Interlayer Tunneling Mechanism for High-T_c Superconductivity: Comparison with *c* Axis Infrared Experiments

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Recent *c* axis–polarized infrared measurements in the high–transition temperature (high-T_c) cuprate superconductor $(La,Sr)_2CuO_4$ can be interpreted on the basis that the entire condensation energy comes from the interlayer Josephson coupling. This gives a parameter-free determination of penetration depth λ and coherence length ξ for this superconductor that are in agreement with experiment.

Accurate measurements have been made of the far-infrared spectrum of $(La_{1.85}Sr_{0.15})CuO_4$ for light polarized along the *c* axis (perpendicular to the planes) (*1*). These data have recently been confirmed and even more carefully quantified (*2*, *3*), and the frequency range has been extended (*4*). The original measurements (Fig. 1) contain two anomalies relative to a usual (Fermi liquid) metal with a Bardeen-Cooper-Schriffer (BCS)–type attractive interaction.

1) The first anomaly is that in a conventional metal, the plasma edge observed near 50 cm^{-1}, below T_c, is present both in the normal and in the superconducting metal, reflecting the sum rule on conductivity σ over frequencies ω

$$\int \sigma(\omega)d\omega = \text{constant} \qquad (1)$$

which can be satisfied in detail in conventional BCS superconductors if the conductivity in the gap region is moved into a δ function at zero frequency, leaving the dielectric constant not much changed for frequencies near and above the energy gap because of the Kramers-Kronig relation between the two. The relaxation rate of the carriers in the normal metal is known from the resistivity in the *ab* plane (perpendicular to the *c* axis) to be

$$\frac{\hbar}{\tau} \simeq \max(kT, \hbar\omega) \qquad (2)$$

(where k is the Boltzmann constant, T is temperature, $\hbar$ is Planck's constant h divided by 2π, and τ is the relaxation time) which should smear but not remove this plasma edge in the normal state. In short, the reflectivity in the normal state should be 1 below the plasma frequency ω_p, not ~1/2. What is observed above T_c looks like a lossy insulator with a dielectric constant ϵ of about 20. In the original paper (*1*), Tanasaki *et al.* stated that the sum rule can be satisfied by the insertion of a "Drude" peak of conductivity of width ~170 cm^{-1}, of which there is no explicit evidence: It is simply the maximum size the experiments would allow. Later, it became clear (*2*–*4*) that the conductivity in the *c* direction $\sigma_c(\omega)$ extends over a much wider range and can hardly be called a "Drude" peak because its width is orders of magnitude larger than the observed relaxation rate. Timusk (*2*), in particular, showed that not enough σ_c exists in the low-frequency range to satisfy the sum rule. But in a sense, it is the absence of a Drude metallic conductivity that is the first anomaly, not primarily the details of spectral weight transfer, which are quite complex theoretically as well as experimentally.

2) The second anomaly is that the observed ω_p is much lower than expected, given the estimates of *c* axis masses (or hopping matrix elements) obtained from band calculations, either local-density approximation (LDA) or tight-binding, and other experimental phenomena. The *c* direction bandwidth should be of order 500 to 1000 cm^{-1}, which would make ω_p an order of magnitude too high, at least in the range from 500 to 1000 cm^{-1}. The value of ω_p in the *ab* plane is around 10^4 cm^{-1} and ω_p as a function of direction should be simply proportional to the relative bandwidth (which is the hopping integral for an anisotropic Fermi liquid). The rough formula is

$$(\hbar\omega_p)^2 \simeq E_F \frac{e^2}{\epsilon_0 r_s} \qquad (3)$$

where E_F is the Fermi energy, e is the electron charge, ϵ_0 is the dielectric constant, and r_s is the screening radius.

