The Seismic Structure of the Sun

D. O. Gough, A. G. Kosovichev, J. Toomre, E. Anderson, H. M. Antia, S. Basu, B. Chaboyer, S. M. Chitre,
J. Christensen-Dalsgaard, W. A. Dziembowski, A. Eff-Darwich,
J. R. Elliott, P. M. Giles, P. R. Goode, J. A. Guzik, J. W. Harvey,
F. Hill, J. W. Leibacher, M. J. P. F. G. Monteiro, O. Richard,
T. Sekii, H. Shibahashi, M. Takata, M. J. Thompson, S. Vauclair, S. V. Vorontsov

Global Oscillation Network Group data reveal that the internal structure of the sun can be well represented by a calibrated standard model. However, immediately beneath the convection zone and at the edge of the energy-generating core, the sound-speed variation is somewhat smoother in the sun than it is in the model. This could be a consequence of chemical inhomogeneity that is too severe in the model, perhaps owing to inaccurate modeling of gravitational settling or to neglected macroscopic motion that may be present in the sun. Accurate knowledge of the sun's structure enables inferences to be made about the physics that controls the sun; for example, through the opacity, the equation of state, or wave motion. Those inferences can then be used elsewhere in astrophysics.

One of the principal purposes of the Global Oscillation Network Group (GONG) project is to determine the internal structure of the sun. Helioseismology is used to model the stratification of density ρ and the internal stress that supports the star, as well as the relation between them when they are adiabatically perturbed (1). In the absence of a magnetic field or small-scale turbulence, the stress is the gradient of the pressure p. Then perturbations of ρ and p are related via the adiabatic exponent γ_1 (2). The variation of these quantities with position is determined by seismology and constitutes what we call the seismic structure of the sun. One must use properties of theoretical solar models for inferring other quantities such as temperature (2). Theoretical models also provide a reference

D. O. Gough, J. R. Elliott, and T. Sekii are with the Institute of Astronomy, University of Cambridge, CB3 0HA, UK. A. G. Kosovichev and P. R. Giles are with HEPL, Stanford University, Stanford, CA, USA. J. Toomre is at JILA, University of Colorado, Boulder, CO, USA. E. Anderson, J. W. Harvey, F. Hill, and J. W. Leibacher are at the National Solar Observatory, Tucson, AZ, USA. H. M. Antia and S. M. Chitre are at the Tata Institute for Fundamental Research, Bombay, India, S. Basu and J. Christensen-Dalsgaard are at the Theoretical Astrophysics Centre, Aarhus University, Denmark. B. Chaboyer is at the Canadian Institute for Theoretical Astrophysics, Toronto, Canada. W. A. Dziembowski is at the Copernicus Astronomical Center, Warsaw, Poland. A. Eff-Darwich is at the Instituto Astrofisico de Canarias, Tenerife, Canary Islands. P. R. Goode is at the New Jersey Institute of Technology, Newark, NJ, USA. J. A. Guzik is at the Los Alamos National Laboratory, Los Alamos, NM, USA. M. J. P. F. G. Monteiro is at the University of Oporto, Postugal. O. Richard and S. Vauclair are at the Observatoire Midi-Pyrenees, Toulouse, France. H. Shibahashi and M. Takata are in the Department of Astronomy, University of Tokyo, Tokyo, Japan. M. J. Thompson and S. V. Vorontsov are at Queen Mary and Westfield College, University of London, London, UK.

against which to compare the seismic model.

The traditional manner of inferring the structure of the sun is to calibrate a theoretical model; that is, to adjust a set of uncertain parameters that specify the model until a best fit with the data is obtained. Difficulty arises when the model cannot be adjusted to fit all the data within the estimated measurement errors, which indicates a fundamental error in the model. Various techniques, known as inverse methods, consider a wider class of possible structures by relaxing some of the basic assumptions on which the theoretical models are based (3). By so doing, one can come closer to explaining what the data might imply and hence obtain a representation of the sun that is in better accord with those data.

Many inverse methods seek differences between certain aspects of the sun and a theoretical model. If the structure of the model is close to that of the sun, the equations relating the two can be simplified by linearization. By a sequence of refinements, it has been possible to produce a model that in many respects is very close to the sun (4).

