We plot the probability to measure the correct result $P$ after $N$ applications of the gate $C^L_{ab}$ to the initial state $|\Psi(0)\rangle \propto (|0\rangle^L_a|0\rangle^L_b + |1\rangle^L_a|1\rangle^L_b)$. We have chosen the parameters such that a single gate $C^L_{ab}$ fails with probability $p = 0.09$. Without error correction (dashed line), $N_{op} \simeq 1/p \simeq 11$ gates can be performed reliably, where $P = \exp(-N/N_{op})$. Note that we have used the gate $C^L_{ab}$ instead of $C_{ab}$ (Eq. 5) for the uncorrected case to make comparison easier. With error correction we can perform $N_{op} \simeq 4/p^2 \simeq 500$ gates.

The present scheme can be generalized to more realistic situations. First, it is relevant to extend the present scheme to finite temperature reservoirs for the phonon mode. In this case, there are two decoherence channels corresponding to heating and cooling. To distinguish the corresponding quantum jumps, one has to use the third motional sideband and design $\mathcal{M}_1$ to measure the phonon number (*7*). Second, it is possible to correct for the occurrence of $n$ quantum jumps in one decoherence channel (jump operator $a$) during one gate, which would allow $N = o(1/\epsilon^{n+1})$ operations. For this correction, one has to use the $n$+1th sideband and gate symmetrization as in (*8*).

A remarkable feature of our proposal is that error-testing measurements are performed after the gate operation to correct errors that accumulated during the computation. The overhead required by the scheme is rather moderate: each logical qubit is encoded into two qubits that are stored in the same (four-level) ion. This setup has the advantage that one-qubit gates (for which decoherence is assumed to be negligible) are the same with or without the redundant encoding. On the other hand, implementation of the two-qubit gate requires an overhead of four two-qubit subgates. A proof-of-principle experiment to demonstrate the possibility of correcting effects of decoherence in dissipative quantum dynamics could be performed with three trapped ions, which seems to be attainable with present technology (*5*).

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# Flux Line Lattice Melting Transition in $YBa_2Cu_3O_{6.94}$ Observed in Specific Heat Experiments


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**When a magnetic field penetrates a type II superconductor, it forms a lattice of thin quantized filaments called magnetic vortices. Resistance, magnetization, and neutron diffraction experiments have shown that the vortex lattice of high-temperature superconductors can melt along a line in the field-temperature plane. The calorimetric signature of melting on this line was observed in a high-accuracy adiabatic specific heat experiment performed on $YBa_2Cu_3O_{6.94}$. The specific heat of the vortex liquid was greater than that of the vortex solid.**


When the temperature $T$ of a classical type II superconductor is lowered in a constant magnetic field $H > 0$, it enters a mixed state in which the field partially penetrates the sample and forms a lattice of thin quantized filaments called magnetic vortices. Each vortex carries a quantum of magnetic flux $\Phi_0$. Although the mixed state is superconducting, an electric current generates dissipation if the vortex lattice is allowed to flow. Fortunately, this lattice can be pinned and the zero-resistance ($R$) state restored (up to a critical current) by introducing suitable defects. In any case, the magnetization $M = -\partial F/\partial H$ (where $F$ is the free energy) varies continuously at the border of the mixed state, and the transition is therefore of second order. The specific heat $C$ shows a jump without any latent heat.

The magnetic structure that is obtained upon cooling most high-temperature superconductors through the superconducting phase boundary differs from this picture. The vortices first form a liquid. Although the discussion of the normal state to superconductivity is also controversial because of the effect of thermal fluctuations (*1*), the main interest has concentrated on the transition in the superconducting state—the freezing of the vortex lattice.