Both of these anomalies are caused by the "confinement"–interlayer tunneling (*5*) mechanism, and they allow a quantitative, parameter-free check on the accuracy of that theory. The principle of the theory is that, when the normal metal is a Luttinger liquid, coherent single-particle tunneling between layers is blocked (*6*), and the sum rule for conductivity in the *c* axis direction is spread out over a very wide frequency range, rather than having a Drude peak below $\hbar\omega \simeq kT$. At $T = 0$ and $\omega = 0$, $\sigma(\omega)$ would be literally zero in the normal state. On the other hand, Josephson-like pair tunneling is allowable between two Luttinger liquid layers if each layer has self-consistently developed an anomalous self-energy or "gap" Δ. Therefore, the Josephson energy

$$E_J = -\frac{\hbar J_J}{2e}\cos\varphi \qquad (4)$$

(where J_J is the Josephson critical current and φ is the relative phase between layers) is the fundamental interaction that causes high-T_c superconductivity. (The sum-rule argument shows that for normal BCS superconductors, this interaction is not effective in causing condensation because it exactly cancels between normal and superconducting states.) The attractive energy for this interaction comes from a net reduction in kinetic energy of localization along the *c* axis.

On the other hand, one cannot expect the relatively small energy gap Δ to restore the whole of the missing coherent electron motion, or equivalently to restore all of the kinetic energy lost by confinement. The ac conductivity is spread over the entire bandwidth, and the high-energy states involved are not much affected by Δ. To estimate how much Josephson conductivity is restored is, in fact, trivial. The free energy of condensation will be roughly proportional to the cosine of the relative phase between layers and will vanish when the phase angle φ is 90°. At this

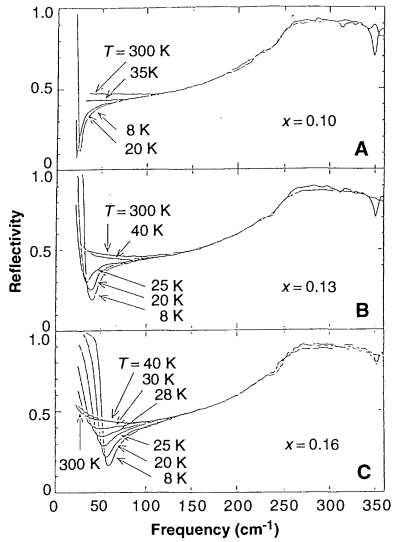

Fig. 1. Infrared reflectivity spectra of $La_{2-x}Sr_xCuO_4$ with polarization perpendicular to the CuO_2 planes ($\vec{E} \parallel c$) above and below T_c for (**A**) $x = 0.10$ ($T_c = 27$ K), (**B**) $x = 0.13$ ($T_c = 32$ K), and (**C**) $x = 0.16$ ($T_c = 34$ K). [Adapted from (*1*), copyright 1992, American Institute of Physics]

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point, we assume an interlayer mechanism: For any mechanism involving coupling pairs within the planes, the entire condensation energy does not depend crucially on φ. The current is $2e$ times the derivative of this energy with phase; hence, the superconducting binding energy is given by

$$F_{cond} = \frac{\hbar J_J}{e} \cos \varphi \quad (5)$$

(accounting for interactions with two neighboring layers). The "Josephson" plasma frequency (7), which in this case is the plasma frequency itself, is given by

$$(\hbar\omega_J)^2 = \frac{\hbar J_J}{e}\frac{2e^2}{\epsilon c} = \frac{2e\hbar}{\epsilon d} J_J \quad (6)$$

(ϵ is ~20, and d is the c-axis lattice constant). From the value 50 cm^{-1} for ω_J, $\hbar J_J/2e \simeq 4/3$ K. This is then equated to the value of F_{cond} per unit cell. In any reasonable theory, and in fact in that of Chakravarty and Anderson (8)

$$F_{cond} \simeq \frac{1}{2} N(0)\Delta^2 \quad (7)$$

simply by energy-entropy balance. With a density of states at $\omega = 0$ of $N(0) \simeq 1/t_{\|} \simeq (1\ eV)^{-1}$ (where $t_{\|}$ is the in-plane element of the kinetic energy matrix $\mathbf{t}$), this gives

$$\Delta \simeq 100\ K \quad (8)$$

which is correct to within a factor 2. That is, the Josephson and condensation energies are the same, at least within the factor 2 accuracy of these estimates.

