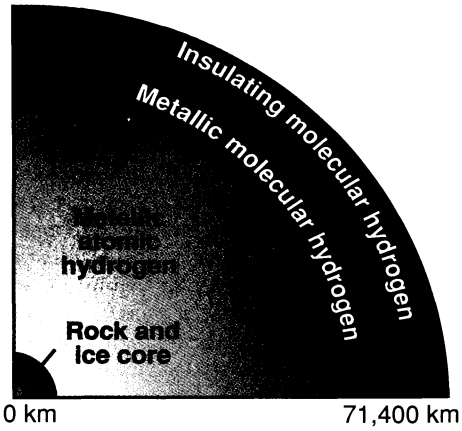
drogen have squeezed it between the tiny chiseled faces of two opposed diamonds in a diamond-anvil cell. The idea was that the hydrogen molecules—pairs of protons bound by two electrons—would come so close to each other that the bonding electrons would be able to slip from molecule to molecule, forming the sea of unbound electrons that is characteristic of a metal.

The diamond-anvil specialists kept their hydrogen cold and solid, at times within a few degrees of absolute zero, reasoning that low temperatures would make the hydrogen easier to confine between the diamonds and the results easier to interpret. But while diamond anvils did succeed in squeezing hydrogen above pressures of 2 million atmospheres (2 megabars), the signs of metalization were suggestive but far from convincing (*Science*, 30 March 1990, p. 1545).

"We did something different," says Nellis: heating the hydrogen to thousands of degrees kelvin. Heat is an inevitable byproduct of their strategy for generating high pressures: firing metal plates from a room-size gun at up to 25,000 kilometers per hour into stationary target samples. In fact, Nellis and his colleagues had to use a target design that kept the temperature of their shocked hydrogen from rising too high and turning it into an ionized plasma.



Showing its metal. Metallic hydrogen prevails out to 88% of Jupiter's radius.

They placed a thin layer of liquid hydrogen between two sapphire plates so that rather than delivering one strong shock to the hydrogen, the apparatus would generate a weaker shock wave that would bounce back and forth between the plates. The reverberating wave would drive up the pressure bit by bit to as much as 1.8 megabars while heating the sample to only a few thousand degrees kelvin.

Nellis and his colleagues didn't expect that this experiment would metalize hydrogen. Theory had been pointing to a metalization pressure of 1.5 to 3 megabars, says Nellis—and that was for solid hydrogen at absolute zero. Hot, fluid hydrogen would presumably be even harder to metalize. Instead, his group just wanted to explore the electrical behavior of fluid hydrogen under typical Jovian conditions. "All the hydrogen at megabar pressures in nature is at high temperature, as in Jupiter and Saturn," he explains.

But when Nellis and his colleagues examined records of conductivity from their experiments, "we were actually surprised to find we had in fact metalized" hydrogen, says Nellis. At pressures of between 0.9 and 1.4 megabars—while the hydrogen was still in its molecular form—conductivity surged by four orders of magnitude to a level typical of a melted alkali metal like cesium at 2000 K. Apparently, says Nellis, metalization in the solid, but not the liquid, state is inhibited because the solid material can accommodate increasing pressures by rearranging its crystal structure and adjusting itself in other ways short of metalizing.

The result implies, says Nellis, that "you get conducting material much closer to the surface" of Jupiter than was thought. Theorists had put the outer edge of the metallic hydrogen zone at a depth of about 17,000 kilometers, but the new result brings it up to 8500 kilometers. The closer the magnetic dynamo of churning metallic hydrogen comes to the surface, the stronger the magnetic field will be there. As a result, the new measurement could help explain a long-standing puzzle: why Jupiter's magnetic field is so strong. Jupiter's field is powerful enough, for instance, to fend off the wind of charged particles from the sun out to a distance as far as 100 times its own radius; Earth manages just 10 times its radius.

Planetary physicist David Stevenson of the California Institute of Technology sees other possible implications for both Jupiter and Saturn. Because the solubility of helium, another Jovian constituent, is much lower in metallic hydrogen than in ordinary, insulating hydrogen, the shock-compression results would also increase the volume of the planet in which helium would come out of solution to form "raindrops," he notes. That would help explain the excess heat coming from Jupiter's interior, because the drops would release gravitational energy as they fell.