Fig. 1. The first two images depict acoustic ray paths for modes with n = 20, $\ell = 1$ (**A**) and n = 20, $\ell = 2$ (**B**), which have similar frequencies but penetrate to different depths. The extent to which the structure of the sun influences their free



quencies is represented by the intensity of the brown and red shading. (C) Is the result of subtraction of the red shading from the brown, the green intensity representing the sensitivity to the frequency difference. The greatest intensity is in the region between the lower limits (caustics) of the two ray paths in (A) and (B).

That model now serves as a reference with which to compare the sun and as a guide to interpretation of the differences we find.

Analysis of the frequencies of modes of oscillation is one means whereby the structure of the sun can be inferred. One can also study the shapes of the oscillating disturbances—the so-called eigenfunctions—or one can investigate, over a limited region of the sun, the propagation of the component waves that constitute those eigenfunctions (5). Such studies are best suited to the investigation of lateral inhomogeneity. Nevertheless, most of the inferences to date have been obtained from global mode frequencies, and it is accordingly to these that we restrict our attention in this article.

Seismic Waves

Acoustic seismic waves propagate through the solar interior along ray paths that are almost in planes through the center of the sun (Fig. 1). After reflection at the surface, the waves propagate downward, to be refracted by the sound-speed gradient back to the surface, where they are reflected again. The paths are not closed, so that after many reflections the waves sample essentially the entire region outside a central zone of avoidance and therefore provide a global diagnostic of that region. Indeed, if there were no attenuation by dissipation, a single ray would fill the accessible space. The waves of particular interest are those whose frequencies are such that on neighboring paths the waves are essentially in phase. This leads to constructive interference and the formation of a resonant mode of oscillation with a well-defined frequency (6).

For a wave to be observed, the disturbance must pass from the interior of the sun to the photosphere through the ill-understood surface layers that influence the oscillation frequencies in a substantial but partly unknown way. That influence must be eliminated from the data.

The detailed geometry of the ray paths, and consequently the values of the resonant frequencies, are determined principally by the variation of the sound speed c through the sun (7). Broadly speaking, the extent to which any given region of the sun influences the resonant frequency is proportional to the time spent in that region by an imaginary point traveling with the wave (8). This is represented in Fig. 1, A and B, by the intensity of the brown and red shading. The shading is dense near the surface of the star, where the wave speed is low. It is also relatively dense near the edge of the zone of avoidance, for although the wave speed is relatively high, the ray density is large because the imaginary point passes through the region many times. Indeed, the ray density is formally infinite at the boundary of the zone of avoidance, to which the rays are tangent. This boundary is called a caustic.

The difference between a measured oscillation frequency of the sun and that of a corresponding mode of a theoretical model can be represented as an average of the difference between the solar and the model wave speeds (9). However, the implications of such averages are not easy to comprehend, because they are made up of contributions from many parts of the sun. What we would prefer is to be told the actual sound-speed difference δc at each point, but that is not possible. What is possible, however, is to be given a sequence of averages, each of which is localized in space. To understand how such an average is obtained, consider the difference between the frequencies of the modes in Fig. 1, A and B: It provides an average of the sound-speed difference weighted with the difference between the corresponding weight functions (Fig. 1C). The new weight function is greatest in the vicinity of the caustics, there having been substantial cancellation elsewhere. Thus, the frequency difference provides localized information about the sound speed. A sequence of such averages can be thought of as a blurred representation of the function δc . More highly localized averages can be obtained from appropriate combinations of a greater number of frequencies. With enough modes, one can eliminate the contribution from the surface, whereas in Fig. 1C cancellation was far from complete. Some examples of well-localized weight functions are shown in Fig. 2A.

One can instead represent the soundspeed difference in terms of a predetermined set of functions, choosing the linear combination that best fits the data. Because the emphasis here is on reproducing the data, the method is akin to the calibration of solar models, except that the representation is not constrained to satisfy the equations that govern those models. Moreover, the number of adjustable parameters is typically much greater. When one has a wide range of modes such as those in the GONG dataset, the value of the resulting function at any point is an average of the actual sound-speed difference, which is usually localized in the vicinity of that point (10).