The anisotropy of the penetration depth will, in these materials, be greater than expected according to band theory and will give too large a value of the band mass relative to band calculations. The constant corresponding to n/m in the expression

$$\frac{1}{\lambda^2} = \frac{4\pi n e^2}{mc^2} \quad (9)$$

which is proportional to $t_\perp{}^2/t_{\|}$ in conventional theory ($t_\perp$ is the out-of-plane element), will be more like $t_\perp{}^4/t_{\|}{}^3$. Here I have observed that both the coherence length ξ and the penetration depth λ, the two parameters of Ginsberg-Landau theory, are uniquely determined in the c direction by the assumption of interlayer coupling as the mechanism for T_c if the material has only one type of layer. The coherence length (from the assumption in Eq. 5) is simply d (actually $d/\sqrt{2}$ or ~4 Å), and the penetration depth c/ω_J is determined from Eq. 6 [$\sim 2 \times 10^{-2}$ cm, in $(LaSr)_2CuO_4$]. The coherence length is in fair agreement with direct experiments (9).

I can see no alternative explanation of these very striking experimental data. Other single-layer materials, especially the Tl and Hg compounds, should be studied in the same way. Multilayers such as YBCO (yttrium barium copper oxide) will have much more complex infrared spectra, which nonetheless would be of interest to probe carefully. In these cases, there should be two Josephson-type frequencies, corresponding to the two different interlayer matrix elements $t_{\perp>}$ and $t_{\perp<}$, with the smaller one corresponding to the penetration depth and coherence length (because phase stiffness adds by inverses) and the larger one to T_c or Δ, because T_c adds directly [evidence of such a second frequency is suggested in some experiments (2)]. By this reasoning, the coherence length should satisfy

$$\xi \simeq \frac{d\, t_{\perp<}{}^2}{(t_{\perp>}{}^2 + t_{\perp<}{}^2)} \quad (10)$$

where $t_{\perp<}{}^2$ and $t_{\perp>}{}^2$ are, respectively, the larger and the smaller of the interlayer tunneling coupling constants. This result is in accord with the existing observations. Multilayers will have a corresponding increase in λ_c (penetration depth for current parallel to the c direction) as well.

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Water on the Sun

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High-resolution infrared spectra of sunspot umbrae have been recorded with the 1-meter Fourier transform spectrometer on Kitt Peak. The spectra contain a very large number of water absorption features originating on the sun. These lines have been assigned to the pure rotation and the vibration-rotation transitions of hot water by comparison with high-temperature laboratory emission spectra.

The hot water molecule is the most important source of infrared opacity in the spectra of oxygen-rich late-type stars (*1*). Water is particularly prominent in the spectra of variable red giant stars (Mira variables) (*2*) and in other cool M-type stars (*3*). For very cool stars ("brown dwarfs"), water is predicted to be the most abundant molecule after hydrogen (*1*, *4*).

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The water molecule was discovered in molecular clouds in 1969 on the basis of the maser transition from 6_{16} to 5_{23} at 22 GHz (*5*). In spite of the large rotational constants, which place most transitions in the submillimeter region, many masing transitions are known for numerous objects (*6*). The water masers are thought to be collisionally pumped, and they are often associated with star-forming regions (*6*).

Because water is a strong absorber in Earth's atmosphere, there is a severe problem in detecting cool extraterrestrial water from the ground. Fortunately, the pure rotation and the vibration-rotation lines of hot water can pass through Earth's atmosphere because the large rotational constants of water (A_0 = 27.877 cm^{-1}, B_0 = 14.512 cm^{-1}, and C_0 = 9.289 cm^{-1}) shift the highly excited transitions well away from the telluric absorptions of cooler water.