Stevenson adds that "we may need to be concerned about magnetic field influences on fluid motions out to a greater radius than some people previously supposed." Besides generating the magnetic field, the churning metallic hydrogen flows under the influence of the field. The closer the metallic region extends to the surface, the more likely it is to pass some of its momentum on to shallower, nonconducting layers, which in turn would influence the seething of Jupiter's dense atmosphere. So it's conceivable, Stevenson says, that about the same time as Nellis and his colleagues were making metallic hydrogen on Earth, the Galileo probe was feeling its effects too, when strong winds buffeted the probe well below the visible clouds.

–Richard A. Kerr

MATHEMATICS

Fermat Prover Points to Next Challenges

With the proof of Fermat's Last Theorem now on the books, what's left for number theorists to do? Plenty, says Andrew Wiles, the Princeton University mathematician who knocked off mathematicians' favorite unsolved problem. In a series of talks earlier this year at the joint meetings of the American Mathematical Society and the Mathematical Association of America in Orlando, Florida, Wiles laid out a string of related questions that remain unanswered. Among them are problems that are mathematically more significant—if less notorious—than Fermat's famous challenge.

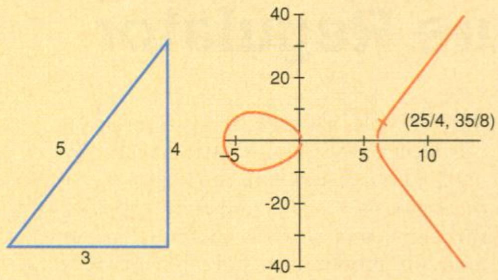
"The problems that remain unsolved are very natural ones," says Wiles, having to do with the properties of the simple algebraic equations that are ubiquitous in mathematics. Like questions about the physics of water or the basis of gravity, they concern an everyday medium—and are exceptionally hard to answer. "What's so beautiful for mathematicians is that the questions are so simple and natural, and yet the answers are so demanding—and so rewarding," he says. And for one of the most important of those problems the rewards may now be much closer, as Wiles explained in his talk.

The question has to do with elliptic curves, which were at the heart of Wiles's attack on Fermat's problem. Elliptic curves consist of solutions of cubic equations in two variables, typically of the form $y^2 = x^3 + Ax + B$, with integer coefficients A and B. Fermat himself was interested in finding rational numbers that could solve such equations—or showing that none exist. Indeed, they may be responsible for the Last Theorem's notoriety. Fermat's famous marginal comment that he had a proof for his own theorem—that the equation $x^n + y^n = z^n$ has no solutions for n greater than 2—may have been based on an overestimate of the power of his methods for studying cubic equations.

Now Wiles's proof of Fermat's Last Theorem has given theorists new tools for attacking the central—and still unsolved—challenge of elliptic curves: taking an arbitrary cubic equation and finding all of its rational solutions. "We don't know how to do that," notes Wiles. Although theorists have come up with methods that work for particular elliptic curves—Fermat himself, for example, proved that the equation $y^2 = x^3 - x$ has exactly three solutions ($y = 0$ and $x = 0, 1,$ and -1)—they don't have a general method that is guaranteed to work on every cubic equation. "There are lots and lots of interesting things [about elliptic curves] that seem to be true but we can't prove," notes

A Proof to Please Pythagoras

Number theorists wielding the tools that Princeton University mathematician Andrew Wiles used to prove Fermat's Last Theorem are now hoping to clear up some modern mysteries surrounding the objects called elliptic curves (see main text). If they succeed, notes Wiles, they will also get the answer to a simple-sounding question that predates even the Fermat theorem—a problem Pythagoras might have pondered.



Rational approach. A point on the elliptic curve $y^2 = y^3 - 36x$ *(right)* shows that the integer 6 is the area of a right triangle with rational sides.

The question is whether or not a given integer can possibly be the area of a right triangle, all three of whose sides are rational. The number 6, for example, is the area of the familiar 3–4–5 right triangle. Less obvious is the number 5: It's the area of the 3/2–20/3–41/6 right triangle. Still less obvious is the fact that 1, 2, 3, and 4 are *not* the area of any right triangle with rational sides.

Number theorists recognized long ago that the secret of this problem lies in the theory of elliptic curves. Each right triangle with rational sides and area N corresponds to a rational solution of a standard elliptic curve, the equation $y^2 = x^3 - N^2x$. If a, b, and c are the sides and hypotenuse of such a triangle, then some slightly messy algebra shows that $x = (c/2)^2$, $y = (a^2 - b^2)c/8$ is a point on the elliptic curve—a solution to the equation. For example, the 3–4–5 triangle corresponds to the solution $x = 25/4$, $y = 35/8$ of the equation $y^2 = x^3 - 36x$. But number theorists still have no general method for deciding whether such an equation has rational solutions or not.

