Unification of Synchrotron Radiation and Inverse Compton Scattering

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A new approach to radiation theory was published recently by Lieu and Axford in the *Astrophysical Journal* (*1*). The method is elegant in that it uses the concept of inverse Compton scattering to explain with unprecedented simplicity all the classical and quantum electrodynamic properties of synchrotron radiation, thereby unifying two fundamental processes in physics.

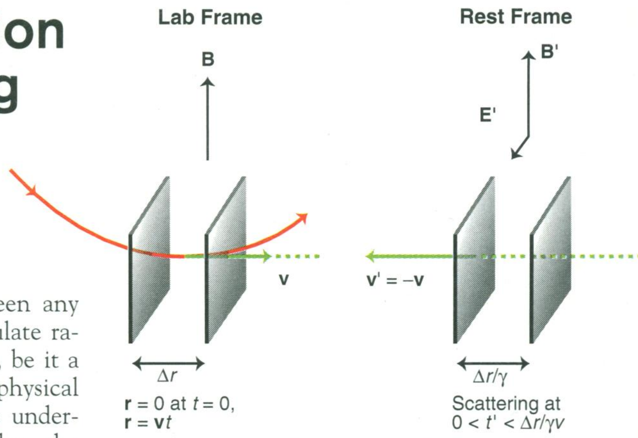
Synchrotron radiation, which is important in astrophysics and laboratory physics, is emitted by charged particles (usually relativistic electrons or positrons) moving in a magnetic field; it is associated with the curvature of the particle trajectory. Inverse Compton scattering, which is the source of diffuse gamma-ray background in space, is the result of Compton interaction of low-energy (starlight) photons with energetic electrons. Within the classical limit ($\hbar \equiv 0$, where $\hbar$ is Planck's constant h divided by 2π), three striking similarities between them have long been noticed: (i) the radiation is emitted with a dipolar angular distribution when the charge is moving at nonrelativistic speeds, (ii) one observes in both cases broadband spectra with a characteristic frequency $\sim\gamma^2\omega_0$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ at relativistic speeds v, and (iii) mathematical expressions for the energy loss rate of the charge are in each case formally the same at all speeds.

These similarities remind us of relativistic bremsstrahlung, which is radiation from a fast moving electron accelerated by the electrostatic field of a nucleus. The phenomenon has been explained approximately but quite satisfactorily (*2*) as inverse Compton scattering of equivalent photons by Fermi, von Weizsäcker, and Williams (hereafter FWW), who noticed that the Coulomb field of a relativistic proton appears in the electron rest frame like an "equivalent" electromagnetic pulse scattering the electron to produce the outgoing (observed) radiation. Moreover, the analogy with synchrotron radiation has long been appreciated by Russian scientists who termed the latter "magnetic bremsstrahlung."

Despite the simplicity of the FWW procedure, however, there has not been any obvious way of applying it to calculate radiation properties of a synchrotron, be it a laboratory light source or an astrophysical emission region, where the charge undergoes continuous acceleration so that the trajectory cannot be approximated by a straight line. The greatest hurdle concerns the nature of the equivalent photons appropriate for synchrotron radiation. In an instantaneous rest frame of the moving charge, the static electric and magnetic fields resemble those of an electromagnetic wave, but it is another matter to describe this within the context of scattering, and to link the different rest frames without using a noninertial system.

The situation has been changed dramatically by Lieu and Axford (*1*). They have succeeded in applying an improved version of the FWW approach to synchrotron radiation, exactly and quite generally, and in so doing have been able to treat it as an inverse Compton effect. They have derived all the standard results of classical radiation (namely, spectrum, polarization, and angular distribution) using coherent addition of Thomson scattered amplitudes. However, the most remarkable aspect of their work lies in its ability to go beyond the classical limit, almost trivially, to obtain corrections of the radiation properties in the limit of high γB (B is the magnetic field), which were previously treated in a complicated way by quantum electrodynamics (QED). Because the Compton cross section that they used can be calculated without second quantization, one concludes that genuine QED effects do not play an important role in synchrotron radiation, as vaguely alluded to by Schwinger (*3*) and by Baier and Katkov (*4*).