A third technique has the advantage of not relying on a reference model (Fig. 2B). If the structure of the sun were known above the caustic of the shallower mode, the frequency of that mode could be calculated. The structure of the more deeply penetrating wave could be calculated above the caustic of the shallower mode, and its frequency would then be calculable in terms of the unknown sound speed in the thin region between the two caustics. A measurement of the frequency would therefore determine the sound speed averaged over this thin region. From the frequencies of a succession of modes that penetrate more and more deeply, it is evident that in principle one can build up a picture of the sound-speed variation (11).

Inferences from GONG Data

In Fig. 3, we plot the square of the sound speed in the sun against radius r, obtained from GONG data, as well as that in a reference model (12). The agreement is



Fig. 2. (A) Localized averaging kernels, which weight averages of sound-speed differences between the sun and a theoretical reference model. (B) Portions of the ray paths depicted in Fig. 1A (blue) and Fig. 1B (red). The dotted circles are the corresponding caustics.

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close. Only near the base of the convection zone and in the energy-generating core are there discrepancies. The former is evident in the enlargement of the vicinity of the base of the convection zone (inset, Fig. 3).

The adiabatic stratification in the sun appears to penetrate about 0.002R more deeply than in the model (13) (R is the radius of the sun). Also, the values of u = p/ ρ , and consequently c^2 (2), converge quickly at greater depths (Fig. 4). Part of the difference in u between the sun and the model could be associated simply with the fact that the model convection zone may be too shallow. However, the excess u caused by that property is of lesser magnitude than that in Fig. 4 and extends more deeply (3). The small positive value of $\delta u/u$ between 0.3R and 0.6R might therefore be accounted for in this way, but the relatively sharp bump between 0.6R and 0.7R cannot. The decrease in *u* locally may indicate that immediately beneath the convection zone the accumulation of helium, which augments the mean molecular mass μ (2), has been overestimated in the reference model. This is consistent with recent computations (14-17), some results of which are compared with the sun in Fig. 5. The bump could in principle have been produced by an opacity error that drops abruptly to zero immediately beneath the base of the convection zone. However, such a fortuitous occurrence is unlikely.

The discrepancy in the core is the third prominent feature. Most secure is the negative region of $\delta u/u$ between about 0.1R and 0.2R, which implies that the variation of uitself is flatter than in the model (see Fig. 3). Once again, this would be a symptom of



Fig. 3. The dashed curve is the square of the spherically averaged sound speed in the sun. The solid curve corresponds to a standard theoretical model. The magnitudes of the slopes of the curves are lower immediately beneath the convection zone, where the temperature gradient is too small to drive the instability. The inset shows that the convectively unstable region of relatively high slope extends somewhat more deeply into the sun than it does in the model.

there being too steep a composition gradient in the model. The density inversion (Fig. 4) is consistent with this interpretation: The regions of relatively steep positive slope in $\delta\rho/\rho$ in the core and immediately



Fig. 4. Relative differences $\delta u/u$ (**A**) and $\delta \rho/\rho$ (**B**) between the sun and the standard model with gravitational settling of helium and heavy elements (3), where $u = \rho/\rho$.

beneath the convection zone imply that the magnitude of the (negative) gradient of density is too high in the model (18).

A certain amount of turbulent mixing reduces the discrepancy immediately beneath the convection zone (Fig. 5, A and B). We have constructed models with somewhat different helium redistribution that reduce the discrepancy further, but this does not imply that mixing is necessarily the solution. A model that also well represents the sun near the base of the convection zone (Fig. 5C) has no such mixing but instead suffers mass loss during the course of its evolution. The upward flow of material into the convection zone, and subsequently out into the solar wind, counteracts the settling of helium, leaving a smaller helium concentration between 0.6R and 0.7R. We estimate the helium abundance to be 0.248 \pm 0.005 in the convection zone where the stratification is adiabatic. This value is similar to previous estimates (19).

