BOOK REVIEWS

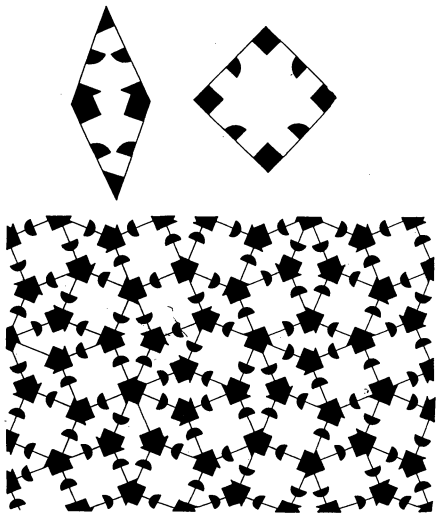
Crystallography Expanded

Quasicrystals and Geometry. MARJORIE SENECHAL. Cambridge University Press, New York, 1995. xvi, 286 pp., illus. $59.95 or £40.

Exploring the consequences of altering the fundamental axioms and definitions of a field can be a very rewarding enterprise for mathematicians. At its best, this is a quest for abstractions with wide-ranging implications that is both a cerebral and an aesthetic joy.

On occasion this activity is triggered by a physical observation. Such was the effect of the report in November 1984 of the discovery of quasiperiodic crystals—quasicrystals for short. The discovery raised many mathematical questions and sparked intense activities in many aspects of geometry: what constitutes an orderly spatial arrangement of points or subdivision of space, and what the definition of a crystal should be. This ferment is still going on.

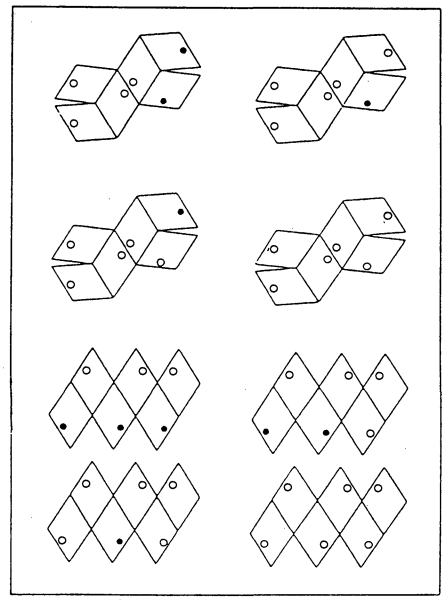
In this excellent monograph Marjorie Senechal, a mathematics professor at Smith College, gives us insight into what occurred when established ideas had to be reexamined, modified, or overturned. There were, and still are, many alternative routes, different axioms and definitions, to be explored. Because the development of each is logical and rigorous, none is wrong; each has different implications, and some have more interesting consequences than others. The choice of which route to take is clearly subjective and aesthetic; the logical arguments and the rigorous mathematics follow from such choices. Senechal's book alternates between raising new possibilities and subjective questions and presenting dry and logical mathematical developments that are all the more fascinating because the author lets us understand the goals. This is unusual for a mathematics book; before each mathematical section there is a glimpse of the goals and then a telling of what has been achieved.



A portion of an octagonal tiling. Octagonal tilings "are one of a series of aperiodic tilings discovered by Ammann." The example shown here "has two prototiles: a decorated rhombus (with small angle 45°) and a decorated square; the decorated square is its own mirror image, and mirror images of the rhombus are also allowed in the tiling. The name 'octagonal' reflects the fact that eight rhombs fit together at a vertex." [From *Quasicrystals and Geometry*]

Retaining the periodicity axiom of crystallography, or defining crystals as composed of periodically repeating structural units, would have simply excluded the quasicrystals from crystallography by definition. Senechal takes us along the historical path to show how periodicity became entrenched in crystallography, but also how alternative axioms were being explored long before quasicrystals. In 1992 quasicrystals became crystals when a commission of the International Union of Crystallographers with Senechal as a member recommended defining crystals as having discrete diffraction patterns. Such a definition has far greater implications. By now mathematicians have imagined whole new sets of strange objects that would be crystals if they had physical realizations and closely related other sets that would not be. Whether or not some cleverly contrived objects, such as Conway's pinwheel tiles, diffract has still not been decided.