The properties of the elliptic curves $y^2 = x^3 - N^2x$ offer the beginnings of a strategy, however. First, each one has either no rational solutions with nonzero y or infinitely many such solutions. Second, because these equations have the property known as complex multiplication, what is called the Coates-Wiles theorem applies: If a curve has infinitely many solutions, an associated function called the L-function takes the value 0 at a special point. If the L-function's value is something other than 0, the corresponding curve has no solutions, which means N is *not* the area of a right triangle with rational sides.

In 1983, Jerry Tunnell, then at Princeton University, provided a quick and easy way to perform this test. While complicated to state (as Wiles puts it, "No one could possibly have guessed this theorem"), Tunnell's formula reduces the evaluation of the L-function to a straightforward counting problem, easily computed for any value of N.

But while this method can rule out certain integers as the areas of right triangles with rational sides, it can't rule in others. That's because the Coates-Wiles theorem can't be reversed: No one has proved that every curve whose L-function falls to 0 has infinitely many rational solutions. With the new tools provided by Wiles's proof of Fermat's Last Theorem, however, number theorists can at least set their sights on such a proof, which would settle the right-triangle problem completely. Pythagoras, not to mention Wiles, would be thrilled.

–B.C.

Joseph Silverman of Brown University.

Even the first step toward understanding the equations—knowing whether an equation has just a finite number of rational solutions or an infinite number of them—can be extremely difficult. Trial-and-error, computational searching cannot decide the point: No matter how many (or few) solutions you do or don't find, there may or may not be infinitely many more.

Number theorists do, however, have an unproven method for analyzing elliptic curves. In the early 1960s, Bryan Birch at Oxford University and H.P.F. Swinnerton-Dyer at Cambridge University went out on a limb with a daring conjecture about the behavior of elliptic curves. By computing many examples, they discovered a striking coincidence between the number of solutions to an elliptic curve and the behavior of an associated analytic function known as an L-function, which, roughly speaking, blends arithmetical information in the Mixmaster of calculus. In all the examples they looked at, if the curve had infinitely many solutions, then its associated L-function had the value 0 at a particular point, and vice versa.

The discovery suggested a convenient way to tell whether or not an elliptic curve has an infinite number of rational solutions or only finitely many: Just evaluate its L-function at a particular fixed point and check whether you get 0 or not. Birch and Swinnerton-Dyer also found that they could extract additional clues to the solutions of an elliptic curve by studying the associated L-function around as well as at the special point.

This structure of conjecture, however, is still waiting for the cement of proof. Wiles had provided one bit of mortar in 1976, when he and his adviser, John Coates at Cambridge University, proved part of Birch and Swinnerton-Dyer's original conjecture for one particular class of elliptic curves—those with a special property known as complex multiplication. Among these curves, they showed, those with an infinite number of solutions always have an L-function that reaches 0 at the appropriate point. Coates and Wiles were not able to prove the converse, however—that every curve whose L-function is 0 at that point has an infinite number of solutions.

More recently, Viktor Kolyvagin at the Steklov Institute in Moscow extended the Coates-Wiles theorem to a broader class of elliptic curves, known as modular curves. That's where Wiles's recent work comes in. In the course of proving Fermat's Last Theorem, Wiles showed that a large class of elliptic curves is indeed modular (*Science*, 2 July 1993, p. 32). The powerful techniques he introduced for dealing with elliptic curves have made number theorists optimistic that a proof that all elliptic curves are modular—an assertion known as the Taniyama-Shimura conjecture—may be close, an opinion no one would have hazarded 3 years ago, says Silverman.

Thanks to Kolyvagin's result, a proof of the Taniyama-Shimura conjecture would establish that every elliptic curve with infinitely many solutions has an L-function that reaches zero at the appropriate point. It wouldn't prove that number theorists can always go safely in the opposite direction, however, and draw conclusions about a curve whose L-function reaches 0. "The other direction is much harder," opines Wiles. "My hunch is that using these modular curves is going to be important" for completing the proof, he says.

But maybe more is needed. "When you're faced with ignorance," says Wiles, "it's very hard to know whether we've got the tools now and the answer is around the corner, or whether we need tools that are completely different." Even after 350 years, "we are far from understanding elliptic curves."

–Barry Cipra