Deviations from spherical symmetry split the degeneracy of the mode frequencies with respect to azimuthal order m. The component of the splitting that is odd in mis produced by those aspects of rotation that depend on odd powers of the angular velocity Ω , whereas the even component is produced by everything else (including phenomena such as centrifugal force, which depend on even powers of Ω and which cannot distinguish east from west). Here we consider only the even splitting, leaving discussion of rotation to (20).

We determine the component of the deviation from spherical symmetry that is axisymmetric about the rotation axis. Only processes that produce a deviation in the wave propagation speed, or a distortion to the shape of the cavity within which the waves propagate, influence the frequencies of the modes. Wave speed can be modified by sound-speed deviations and by a magnetic field, or the wave can be advected by the component of large-scale material flow in meridional planes (21). Unfortunately, one cannot distinguish between them by seismic frequency analysis alone (22). Therefore, we simply express the outcome of our analysis as a scalar wave-speed variation. Inversions are carried out in a manner analogous to those for the spherically averaged structure, except that now the latitudinal dependence of the waves must be taken into account (23).

The only significant aspherical variation is confined to a shallow layer immediately beneath the solar surface (Fig. 6). This finding is consistent with previous inferences (24). Moreover, the variation of wave speed with latitude is very similar to the brightness temperature of the solar atmosphere, which confirms previous findings (24). The wave-speed variation (Fig. 7) is less well resolved than the spherically averaged sound speed (Figs. 3 through 5) and is therefore less reliable. This is because it depends on very small frequency differences, which are of order 0.1 mHz, rather than on the full mode frequencies. As GONG continues, the results will improve. However, the precision will be limited by the structural changes that the sun undergoes in the course of the solar cycle.

There is no significant asphericity (Figs. 6 and 7) beneath $r \simeq 0.9R$. Therefore, we find no evidence for a deeply seated thermal perturbation or magnetic field. If there were a field concentrated in a layer of thickness 0.1R, say, as some dynamo theorists have postulated (25), we could set an upper bound to its average intensity over that layer of a few tens of teslas. That bound is not inconsistent with dynamo models (25).



Fig. 5. Relative differences $\delta u/u$ between *u* in the sun and in various theoretical models. The theoretical models are (**A**) a model with weak mixing (*15*), presumed to be generated by rotationally induced turbulence; (**B**) a similarly mixed model (*16*); and (**C**) a model with mass loss (*17*). In order to produce a homogeneous comparison, the inversions are of the frequency differences v_{∞}

 $-\nu_{\rm s} - (\nu_m - \nu_{\rm o})$, where ν_{\odot} is solar frequencies, $\nu_{\rm s}$ is the frequencies of the standard solar model used in Fig. 4, and ν_m and ν_o are frequencies computed with the same computer code with and without mixing (or mass loss), respectively. Thus, they represent the effect incorporating mixing (or mass loss) into the reference model used in Fig. 4.

We have made two suggestions that might possibly account for the discrepancies in the sound speed: mass loss and material mixing. Both require that material now in the convection zone was previously at higher temperature. That could have caused the light chemical elements Li and Be to have been partially destroyed by nuclear reactions. Both possibilities have been proposed before to explain the observed Li and Be deficiencies in the atmospheres of the sun and other similar stars; now we have evidence that one of them might be correct. The model in Fig. 5C has lost 10% of its mass, most of it in the first 109 years, and it reproduces the observed Li deficiency in the solar photosphere. However, there are also plausible mechanisms for material mixing, including weak convective overshooting (26), nonlinear wave transport (27), rotationally induced shear turbulence (28), and



-0.0002 -0.0001 0.0000 0.0001 0.0002 Sound speed perturbation (δ*c*/*c*)

Fig. 6. Effective sound-speed deviation from the spherical average.



Fig. 7. Relative sound-speed deviation from the spherical average, plotted against radius, at the equator (continuous curve), latitude 45° (dotted lines), and the poles (dashed lines). Thick lines represent the expected solutions; thin lines deviate from those solutions by ± 1 SD of the estimated data errors.

Ekman circulation (29). Distinguishing the different possibilities will require a more highly resolved picture of the transition at the base of the convection zone (30), including the shear in the angular velocity (20).