Vortex lattice freezing or melting has been investigated by several groups using mainly resistance, magnetization, and neutron diffraction experiments (*2–7*). Phase diagrams were constructed in the ($H$,$T$) plane and correspond to the usual ($p$,$T$) phase diagram (where $p$ is pressure) for solids and liquids made of atoms or molecules. An interesting difference is the possibility of varying the density of vortices over orders of magnitude by simply changing the magnetic field. The melting line for $Bi_2Sr_2CaCu_2O_8$ (Bi-2212) was found to join smoothly the superconducting transition temperature [$T_c = 90$ K, induction field $B = 0$] and a critical point ($T_{cr} = 37.8$ K, $B_{cr} = 380$ G) (*3*). Hall probes revealed a jump in $M$ that increased from $4\pi\Delta M \approx 0.2$ G just above $T_{cr}$ to $\approx 0.4$ G just below $T_c$. In $YBa_2Cu_3O_7$ (Y-123), the melting line could be followed from the superconducting transition temperature ($T_c = 93$ K, $B = 0$) up to the point ($T = 78$ K, $B = 8$ T) (*5, 6*). The possible existence of a critical point at higher fields was not clarified. A jump in $M$ was also observed with SQUID (superconducting quantum-interference device) magnetometry that decreased smoothly from $4\pi\Delta M \approx 0.3$ G at 82 K to zero at $T_c$ (*5, 6*). These jumps in $M$ in both Bi-2212 and Y-123 were considered as a thermodynamic argument in favor of a first-order transition.

The full proof for the existence of a first-order transition at the melting temperature $T_m(H)$ must include the observation of the corresponding discontinuity in the entropy $S = -\partial F/\partial T$, or equivalently, of a peak in $C$ with an integrated latent heat $\Delta Q = T_m\Delta S$. Because the number of vortices is directly proportional to the induction field $B$, then, due to the much higher melting fields in Y-123, it should be possible, for a given temperature near $T_c$, to observe an effect that is about two orders of magnitude greater for Y-123 than for Bi-2212; therefore, we focused on Y-123. The Clausius-Clapeyron equation, $\Delta S/\Delta M = -dH_m/dT$,


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together with the measured slope of the transition line $H_m(T)$, predicts a melting entropy $\Delta S$ on the order of $0.6k_B$ per vortex per layer (6), where $k_B$ is Boltzmann's constant. The $\Delta S$ that should be observed in calorimetrists' units is $\approx 10^{-4}$ J/(K gram atom) for $B = 4$ T, taking a separation of 12 Å between $CuO_2$ bilayers and a volume of 8 $cm^3$ per gram atom. Because the total $C/T$ of Y-123 at $T_m$ is about 0.1 J/($K^2$ gram atom), the net expected effect is a peak rising 1% above the background if the transition is smeared over 0.1 K. State-of-the-art experiments have a resolution of 0.01%, but no positive result has previously been reported.

We present here a series of high-accuracy specific heat measurements performed on 0.3 g of twinned crystal of Y-123 in magnetic fields from 0 to 14 T. No first-order peak of the predicted amplitude was found, but a step was observed at the temperature $T_m(H)$. Its amplitude represents $\approx$0.1% of the total specific heat and constitutes the thermal signature of melting.

The Y-123 crystal was synthesized by the traveling solvent floating zone melting technique. The adiabatic calorimeter works in the continuous-heating mode, with typical heating rates of 10 to 15 mK/s. The smoothness of the data is on the order of 0.01%, and the reproducibility with respect to the field is better than 0.05%. Typical $C/T$ curves for Y-123 in the region of $T_c$ are shown in Fig. 1. The full set of data includes the fields 0 to 8 T in increments of 0.5 T, and 14 T. The field, which is parallel to the crystallographic c axis, is varied only in the normal state. The main feature at $T_c = 92.5$ K is due to the superconducting transition; note that it represents no more than 3 to 4% of the total $C/T$. This transition is progressively broadened as the field is increased. It is also shifted, as shown by the characteristic temperatures where $|\partial(C/T)/\partial T|$ is maximum and $|\partial C/\partial H|$ is maximum (see Fig. 3, which is discussed below). We focus on the left part of Fig. 1 (shown by arrows) where the vortex lattice melts. The difference $C(H,T) - C(0,T)$, where neither $C(H)$ nor $C(0)$ have been smoothed or corrected for experimental drifts, is expanded in Fig. 2A. For a given field, this quantity remains flat and structureless on the low-temperature side up to a point where it suddenly rises (over a ~0.5 to 1 K interval) up to what might be called a plateau. The amplitude of the step is on the order of 0.1% of the total C. The difference then goes strongly negative at the superconducting transition.

