The Case for a Hubble Constant of 30 km s^{-1} Mpc^{-1}

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Although recent determinations of the distance to the Virgo cluster based on Cepheid variable stars represent an important step in pinning down the Hubble constant, after 65 years a definitive determination of the Hubble constant still eludes cosmologists. At present, most of the observational determinations place the Hubble constant between 40 and 90 kilometers per second per megaparsec (km s^{-1} Mpc^{-1}). The case is made here for a Hubble constant that is even smaller than the lower bound of the accepted range on the basis of the great advantages, all theoretical in nature, of a Hubble constant of around 30 kilometers per second per megaparsec. Such a value for the Hubble cures all of the ills of the current theoretical orthodoxy, that is, a spatially flat universe composed predominantly of cold dark matter.

The hot big-bang model is enormously successful. It provides the framework for understanding the expansion of the universe, the cosmic background radiation (CBR), and the primeval abundance of the light elements, as well as a general picture of how the structure seen in the universe today (galaxies, clusters of galaxies, superclusters, walls, and voids) formed (*1*). Further, some would argue that the current orthodoxy, a spatially flat universe composed primarily of cold dark matter (*2*), is close to bringing the model to an even higher level of success by extending our understanding of the universe from the current limit of about 1 s after the big bang back to around 10^{-34} s (the spatially flat universe is the dividing line between universes that recollapse and those that expand forever).

The latter opinion is not shared by all cosmologists (*3*); some would argue that current challenges to the orthodoxy will upset it and perhaps even lead to the demise of the big-bang model itself (*4*). Those challenges include a resolution of the age or Hubble constant dilemma, determination of the composition and quantity of dark matter, and the formulation of a coherent and detailed picture of the origin of structure in the universe.

Most of the problems associated with the orthodoxy become successes should the Hubble constant H_0 be found to have a value of around 30 km s^{-1} Mpc^{-1}, a value outside of the range of current measurements (*5*) and considerably smaller than the recent determinations based on distances to Cepheid variable stars in the Virgo cluster ($H_0 = 80 \pm 17 \pm 6$ km s^{-1} Mpc^{-1}) (*6*).

The Hubble constant is perhaps the most fundamental cosmological parameter, setting both the time and distance scale for the universe. However, as 65 years of effort testify, it is difficult to measure accurately. In large measure, this owes to the fact that cosmology is an observational rather than experimental science. We take the view that, in instances like this where a key fundamental physical parameter is difficult to determine, theoretical arguments concerning its value can be of some use. In the end, observation will, as it must, have the final say about the value of the Hubble constant.

The Ages of the Universe

Consider first the question of the age of the universe. In the absence of a cosmological constant, the time t_0 since the big bang (expansion age, measured in billions of years) and the Hubble constant are related by

$$t_0 = f(\Omega_0)H_0^{-1} \simeq 9.8f(\Omega_0)(H_0/100)^{-1} \quad (1)$$

Here, $f(\Omega_0)$ is a monotonically decreasing function of the mean density of the universe [obtaining a value of 1 for an empty universe ($\Omega_0 = 0$) and a value of 2/3 for the theoretically favored flat universe ($\Omega_0 = 1$)], Ω_0 is the ratio of the mean density ρ_{mean} to the critical density, $\Omega_0 \equiv 8\pi G\rho_{mean}/3H_0^2$, and G is the gravitational constant. Accurate determinations of the age of the universe are difficult, but recent values based on the ages of the oldest stars are uncomfortably high, 15 ± 3 billion years (*7*), when compared to the expansion age unless the Hubble constant is low.

If the universe is flat and the Hubble constant is at the lower extreme of the currently accepted range, then the expansion age is barely long enough to be consistent with the ages of the oldest stars. Even in the case of an open universe, consistency requires the Hubble constant to be at the low end of the currently accepted range. For example, the age of the universe for a model with $\Omega_0 \approx 0.2$ and $H_0 \approx 70$ km s^{-1} Mpc^{-1} is only about 12 billion years. This all becomes more severe if the oldest stars formed at modest redshifts, say $z \sim 1$ to 2, as might be expected in the cold dark matter model, because it would require the addition of about 3 billion years to the previous estimate for the age of the oldest stars to obtain the age of the universe.

There is another, less direct indication that the age problem is a severe one, requiring a very small value of the Hubble constant. It involves "very red" galaxies observed at redshifts of order unity, for example, the extreme case of the most distant galaxy known, with redshift $z = 4.25$ (*8*). The colors of these galaxies are indicative of an old stellar population, implying substantial evolution. Such colors are hard to accommodate unless the Hubble constant is small because the age of the universe (in billions of years) at redshift z is $t(z) = 2H_0^{-1}/3(1+z)^{3/2} \simeq 1.8\ (30/H_0)$, taking $\Omega_0 = 1$ and $z = 4.25$.

