N. Y. Gnedin, and J. P. Ostriker, *Astrophys. J.* **417**, 387 (1993).
74. Supported in part by DOE (at Chicago and Fermilab) and NASA (at Fermilab through grant NAG 5-2788). We thank M. White, A. Liddle, and other participants of the 1995 Aspen Center for Physics workshop, *Inflation: From Theory to Observation and Back*, for many lively and useful discussions.