The discrepancy in the energy-generating core might also be a symptom of macroscopic motion, which transports the products of the nuclear reactions from their sites of production. That would modify the neutrino emission rates and thereby change the status of the solar neutrino problem, despite evidence that at least part of the problem lies in elementary-particle physics (31). It would also lengthen the life expectancy of the sun, by replenishment of spent hydrogen fuel. The implications are far-reaching; for example, if other stars behave similarly, the conflict between the age estimates for globular clusters and some lesser age estimates for the universe would be exacerbated. Such motion would also transport angular momentum and would therefore leave a signature in the variation of angular velocity in the core.

REFERENCES AND NOTES

- Except in the surface layers of the sun, the characteristic cooling time is much longer than the periods of the selsmic waves, so the wave motion is essentially adiabatic. Near the surface, the sun is highly turbulent and is not well understood.
- 2. The adiabatic exponent γ_1 is the thermodynamic quantity ($\partial \ln p/\partial \ln p \right)_s$, the partial derivative being taken at constant specific entropy s. It determines the so-called sound speed *c* according to $c^2 = \gamma_1 u$, where u = p/p. For a perfect gas, which provides a guide to the equation of state of solar material, $u = T/\mu$, where *T* is temperature and μ is the mean "molecular" mass of the material (that is, mean mass per particle—atom or ion—measured in atomic units). Because chemical composition, and therefore μ , are not well determined in the sun, in order to infer *T* one must consider the balance of thermal energy production against energy transport, neither of which is reliably understood, particularly the latter. In Figs. 4 and 5 we display relative differences in *u*, except in the ionization zones of hydrogen and helium near the surface of the sun ($r \ge 0.95R$), $\gamma_1 \simeq 5/3$, and $\delta u/u \simeq \delta c^2/c^2$.
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- 6. J. B. Keller and S. I. Rubinow, Ann. Phys. 9, 24 (1960); D. O. Gough, in Astrophysical Fluid Dynamics, J. P. Zahn and J. Zinn-Justin, Eds. (North-Holland, Amsterdam, 1993), pp. 399–560. In a sphere, such as the sun, the phases of waves on adjacent planes containing the center of the sphere must also be in appropriate relative phase for resonance to occur.
- 7. Frequencies also depend on the variation of density

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but to a lesser extent than they depend on sound speed [see (9)]. That time is inversely proportional to the magnitude

