MATHEMATICS

A New Theory of Turbulence Causes a Stir Among Experts

Everyone who has looked over the side of a boat or suffered through a choppy flight is well acquainted with turbulence. But mathematically speaking, it's still a mystery. The effects of turbulence are extremely difficult to calculate and seemingly impossible to derive from first principles. So for decades engineers have based their designs of aircraft, pipelines, and other structures that operate in a fluid environment on empirical laws that fit simple formulas to experimental data. But how well founded are these laws?

Not very, according to two mathematicians at the University of California, Berkeley. In a paper to appear this month in the *Proceedings of the National Academy of Sciences*, Grigory Barenblatt and Alexandre Chorin report a new analysis showing that one of the key formulas of turbulence is off by as much as 65%. The discrepancy, which shows up in a thin layer of highly turbulent flows, has gone unnoticed, they say, because experimental data have not been precise enough to reveal it, except perhaps in hindsight. Now, says Chorin, "many textbooks will have to be revised."

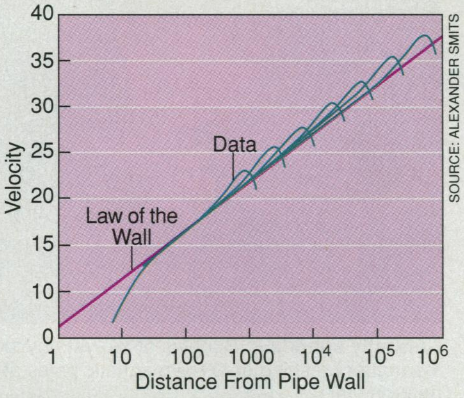
That claim is causing its own stir among turbulence experts, however. Some researchers say the new analysis could be a first step toward putting the study of turbulence on a firmer theoretical basis. But many agree with Paul Dimotakis, a professor of aeronautics and applied physics at Caltech, who says the formula in question "has not been found wanting."

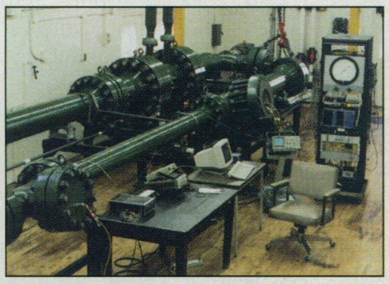
First proposed in the early 1930s by pioneering aerodynamicists Theodor von Kármán and Ludwig Prandtl, the formula, known as "the universal logarithmic law of the wall," describes shear forces exerted by turbulent flows at boundaries such as wings or fan blades or the interior wall of a pipe. What generates these shear forces is the change of fluid velocity as it nears the wall. In principle, a system of partial differential equations known as the Navier-Stokes equations describes the exact behavior of the fluid flow in this so-called "boundary layer," but solving these equations remains beyond the scope of current theory or computation. The law of the wall provides a convenient shortcut.

The law asserts that the fluid's average velocity in the boundary layer increases linearly with the logarithm of the distance from the boundary. While the thickness of the boundary layer depends on details such as the fluid's viscosity and its overall average velocity, the slope of the line relating velocity and distance in the boundary layer is universal—or so says the law of the wall.

That assertion is based on certain "similarity" arguments, which describe how flow patterns right next to the wall give way to other patterns farther away. While plausible, these arguments have never been rigorously established—and perhaps for good reason. According to Barenblatt and Chorin, a more detailed mathematical analysis reveals a sharp departure from the simple picture offered by the law of the wall.

Their theory, Chorin explains, combines Barenblatt's studies of "scaling" laws—principles that relate large-scale and small-scale phenomena in turbulent flows—and his own analyses of how turbulence behaves as the viscosity drops to zero. The scaling laws helped Barenblatt and Chorin dissect the assumptions that underlie the similarity arguments, while the zero-viscosity limit helped them analyze the high-speed, low-viscosity flows most likely to be turbulent.

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Test of turbulence. The Superpipe *(left)* measures velocity changes in the boundary layers of various flows.

Instead of a single straight line for all flows, they calculate a family of curves. Each curve corresponds to a different value of a variable known as the Reynolds number, which combines the dimensions of the flow, the average velocity of the fluid, and the fluid's viscosity to give a measure of how prone it is to turbulence. One section of each curve is indistinguishable from the law of the wall's straight line, but another is significantly steeper, by a factor of 1.65, implying shear forces larger than the law of the wall predicts.

"For years, people have looked at turbulent flow without understanding what they saw," Chorin says. "The theory we developed led to predictions of a beautiful and complex structure in turbulence near walls." Those complexities, he says, "could be seen in the experimental data"—experiments done last year at Princeton University's "Superpipe" facility (*Science*, 8 September 1995, p. 1361). Superpipe creates highly turbulent but carefully controlled flows of compressed air. The individual curves corresponding to different Reynolds numbers and the distinctive steepening are clearly visible in the data, the researchers say.

"The law of the wall was viewed as one of the few certainties in the difficult field of turbulence, and now it has been dethroned," says Chorin. "Generations of engineers who learned the law will have to abandon it," he predicts.

If he's right, the revisions could have implications for designs of high-pressure pipelines or structures such as oil-rig platforms, which are subject to turbulent flows with extremely high Reynolds numbers. The practical effects for most structures might be modest, however, because the design implications of a revision in the law of the wall would be dwarfed by other design considerations—including substantial margins of safety.

And all this assumes that turbulence experts will jettison one of their most successful principles—which few of them are prepared to do. Dimotakis, for instance, says the experimental evidence does not yet demand any revision in the law of the wall. In short, he says, "it ain't broke." The Superpipe researchers agree. "The experimental results are not particularly in agreement with the Barenblatt theory," says Princeton's Steven Orszag. The new theory does fit portions of the data, he notes, but he argues that the improvement is not enough to justify a radical departure from the law of the wall.

Others are more favorably inclined toward Barenblatt and Chorin's analysis. "I think their results are very interesting," says aeronautics and astronautics professor Brian Cantwell at Stanford University. "The assumptions that underlie their derivation of the pipe-flow profile are less restrictive and more general than the assumptions that underlie the arguments that lead to a logarithm," he explains. "If they are correct, then there are important implications for our theoretical understanding of turbulent flow."

For his part, Chorin says that the engineering community will eventually be convinced. And if the issue is slow to be resolved, that's only to be expected. One thing everyone working on turbulence knows is that it takes a long time for the dust to settle.

–Barry Cipra