Tilings are one way of making an orderly subdivision of space and have been crucial in the understanding of aperiodicity. The variety of fundamentally different tilings discovered is astonishing, prompting a chapter called "The aperiodic zoo." But atoms, unlike tiles, do not need to fill space without any gaps. With quasicrystals in mind Senechal takes us through other concepts of orderly spatial arrangements. There are chapters on order on a line and on arrangements of sets of points in space, such as Delone sets, and associated



"Katz's matching rules for the 'Penrose' rhombohedra." The prototile shapes "are two rhombohedra, one thick and one thin; all of their facets are congruent. . . . When decorated, the number of prototiles increases from two to 22. Nets for four of these tiles are shown [here]; the others are obtained from these by reflection and inversion. To construct the tiling, black dots must be matched to white. It is evidently not known whether these rules are local or nonlocal, or whether the tiling has the substitution property." [From *Quasicrystals and Geometry*]

Voronoï cells. Fibonacci sequences are explained. Special numbers and number theory are brought in. Periodic objects in higher dimensions, where new symmetries are allowed, are related to quasiperiodic objects in lower dimensions. There is a chapter called "Introduction to diffraction geometry" and, since diffraction is a way of forming a transform, a later chapter, "Atlas of tiling transforms," that shows the diffraction patterns of many different types of aperiodic tilings of the plane.

Still this is a mathematics book; the mathematical sections are typically terse. This is somewhat alleviated by a helpful appendix called "A mathematical toolbag," but even it is terse. Definitions, theorems, and lemmas are precise and concisely stated; all too often these gained meaning for me only in the context of subsequent applications. Reading this book in depth requires familiarity with modern mathematical notation well beyond what most readers of *Science* would have. Still, there is much of interest that can be gained without tackling the difficult parts.

The author is well qualified for the task she has set for this book. Prior to 1984 she had been active in a number of areas of mathematical crystallography. She is knowl-

Vignettes: Dinners with Notables

During a dinner at the Huxleys', Grace [Mrs. Edwin Hubble] was seated next to the novelist's brother Julian . . . . Edwin was given the place of honor beside Bertrand Russell, who confessed to a weakness for Agatha Christie mysteries. . . . Hoping to pass the evening on a high intellectual plane, the Hubbles were disappointed by Russell's preoccupation with his fate. The long-standing pacifist and his wife would soon have to go at great expense to Mexico, provided the authorities would admit them, and then, if they got there, the United States might not let them come back. Grace offered to contact a certain well-connected friend on the Russells' behalf, but the annoying lament continued. . . . She followed Edwin's longing gaze across the room where the actor Ronald Colman, who had a head cold from sitting in a massive refrigeration unit during filming, was reverently listening to a discussion dominated by Aldous.

—*Gale E. Christianson*, in *Edwin Hubble: Mariner of the Nebulae* (Farrar, Straus, and Giroux)

We had some time to wait before dinner for Dr. Fitton, which is always awful and, in my opinion, Mr. Lyell is enough to flatten a party, as he never speaks above his breath, so that everybody keeps lowering their tone to his. Mr. Brown, who Humboldt calls "the glory of Great Britain," looks so shy, as if he longed to shrink into himself and disappear entirely; however, notwithstanding these two dead weights, viz. the greatest botanist and the greatest geologist in Europe, we did very well and had no pauses.