Consider first the radiation emitted when a charge travels transversely across an elementary plane slab of magnetic field. In the frame of the charge, the interaction is due to scattering with an equivalent electromagnetic wave in the form of a pulse, the duration of which is determined by the (infinitesimal) laboratory width of the slab. The figure illustrates the interaction. Because Compton scattering is inherently phase-coherent, the scattered pulse is well defined, while the electron trajectory is slightly deflected by a known amount. The properties of the emitted radiation when



Synchrotron radiation from an elementary segment of the charged-particle orbit may be considered as inverse Compton scattering of equivalent photons. The segment is bounded by two planes perpendicular to the instantaneous velocity of the charge. When viewed in the particle frame, such an imaginary system appears like an electromagnetic pulse propagating in the $-\mathbf{v}$ direction. The amplitude of the resultant (Compton) scattered radiation is determined by the field and orbit. This can be transformed to the laboratory frame and coherently added with amplitudes from other segments of the orbit to produce the total synchrotron radiation that we observe.

the particle is made to interact with a continuous sequence of such slabs along its trajectory is obtained by adding the elementary contributions constructively after they have been transformed to the laboratory frame (because there is not a unique rest frame). Thus, in this approach, the Fourier time integral has a new interpretation: It represents a coherent addition of infinitesimal inverse Compton radiation amplitudes from all parts of the particle trajectory. Because each elementary slab is treated independently, there is no reason why the magnetic field should be uniform along the orbit.

For the problem of classical radiation from a charge moving at arbitrary velocity, Lieu and Axford followed the above prescription using the Thomson cross section to derive exact formulas for the differential emission rate. Thus, the cyclotron limit of $\gamma \sim 1$ is also properly accounted for. Recently, Lieu and Axford have produced a more compact version of the derivation (*5*).

Now the interesting question is whether the technique can also be used to calculate synchrotron radiation properties in the limit where the particle orbit remains classical but the emitted photon energy is large: $\hbar\omega \rightarrow \gamma mc^2$ (m is the mass of the particle). The limit is reached whenever

$$\gamma \gg 1$$

and

$$\gamma B \rightarrow B_c = m^2c^3/e\hbar \cong 4.4 \times 10^{13}\ \mathrm{G} \quad (1)$$

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Previously, this situation was treated exactly by QED, but only for the case of circular orbits (*6*).

Once again, Lieu and Axford enlisted FWW but introduced Compton corrections. First, momentum conservation requires that a scattered photon of laboratory frequency ω is related to an incident photon of frequency ω/η, where η is given to an accuracy of $1/\gamma^2$ by

$$\eta = 1 - \hbar\omega/\gamma mc^2 \qquad (2)$$

Note that η does not depend on any laboratory angle. Moreover, a scattered photon energy of $\hbar\omega \rightarrow \gamma mc^2$ results from an incident photon of infinite energy, implying that a spectral cutoff must exist at $\hbar\omega = \gamma mc^2$

Second, the Compton cross section must be used. Here a distinction is made between spinless and spin-½ particles. Lieu and Axford (*1*) published results that are valid for arbitrary planar orbits in a nonuniform magnetic field, with generalization to include three-dimensional orbits (5). A particularly fascinating point is that for $\hbar\omega/\gamma mc^2 << 1$, the differential number of emitted photons is simply given by its classical formula with the replacement $\omega \rightarrow \omega/\eta \approx \omega(1 + \hbar\omega/\gamma mc^2)$. This well-known first-order quantum correction (*3*) can now be identified with electron recoil accompanying the scattering of equivalent photons. For a full comparison with QED, however, it is necessary to adopt the restricted case of a circular orbit, because QED has no exact results for more general situations.