The steps are easily recognized in Fig. 2A, but their positions can be refined by looking at the peak of the differential $\delta C = C(B + \delta B/2) - C(B - \delta B/2)$ with $\delta B = 0.5$ T (Fig. 2B). This peak defines the temperatures $T_m(B)$ that are plotted in Fig. 3. Preliminary M measurements convinced us that $T_m(B)$ lies in the reversible regime within the resolution of our SQUID magnetometer. The test of Maxwell's relation $\partial S/\partial H = \partial M/\partial T$ with various superconductors cautioned us, however, that the notion of reversibility in the ideal thermodynamic sense may be a delicate issue (8).

The lower full line in Fig. 3 is the melting line strictly as reported by Welp *et al.* (6), $B(T) = 99.7[1 - T_m/T_c(0)]^{1.36}$. The latter, which is quite similar to that given by Liang *et al.* (5), is based on measurements of R and M. The coincidence of data convincingly shows that the same phenomenon causes the anomalies in M and C.

At variance with the conclusion drawn from most M measurements, the simplest interpretation of the step in $C(H,T)$ at $T_m$ is in terms of a second-order transition. The sign of the step indicates that the C of the vortex liquid is greater than that of the vortex solid. This is a common property for melting, caused by the onset of additional degrees of freedom (anharmonic modes) in the liquid phase.

It is believed that disorder might drive the liquid→lattice, first-order transition predicted by theory into a liquid→glass, second-order transition (9). Our results could be made compatible with this picture if our sample contained enough pinning centers. Detwinning of Y-123 crystals was claimed to be essential for a successful observation (5, 6). However, preliminary experiments on one untwinned, high-purity flux-grown crystal have also not shown any latent heat (*10*). One might argue that the field is radially distributed over the sample because of geometrical barriers (*11*), thereby preventing the observation of a well-defined latent heat. The resulting correction, on the order of the lower critical field $H_{c1}$, is not believed to be significant in the large fields used for Y-123. Finally, one might wonder about the effect of a gaussian distribution of $T_m$ caused by inhomogeneity on a supposedly first-order transition. The step in $M(T)$ would be broadened. The slope $\partial M/\partial T$ near $T_m$ would reflect the shape of the distribution. The

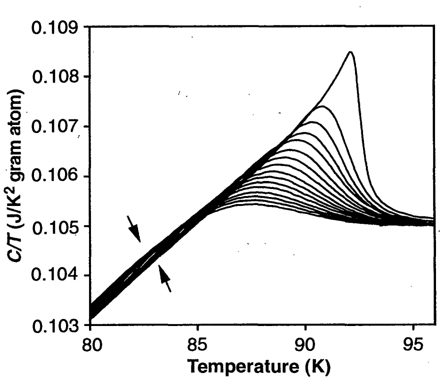


**Fig. 1.** Total specific heat $C$ of $YBa_2Cu_3O_{6.94}$ divided by temperature $T$ as a function of $T$ in the vicinity of the transition temperature $T_c = 92.5$ K, measured in magnetic fields of 0 to 8 T in increments of 0.5 T. Note the offset of the scales. The arrows point to the variations that are expanded in Fig. 2.