A cosmological constant can ease the age problem to a degree. Representing a repulsive "force" in the Einstein equations, the cosmological constant reduces the deceleration of the universal expansion and thus increases the age of the universe for a given value of the Hubble constant. In a flat model where the cosmological constant accounts for 80% of the mass density, $H_0t_0 \approx 1.1$ and $t_0 \gtrsim 15$ billion years for $H_0 \lesssim 70$ km s^{-1} Mpc^{-1}. Unfortunately, the modern physical interpretation of the cosmological constant, that it arises from zero-point oscillations of quantum fields, gives little support to the idea that a cosmological constant contributes a significant fraction of the critical mass density today. On the contrary, the simplest arguments suggest a value that is many, many orders of magnitude larger than is acceptable, leading to what is referred to as the cosmological constant problem. The severity of this problem suggests that some kind of cancellation mechanism is at work; if that is so, it is difficult to understand how it would give the very tiny value required rather than zero. Although one can solve the age problem by introducing a cosmological constant, it is far from being an an attractive solution.

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Structure Formation and Cold Dark Matter

The detection of variations in the temperature of the cosmic background radiation across the sky (CBR anisotropy) by the Differential Microwave Radiometer (DMR) on the Cosmic Background Explorer (COBE) satellite 2 years ago provided the first evidence for the density inhomogeneities that are believed to have seeded all of the structure seen today (9). Subsequent detections of CBR anisotropy on angular scales from 0.5° to 90° provide further evidence (*10*) and together provide a general confirmation of the gravitational-instability theory of structure formation. According to this theory, tiny ($\simeq 10^{-5}$) inhomogeneities in the matter density were amplified by gravity, producing the plethora of structure in the universe today, including galaxies, clusters of galaxies, superclusters, walls, and voids. The cold dark matter (CDM) model is the most detailed, most studied, and perhaps most successful attempt at constructing a coherent picture of structure formation. However, as the quality and quantity of the data that probe the power spectrum of density inhomogeneities have improved, the case for a discrepancy between the predictions of the simplest version of CDM and the data has grown stronger (*11*) (Fig. 1).

Cold dark matter models are predicated on a flat universe with nearly scale-invariant density perturbations and matter composed mainly of very slowly moving particle relics such as axions or neutralinos. Scale invariant refers to the fact that the perturbations in the gravitational potential have a variance per decade in wave number that is constant. The simplest version of CDM, where the perturbations are precisely scale invariant and the matter content consists exclusively of baryons and CDM particles, cannot simultaneously accommodate the amplitude of the fluctuations as measured by COBE, the large-scale structure as observed by galaxy redshift surveys [such as Cambridge Automated Plate Measuring (APM), Queen Mary College–Durham–Oxford–Toronto (QDOT), and others], the abundance of x-ray clusters, and the small-scale pairwise velocities of galaxies. A low value of the Hubble constant addresses each of these issues.

A quantitative estimate of the problems faced by standard CDM, by which we shall mean CDM with $h = 0.5$ (hereafter $H_0 \equiv 100h$ km s^{-1} Mpc^{-1}), comes from the power spectrum compiled by Peacock and Dodds (*12*) on the basis of the observed galaxy distribution. Their analysis incorporates corrections for redshift-space distortions and nonlinear clustering and uses five different catalogs that probe inhomogeneity on length scales from $10h^{-1}$ to $200h^{-1}$ Mpc. They conclude that the power spectrum of standard CDM has the wrong shape.

To be more precise, although the primeval density perturbations are scale invariant, the fact that the universe made a transition from radiation domination to matter domination at a redshift of about $z_{EQ} \simeq 2.4 \times 10^4 \Omega_0 h^2$ does impose a scale on the power spectrum: $k_{EQ} \simeq 0.5(\Omega_0 h^2)$ Mpc^{-1}, the wave number of the Fourier mode that crossed inside the horizon at matter-radiation equality. The shape of the power spectrum seen at late epochs is determined by this scale times H_0^{-1} (because observations rely on redshift as a distance indicator), leading to the definition of a shape parameter $\Gamma = \Omega_0 h \propto k_{EQ} h^{-1}$ Mpc. In standard CDM, the value of the shape parameter is 0.5; on the other hand, Peacock and Dodds conclude that the data are best fit by $\Gamma = 0.25 \pm 0.05$ (*12*).