In the case of orbital radiation from a spinless charge, there is complete agreement with QED. For spin-½ particles, the agreement is not complete, but the two theories are indistinguishable until $\gamma B/B_c \sim 10$. Moreover, as announced by Lieu at the SPIN94 conference, this discrepancy is now removed (*7*). The source of the problem has to do with spin-flip effects.

It was predicted by Sokolov and Ternov (8) that electrons should become spin-polarized antiparallel to the field as a result of spin-flip synchrotron radiation. The effect was first observed in 1970 (9) and is routinely seen at the storage rings like the Large Electron-Positron Accelerator (LEP) and the Hadron-Electron Ring Accelerator (HERA), where it is now possible to make longitudinally polarized electrons for high-energy experiments (*10*).

Because the electron "spin" is usually taken to mean the spin angular momentum in the rest frame of the particle, it would be illuminating to calculate the entire spin-½ emissivity in this frame with the Lieu-Axford formalism. However, originally Lieu and Axford (*1*) did not calculate spin-flip effects because they used the spin-averaged Compton cross section (*11*). Instead, the correct procedure will involve the spin-dependent cross section, which contains a simple magnetic dipole (that is, $\mathbf{s}\cdot\mathbf{B}$) interaction term, followed by coherent integration of the amplitudes along the orbit. One can then arrive at a general emissivity formula valid for spin-½ particles, which reduces exactly to the QED result for circular orbits.

Apart from the beam polarization effect already discussed, another potentially important application of this approach has to do with the so-called "beamstrahlung" in high-energy linear colliders, where one beam is deflected significantly by the very strong collective fields of the colliding beam, with consequent radiative deceleration. The fields of concern reach the quantum limit defined in Eq. 1. In particular, because the field gradient can be large, the standard QED formula for circular orbits is not very useful. Perturbation methods have been used to treat small gradients (*12*), but the general formula in (*1*) provides, in principle, a new framework for going beyond this result. There is also relevance to astrophysics because the quantum limit of concern can be reached in pulsars and possibly in the jets of active galactic nuclei (AGN). In the former, Eq. 1 becomes relevant at the pulsar surface, where the magnetic field reaches 10^{12} G, and further out, where the field remains quite high and γ may become large. In the latter, the limit given by Eq. 1 is likewise reachable for AGN jets near the central engine (believed to be a black hole), where fields on the order of 10^3 G are expected and electrons of energy approaching 10^{18} eV could arise by pion production from a directly accelerated proton, if electrostatic potentials as estimated by Lovelace *et al.* (*13*) occur. Finally, the work of Lieu and Axford may lead to new insights on the problem of radiation from charges accelerated by nonelectromagnetic forces.

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A Rising StAR: An Essential Role in Cholesterol Transport

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The biosynthesis of the steroid hormones—estrogens, glucocorticoids, mineralocorticoids, and androgens—starts with cholesterol. Within the mitochondria of the adrenal cortex, gonads, and placenta, cholesterol is converted to pregnenolone by the side chain cleavage, cytochrome P450 (P450scc) This side chain cleavage reaction is the rate-limiting step; however, it is not the complex catalytic mechanism that is rate limiting. Rather, mobilization of cholesterol from lipid stores to the vicinity of P450scc in the inner mitochondrial membrane controls steroid synthesis. In this issue of *Science* (page 1828), Lin and co-workers (*1*) report that the underlying mutation in patients with congenital lipoid adrenal hyperplasia is in the recently discovered steroidogenic acute regulatory protein (StAR) (*2*) and that StAR is now implicated in this mobilization process. The black box of cholesterol transport within steroidogenic cells may soon release its secrets.

Steroid hormone biosynthesis couples the action of peptide hormone receptors with that of nuclear, zinc-finger receptors. Peptide hormones from the anterior pituitary regulate steroidogenesis through adenosine 3',5'-monophosphate (cAMP), leading to the production of steroid ligands for the nuclear receptors. For example, adreno-

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