- That time is inversely proportional to the magnitude of the group velocity and is directly proportional to the relative density of ray-path segments.
- Because resonance results from constructive phase coherence, it is the phase speed that is relevant here. In a uniform fluid, the group and phase speeds have the same value, c, which satisfies $c^2 = \gamma_1 \rho / \rho$. But in a medium stratified under gravity, such as the sun, density gradients cause the two speeds to differ. A magnetic field would also contribute to the difference. Here the term "wave speed" always means phase speed, which is higher than the magnitude of the group velocity. Except near the surface of the sun, both the phase and the group speeds are almost the same, and, accordingly, we often use the term "sound speed" here to denote phase speed, because it is more familiar. When comparing the sun with models, we use c^2 or $u = p/\rho$ rather than c because, being proportional to T/μ , it is more readily comprehended
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- This technique has been carried out with only asymptotic representations of the resonance conditions [for example, see J. Christensen-Dalsgaard et al., Nature 315, 378 (1985)].
- As discussed earlier in the text, what the seismic data 12. give us are averages of the structure variables. The difference δv_i between the frequencies of a mode (labeled i) of the sun and of the reference model can be written $\delta v_i = \int K'_{c2,\gamma_i} \delta \ln c^2 dr + \int K'_{\gamma_i,c^2} \delta \ln \gamma_1 dr$. The data kernels K'_{c2,γ_i} and K'_{γ_i,c^2} are functions of the eigenfunctions of oscillation, and examples of the former are represented in brown and red in Fig. 1. Localized averages of the relative differences in c2 are obtained from $\int \mathscr{K}_{c^2,\gamma_1} \delta \ln c^2 dr = \sum \alpha_i \delta \nu_i - \int \mathscr{K}_{\gamma_1,c^2} \delta \ln \gamma_1 dr$ by neglecting the second term on the righthand side, where the averaging kernel $\Re_{c^2,\gamma_1}(r; r_0) =$
 $$\begin{split} &\Sigma \alpha_i r_0 K_{2,\gamma_1}^{(\prime)} \text{ is concentrated near } r = r_0 \text{ and } \mathcal{H}_{\gamma_0,\mathcal{C}}^{\prime} \\ &= \Sigma \alpha_i K_{1,\mathcal{C}}^{\prime,2} \text{ is everywhere small. Reference (3) explains how the coefficients } \alpha_i \text{ are computed. Example:} \end{split}$$
 ples of averaging kernels are shown in Fig. 2A. The subsidiary variable to c^2 , here γ_1 , could be any function of the seismic structure that is independent of c^2 , such as p. We carried out inversions for several different pairs of variables to confirm the robustness of our inferences against contamination by the neglected integral of the second variable. The averaging kernels can be dangerously large very near the surface, where all the data kernels are large, particularly when leastsquares frequency-fitting techniques are used to construct α_i. Indeed, it is partly for this reason that naïve fitting of raw frequencies can be misleading. To obviate contamination by surface effects, one subtracts from all δv_i an arbitrary function of v_i divided by the modal inertia, which is the functional form of any sur-
- face uncertainty [see (3)].
 13. That sets the radius of the base of the adiabatically stratified part of the convection zone at about 0.709*R*. This value is somewhat less than the value 0.713*R* obtained previously by J. Christensen-Dalsgaard, D. O. Gough, and M. J. Thompson [*Astrophys. J.* 378, 413 (1991)], and by A. G. Kosovichev and A. V. Fedorova [*Sov. Astron.* 35, 507 (1991)]. A study of the transition at the base of the convection zone [see (30)] is consistent with the new result, although a repeat of the analysis with the use of the reference models of Richard *et al.* (*15*) yields 0.714*R*.
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- 18. In the cores of solar models, the values of ρ and T are more robust than ρ and μ because pressure supports the weight of the star, which is determined principally by the mass, which is a known quantity; and because the nuclear reaction rates, which are observationally constrained by the sun's radiative luminosity, are sensitive to *T*. Therefore, ρ and *T* are well constrained by observation. Because $\gamma_1 \approx 5/3$ in the essentially fully ionized core, $\delta \ln c^2 \propto -\delta \ln \rho \propto -\delta \ln \mu$: a local increase in c^2 tends to be associated with corresponding relative decreases in ρ and μ .

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- 21. The wave is also advected by azimuthal flow, but the axisymmetric component of that flow is rotation, which is addressed in (20).
- 22. E. G. Zweibel and D. O Gough, in *Proceedings of the Fourth SOHO Workshop: Helioseismology*, J. T. Hoeksema et al., Eds. [European Space Agency (ESA) Special Publication (SP)–379, Noordwijk, Netherlands, 1995]. Although one cannot distiguish between possible sources of asphericity by analyzing frequencies alone, the different anisotropies of the wave-speed perturbations from different sources renders it possible in principle to distinguish them by their eigenfunctions. That might be possible in the future with the use of techniques such as time-distance seismology.
- 23. The averaging kernels in this case are quadratic in the horizontal structure of the eigenfunctions. They are therefore even functions of latitude and are sensitive only to the north-south symmetric component of the asphericity. It requires some knowledge of the form of the eigenfunctions to determine the asymmetric component.
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- 30. We have investigated the structure of the transition between the convection zone and the radiative interior by first considering how mode frequency varies with the phase difference between the surface of the sun and the base of the convection zone, after having filtered out surface effects, and then comparing it with theoretical models (32). The signature of the transition oscillates with phase, in step with the eigenfunctions, with an amplitude that is smaller than that of the reference model, confirming that, if spherical, the transition is smoother than that of the model The apparent smoothness might have come about because what has been observed is actually the spherical average of an aspherical structure. Adiabatic convective overshooting is likely to increase the amplitude; therefore, if such overshooting occurs in the sun, the physical discrepancy is actually greater than it appears at first sight. There is some indication that the amplitude of the oscillatory signal varies with the ratio m/ℓ of azimuthal order to degree. This suggests that the structure of the lower boundary layer

of the convection zone might vary with latitude. Although at present this is no more than a hint, it points to an exciting direction of research with further longterm seismic data.