The simplest way of achieving this is a low value for the Hubble constant. It has the additional effect of increasing the baryon fraction predicted by the theory of primordial nucleosynthesis (see below), which in turn further alters the shape of the power spectrum by suppressing power on galactic scales. When the higher baryon fraction is taken into account, a Hubble constant of about 30 km s^{-1} Mpc^{-1} provides a very good fit to their data (Fig. 1).

Other fixes—such as mixed dark matter, where the dark matter is composed of a roughly 30% neutrino and 70% CDM mix, the addition of a cosmological constant to CDM, raising of the energy level in relativistic particles by adding new massless particles (which delays matter-radiation equality and therefore has the same effect as lowering the Hubble constant), and "tilting" the primeval power spectrum away from its purely scale invariant form—have also been proposed to address the "shape problem" (*13*). The power spectrum in the mixed dark matter scenario falls dramatically on small scales, and so this version of CDM has difficulty accounting for the early formation of objects such as quasi-stellar objects and large, unbound groups of galaxies at high redshift. With the exception of the model containing a cosmological constant, these variants do not address the problem of the dark matter–to–baryon ratio measured in clusters like Coma. In addition to the fact a cosmological constant of the appropriate size is not well motivated, this fix cannot easily account for the large value of Ω_0 inferred from the analysis of peculiar velocities in our local neighborhood (*14*). At the very least, a low value for the Hubble constant is the most economical solution.

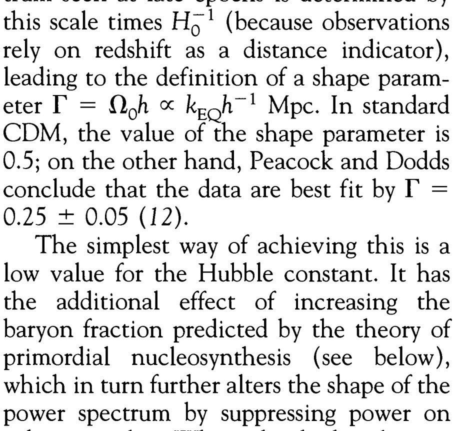

Fig. 1. CDM fluctuation power spectra compared with observations compiled by Peacock and Dodds (*12*) (●). The solid lines show the CDM power spectrum (*40*), normalized to the COBE amplitude (Q_{rms} = 17 μK) for h = 0.2, 0.3, 0.5, and 1.0 (bottom to top) and accounting for the suppression caused by an abundance of baryons consistent with primordial nucleosynthesis. The dashed lines show the CDM power spectrum for h = 0.2 and 0.3 (bottom and top) with $\Omega_B = 0$, showing the importance of the suppression of power on small scales for small values of h. The fluctuation power spectrum is defined as $\Delta^2(k) \equiv k^3|\delta_k|^2/2\pi^2$, where δ_k is the Fourier transform of the density field.

A problem for standard CDM not unrelated to the shape problem is excessive power on small scales. An often-used measure of inhomogeneity on small scales is the variance of the mass in spheres of radius $\sigma_8 = 8h^{-1}$ Mpc; for reference, the variance of optical galaxy counts in such spheres, relative to the mean, is unity (*15*). In standard, COBE-normalized CDM, $\sigma_8 = 1.3$, and a Hubble constant of 30 km s^{-1} Mpc^{-1} results in a significantly lower value, $\sigma_8 = 0.6$. This value agrees with the variance of galaxies detected by the Infrared Astronomy Satellite (IRAS) (Fig. 1) and implies that bright, optical galaxies are a biased tracer of mass whereas IRAS galaxies better trace the mass distribution.

In the certainly oversimplified linear-bias scheme, the distribution of galaxies follows the distribution of mass up to a constant factor know as the bias. For $h = 0.3$, the bias $b \equiv (\delta n_{GAL}/n_{GAL})/(\delta\rho/\rho) = 1/\sigma_8 \approx 1.7$, where n_{GAL} is the galaxy number density and ρ is the density. This agrees with the bias found by several authors using the abundance of galaxy clusters to probe the mass fluctuations on the same scale (*16*). We have used the Press-Schechter formalism (*17*) to calculate the distribution of x-ray emitting galaxy clusters as a function of temperature for several values of the Hubble constant. We compare these results to the data from Edge *et al.* (*18*) and Henry and Arnaud (*19*) (Fig. 2). For COBE-normalized CDM models, $h = 0.3$ provides an excellent fit to the data; it is all the more remarkable considering the extreme sensitivity of the cluster abundance to the Hubble constant, which is caused by the additional suppression of power on small scales because of the higher baryon abundance.