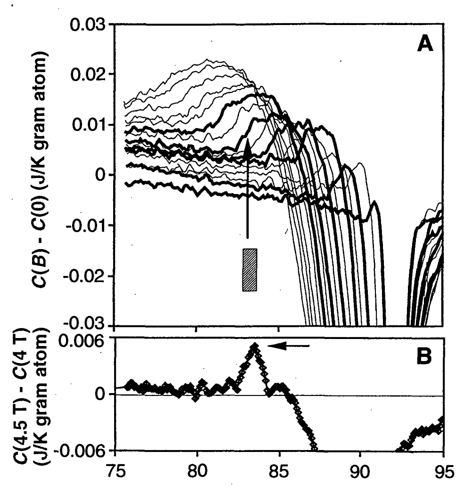


**Fig. 2.** (**A**) Remaining $C$ after having subtracted the zero-field curve $C(0,T)$ from $C(B,T)$ for fields from 0.5 (bottom) to 8 T (top). The fields 0.5, 1.5, 2.5, 3.5, 4.5, and 5.5 T are enhanced for clarity; the arrow points to the step in 4.5 T. The shaded area shows the latent heat (smeared over 1 K) that would be expected according to (6) on top of the step at 4.5 T. (**B**) Incremental variation of $C$ for a field change from 4.0 to 4.5 T. The small positive peak defines the temperature $T_m$ for $B = 4.25$ T. This determination and measurements at other fields are reported in Fig. 3.

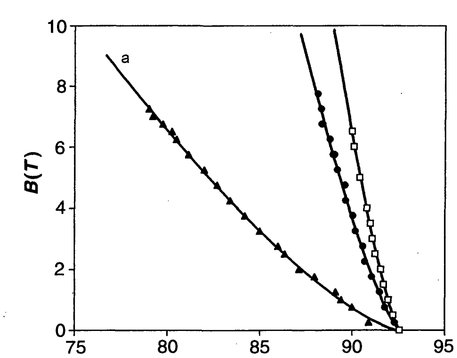


**Fig. 3.** Curve labeled a is the melting line in the $(B,T)$ plane as given by (6) for their crystal. Solid triangles, position of the $C$ steps in the present experiment. Open squares, bulk superconducting transition temperature $T_c(B)$ given by the criterion of the maximum descending slope of $C/T$ versus $T$. Solid circles, alternative estimation of $T_c(B)$ given by the point where $C$ undergoes the largest variation when the field is changed incrementally. All data sets can be described by the critical exponent of the three-dimensional XY universality class, $B \propto [1 - T/T_c(0)]^{4/3}$ (smooth lines through the data).

curvature $\partial^2 M/\partial T^2 = \partial(C/T)/\partial H \cong (T_m \delta H)^{-1}[C(H + \delta H) - C(H)]$ would then show an oscillation with both positive and negative parts. This was not observed (Fig. 2B). The present measurements suggest a second-order transition, in agreement with recent results of torque magnetometry (*12*).

The amplitude of the step is $\Delta C = (6.6 \pm 15\%)$ mJ/(K gram atom). Ehrenfest's relation determines the corresponding change of slope of the equilibrium magnetization by $\Delta(\partial M/\partial T) = -(dH_m/dT)^{-1}\Delta C/T_m$. With $B_m = 4.2$ T, $T_m = 83.5$ K, and $T_c = 92.5$ K, we obtain $4\pi\Delta(\partial M/\partial T) = (0.20 \pm 15\%)$ G/K, to be compared with the experimental value $(0.2 \pm 25\%)$ G/K in the same field (*6*). The break in the slope is therefore confirmed to be an equilibrium property.

The amplitude of the step is nearly constant from 1 to 6 T. The number of vortices is proportional to $B$, and $T_m$ decreases with $B$; therefore, the C jump per flux line increases when $T_m$ approaches $T_c$. This may result from critical fluctuations (*3*), variations of the length of the correlated element in the flux line, or both. Below 1 T, the jump decreases and becomes sharper; above 6 T, it progressively transforms into a smooth crossover. Measurements at 14 T do not show any sharp structure on the extrapolated melting line. This sets an upper limit to the possible existence of a critical point terminating the melting line.