—*Emma Wedgwood Darwin*, 1839, on a dinner party given shortly after her marriage to Charles, as quoted by Janet Browne in *Charles Darwin: Voyaging, Volume 1 of a Biography* (Knopf)

edgeable about the history of mathematical crystallography and gives a sweeping historical view in order to set the stage for the last 10 years. By lucky happenstance she was the co-organizer of a workshop on mathematical crystallography at Institut des Hautes Etudes Scientifiques in France in January 1985 that had long been planned to feature some intense discussions about modulated crystals, generalizations of the aperiodic tilings of Penrose and Ammann, and crystallography in higher dimensions. All were topics of immediate applicability to quasicrystals. A last-minute invitation issued to some of the discoverers of quasicrystals produced an immediate appreciation of the mathematical opportunities. Senechal became an active participant in this research and an able chronicler. Her several published reports and this book convey both the searches for new directions and the excitement of new results and are worthwhile narratives of ten exciting years.

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Other Books of Interest

Frontiers in Mathematical Biology. S. A. LEVIN, Ed. Springer-Verlag, New York, 1994. x, 633 pp., illus. $69 or DM98. Lecture Notes in Biomathematics, vol. 100.

This 100th volume marks the termination of a monograph series of some 20 years' standing. Conceived as an effort to foster the development of a field that at the time seemed peripheral to much of biology, the series comes to its end at a point at which, the editor writes, "mathematical biologists . . . see their work guiding experiments, and shaping the conceptual foundations of almost all areas of biology." For this final volume Levin has invited a number of contributors to the field to "take a step back from their work, and to give their views on the key issues in their subject areas." In all there are 36 papers grouped thematically according to what Levin describes as "a convenient if imperfect hierarchy." The first group, Frontiers in Cell and Molecular Biology, contains discussions of molecular topics ranging from RNA structure to gene mapping by De Lisi and by Waterman and of cellular topics by Oster and Perelson (the mechanics of protrusion) and by Alt (effects of environmental stimuli on movement and orientation). Organismal biology is represented next, with papers on pattern formation in tissue interaction (Murray *et al.*), neurotransmitter release (Segel), mutual synchronization in neurobiology (Strogatz), ventricular fibrillation (Winfree), and immune networks (Rose and Perelson). Turning to issues in evolutionary biology, Ohta considers the evolution of gene families, Ewens gives a "devil's advocate" critique of theoretical population genetics, Otto *et al.* address the advantages and disadvantages of recombination, Bookstein offers an "intellectual history" of the biometric analyis of size and shape, and Mangel and Roitberg consider the consequences of behavior for fitness with respect to infectious disease. In a section on population ecology two papers (Tuljapurkar, Diekmann and Metz) are concerned with life history. Then models for the dynamics of structured populations are discussed by Cushing, social aggregations of animals by Grünbaum and Akubo, and spatial chaos in ecology and evolution by May. Frontiers in Community and Ecosystem Ecology opens with two papers by Cohen, both dealing with food webs—their future as descriptions of community ecology and the contribution of Lorenzo Camerano, a translation of whose 1880 paper on the subject is also included. In other papers Levin discusses the challenge of incorporating physical environmental factors into population biology; De Angelis *et al.* concern themselves with "computational ecology," specifically with population models that simulate many discrete organisms; Getz presents a "metaphysiological approach" to the analysis of trophic systems; Yodzis discusses trophodynamic models of communities, with attention to effects of the way data are aggregated; and Castillo-Chavez *et al.* take up "contact structures"—those by which diseases, genetic characteristics, or cultural traits are transmitted. Categorized as applied biology are papers on spatial structure in conservation (Hastings), the construction of epidemiological models (Hethcote), fisheries management (Ludwig), and ecological risk assessment (Hallam and Lassiter). A final section, Mathematical Challenges, comprises papers on health information in developing countries (Krickeberg), the Belousov-Zhabotinsky reaction (Tyson), model-building as an inverse problem (Capasso and Forte), and estimation techniques for size-structured population models (Banks), along with a discussion of dynamic systems that its author (Akin) describes as a "conservative harumph" at currently trendy approaches. A subject index concludes the volume. Unfortunately lacking is a listing of the previous works that have made up the series.

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