A problem that plagues all unbiased $\Omega_0 = 1$ models is the prediction of galaxy pairwise velocities on scales of around 1 Mpc that are several times larger than observed. The dispersion of the velocity difference of pairs of galaxies, measurable from a catalog of galaxy redshifts, is related to the magnitude of the gravitational attraction of the pairs and thus represents one of the key dynamical probes of the density parameter Ω. Although a bias of 1.7 helps significantly in reducing galaxy pairwise velocities in an $\Omega = 1$ universe, it does not do the whole job. However, Zurek *et al.* (*20*) argue that velocity bias, caused primarily by merging, in which the galaxy velocities do not faithfully represent average particle velocity, reduces the predicted velocities by about 30%. They also suggest that observational bias, which arises in interpreting pairwise velocities in a sample contaminated by Virgo infall (and may be corrected for by treating both data and simulations identically) raises "observed" pairwise velocities by about 50%. In addition, Bartlett and Blanchard (*21*) have shown that the interpretation of the pairwise velocities is sensitive to the unknown distribution of mass around galaxies and, on the basis of a simple model, suggest that this may alleviate the problem even further.

An often-debated issue within the context of CDM models (*3*), especially those with reduced power on small scales, is whether objects seen at high redshift such as quasars can indeed form sufficiently early (*22*). Figure 3 displays redshifts of formation for objects of various mass in COBE-normalized CDM models for different values of the Hubble constant. The formation redshift is the epoch when most of the baryons are in nonlinear objects of the specified mass. Because quasars and even massive galaxies, especially those seen at high redshift, are rare objects, one can multiply the formation redshift (plus one) by, for example, a factor of 3 for 3σ fluctuations because the fluctuation amplitude in the linear regime grows as $(1 + z)^{-1}$. We conclude that even for a Hubble constant as low as 30 km s^{-1} Mpc^{-1}, sufficiently early formation of rare massive galaxies, the likely hosts of quasars, can occur by $z = 5$, the epoch of formation of the first quasars.

Baryon Fraction in Rich Clusters

The final important argument in favor of a low value for the Hubble constant comes from primordial nucleosynthesis and recent determinations of the ratio of dark matter to baryonic matter in rich, x-ray emitting clusters such as Coma. With a single parameter, the baryon-to-photon ratio η, primordial nucleosynthesis successfully accounts for the abundances of the four lightest elements, D, ^{3}He, ^{4}He, and ^{7}Li, provided that $\eta \simeq (2.5 \text{ to } 6) \times 10^{-10}$. This leads to the best determination of the baryon density, $\rho_B = (1.7 \text{ to } 4.1) \times 10^{-31}$ g cm^{-3} (*23*). However, because the critical density depends on the Hubble constant, the fractional contribution of baryons to the critical density also depends on the Hubble constant

$$\Omega_B \equiv \rho_B/\rho_{crit} \simeq 0.009h^{-2} \textit{ to } 0.02h^{-2} \quad (2)$$

The fractional contribution of baryons increases with a lower Hubble constant, although it still must be less than about 20% even if $h = 0.3$. [An even more extreme view than ours has long been advocated by Shanks (*24*), who has argued for a value of H_0 as low as 30 km s^{-1} Mpc^{-1} in order to revive a baryon-dominated universe. However, both primordial nucleosynthesis and recent detections of CBR anisotropy on various angular scales are inconsistent with $\Omega_B \sim 1$ (*25*).]

An accurate determination of Ω_B leads to a test of the orthodoxy that has been much emphasized recently: The ratio of total mass to baryonic mass in a system large enough to represent a fair sample of the universe should be $\Omega_B^{-1} \simeq 50h^2$ to $100h^2$. It has been pointed out that the baryon-to-dark matter ratio in clusters could be problematic (*26*). On the basis of the data of Briel *et al.* (*27*), White *et al.* (*28*) have concluded that the ratio of total mass to baryonic mass in x-ray emitting gas is about $(20 \pm 5)h^{3/2}$ (essentially all the "visible" mass in baryons is in x-ray emitting gas; in this analysis it is assumed that the dark matter in clusters is not comprised of dark baryons).