One of the signatures of a first-order transition is given by the phenomenon of superheating and supercooling. An observation of the heating and cooling rates in our experiment sets an upper limit of 0.2 K for the hysteresis. Preliminary measurements with an ac technique on an untwinned single crystal bring this limit down by an order of magnitude (*10*). No hysteresis was found in M measurements (*6*). These observations suggest a second-order transition.

The quantity $C(H,T) - C(0,T)$ was measured for Bi-2212 in fields that lie above the critical point [see figure 13a of (*13*)]. These data give an idea of the shape of the background near $T_c$ in the absence of freezing. There is no positive overshoot such as that seen in Fig. 2A for Y-123.

The nature of the thermodynamic transition between the different vortex phases of high-temperature superconductors is still under discussion (*12*). Our measurements show a second-order step on the transformation line that had been determined independently by measurements of $R$ and M. This second-order step constitutes the thermal signature of the melting of a vortex solid. This situation is analogous to the melting of a usual atomic lattice, with vortex modes playing the role of phonons (*14*). Our results establish that the transition line of interest and the break in the slope of M are equilibrium properties (*15*).

*Note added in proof*: Using differential thermal analysis, Schilling *et al.* (*16*) have observed a latent heat on the vortex lattice melting line of an untwinned Y-123 crystal. Using adiabatic calorimetry and a twinned, fully oxidized 18-mg single crystal grown in $BaZrO_3$, we observed a specific heat peak rising up to ≈1% above the $C/T$ background in $B = 8$, 11, and 14 T on the vortex lattice melting line (*17*). The area under the peak corresponds to a melting entropy on the order of 0.5 $k_B$ per vortex per layer up to the highest field. This result might confirm and extend Schilling's differential thermal analysis experiments.

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$$\int_T^{>T_c} dT[C(H) - C(0)]/T \text{ and } \int_0^H dH(\partial M/\partial T)$$

agree from $T = 120$ to 95 K in $\mu_0 H = 5.5$ T for the 116-K superconductor Tl-1223 [figure 27 of G. Triscone, A. Junod, R. E. Gladyshevskii, *Physica C* **264**, 233 (1996)], as expected in thermodynamic equilibrium. However, both estimations differ at lower temperatures, although $M$ remains apparently reversible down to 65 K [G. Triscone and A. Junod, unpublished material]. The latter condition is not sufficient. This is also observed for the classical superconductor $Nb_{77}Zr_{23}$ [B. Revaz, unpublished material].
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# Allelic Diversity and Gene Genealogy at the Self-Incompatibility Locus in the Solanaceae


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The self-incompatibility (*S*) locus of flowering plants offers an example of extreme polymorphism maintained by balancing selection. Estimates of recent and long-term effective population size ($N_e$) were determined for two solanaceous species by examination of *S*-allele diversity. Estimates of recent $N_e$ in two solanaceous species differed by an order of magnitude, consistent with differences in the species' ecology. In one species, the evidence was consistent with historical population restriction despite a large recent $N_e$. In the other, no severe bottleneck was indicated over millions of years. Bottlenecks are integral to founder-event speciation, and loci that are subject to balancing selection can be used to evaluate the frequency of this mode of speciation.


Balancing selection can maintain large numbers of alleles within populations (*1*, *2*), and this polymorphism persists much longer than does selectively neutral genetic variation (*3*, *4*). At the *S* locus, the transmission rate of an allele is inversely proportional to its frequency, and populations commonly harbor as many as 30 to 50 alleles (*5*, *6*). Alleles at this locus can be extremely old, as reflected by their extreme sequence variability (*7*) and by phylogenetic analyses that have found that an allele from one species may be more closely related to an allele from another species or genus than to other alleles from the same species, a pattern called trans-specific evolution (*3*, *8*). Thus, historical events such as changes


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