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Differential Rotation and Dynamics of the Solar Interior

M. J. Thompson, J. Toomre, E. R. Anderson, H. M. Antia, G. Berthomieu, D. Burtonclay, S. M. Chitre, J. Christensen-Dalsgaard, T. Corbard, M. DeRosa,

C. R. Genovese, D. O. Gough, D. A. Haber, J. W. Harvey, F. Hill, R. Howe, S. G. Korzennik, A. G. Kosovichev, J. W. Leibacher, F. P. Pijpers, J. Provost, E. J. Rhodes Jr., J. Schou, T. Sekii,

P. B. Stark, P. R. Wilson

Splitting of the sun's global oscillation frequencies by large-scale flows can be used to investigate how rotation varies with radius and latitude within the solar interior. The nearly uninterrupted observations by the Global Oscillation Network Group (GONG) yield oscillation power spectra with high duty cycles and high signal-to-noise ratios. Frequency splittings derived from GONG observations confirm that the variation of rotation rate with latitude seen at the surface carries through much of the convection zone, at the base of which is an adjustment layer leading to latitudinally independent rotation at greater depths. A distinctive shear layer just below the surface is discernible at low to mid-latitudes.

 O_n the time scale of stellar evolution, the sun is a middle-aged star. The observed surface rotation rates of young solar-type stars are up to 50 times that of the sun. It is therefore believed that the sun has been losing angular momentum over its lifetime through its magnetized wind, thereby gradually spinning down its outer convection zone and probably the bulk of its interior. The effectiveness of the overall spin-down of the star is difficult to estimate from stellar evolution theory, because delicately balanced circulations and instabilities that would tend to mix the interior-and magnetic fields that may retard or modify such processes—must be taken into account (1). This has led to suggestions that the sun might still have a rapidly rotating core, perhaps highly magnetized, reflecting its primordial past. The apparent deficit of neutrinos coming from the sun's energygenerating core has also prompted ideas for readjusting the chemical composition and stratification in models of the nuclear-burning core, and such adjustments have implications for the mixing of angular momentum in that region (2). Tracking of surface features has shown that the sun does not rotate as a solid body: it rotates once in ~ 25 days near the equator and in \sim 33 days near the poles. Further, the rotation rate of sunspots at low and mid-latitudes is somewhat faster than that deduced from Doppler shifts of the surface plasma; this finding suggests that the magnetic fields of the spots may be

M. J. Thompson and R. Howe are in the Astronomy Unit, Queen Mary and Westfield College, University of London, Mile End Road, London E1 4NS, UK. J. Toomre, M DeRosa, and D. A. Haber are at the Joint Institute for Laboratory Astrophysics, University of Colorado, Boulder, CO 80309-0440, USA. E. R. Anderson, J. W. Harvev. F. Hill, and J. W. Leibacher are at the National Solar Observatory (NSO), National Optical Astronomy Observatories (NOAO), Post Office Box 26732, Tucson, AZ 85726-6732, USA, H. M. Antia and S. M. Chitre are at the Tata Institute of Fundamental Research, Bombay 400005, India. G. Berthomieu, T. Corbard, and J. Provost are at the Observatoire de la Cote d'Azur. 06304 Nice Cedex 4, France. D. Burtonclay and P. R. Wilson are in the School of Mathematics, University of Sydney, Sydney, NSW 2006, Australia. J. Christensen-Dalsgaard and F. P. Pijpers are at the Theoretical Astrophysics Center, Aarhus University, DK-8000 Aarhus C, Denmark. C. R. Genovese is in the Department of Statistics, Carnegie Mellon University, Pittsburgh, PA 15213, USA. D. O. Gough and T. Sekii are in the Institute of Astronomy University of Cambridge, Cambridge CB3 0HA, UK. S. G. Korzennik is at the Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA, A. G. Kosovichev and J. Schou are at Hansen Experimental Physics Laboratory Annex, Stanford University, Stanford, CA 94305-4085, USA. E. J. Rhodes Jr. is in the Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089, USA. P. B. Stark is in the Department of Statistics, University of California, Berkeley, CA 94720-3860, USA.