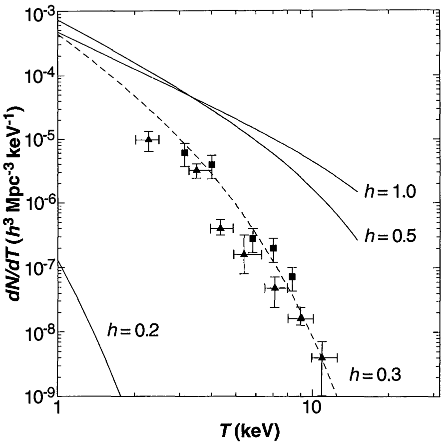

Fig. 2. The distribution of x-ray emitting galaxy clusters as a function of temperature *T*. The curves represent the number *N* predicted in the Press-Schechter formalism (*17*) for several values of *h*. All of the underlying CDM power spectra have been COBE normalized and include the effect of suppression of power on small scales caused by baryons. The data come from Edge *et al.* (■) (*18*) and Henry and Arnaud (▲) (*19*). The dashed line highlights $h = 0.3$, which provides a remarkably good fit to the data.

No value of H_0 within the traditionally accepted range can account for this observation (with $\Omega_0 = 1$). For $H_0 = 30$ km s^{-1} Mpc^{-1}, the orthodoxy is consistent with the data, but just barely; the measured ratio differs from the nucleosynthesis prediction by about two standard deviations. However, it is likely that systematic effects still remain, most of which go in the direction of increasing the total-to-baryonic mass ratio.

For example, the baryon-to-dark matter ratio is likely to be somewhat enhanced because of the settling of baryons caused by the radiative dissipation of some of their infall energy (*28*). Mapping of the mass distribution in clusters by studying the shear of background galaxy images produced by gravitational lensing results in an estimated mass that exceeds that obtained from application of the virial theorem to the hot gas in two separate cases by a factor of about 3 (*29*). This result makes sense if the clusters are not in virial equilibrium or if the gas is partially supported by magnetic fields. The former possibility is inferred to be the case for the hot, x-ray emitting gas that is still undergoing infall according to cluster simulations and is also seen to show substructure (*30*). That this may be a more or less ubiquitous phenomenon is suggested indirectly by the requirement that substantial amounts of intracluster gas must have only merged recently, moving at a speed comparable to the sound velocity, in order to provide enough ram pressure to account for radio-source morphologies (*31*). Ensuing gas clumpiness would also lower the inferred gas mass and, together with upward correction of cluster mass estimates, could

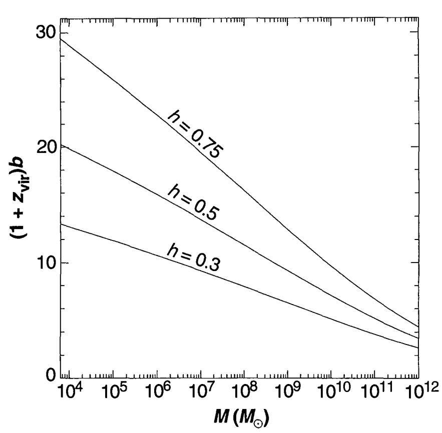

Fig. 3. Virialization redshifts for objects of various mass (measured here in solar masses). The virialization redshift is defined to be the redshift at which the bulk of the matter condenses into objects of a specified mass (baryonic + CDM) multiplied by the bias factor *b* (about 1.7). Details of the calculation are given in (*41*). [This figure was prepared by M. Tegmark.]

comfortably reconcile cluster gas content with nucleosynthesis predictions for $H_0 = 30$ km s^{-1} Mpc^{-1}.

Concluding Remarks

As we noted in the beginning, ultimately the value of Hubble constant will be determined by measurements and not the wishes of theorists. In the near term, the best prospects for indirect confirmation of our provocative suggestion involve measurements of CBR anisotropy (*10*). Compared to standard CDM, degree-scale CBR anisotropy is predicted to be about a factor of 1.5 larger (Fig. 4). Although the experimental situation on the degree scale is not settled at the moment, there is statistical support for the higher range of detections around $\delta T/T \sim 3 \times 10^{-5}$ (*32*), such as those of the MAX (*33*) and Python (*34*) collaborations. These detections are compatible with CDM, provided that $H_0 \lesssim 50$ km s^{-1} Mpc^{-1}.

The ultimate test is a definitive measurement of the Hubble constant itself. It has been argued that a variety of techniques provide strong evidence that the Hubble constant is 80 ± 10 km s^{-1} Mpc^{-1} (*5*). Further, measurement of the distance to the Virgo Cluster by means of Cepheid variable stars made by the refurbished Hubble Space Telescope also lend support to this value (*6*). However, most of the techniques that are converging on this value, including Tully-Fisher, surface-brightness fluctuation, fundamental-plane, and planetary-nebulae techniques, involve the same lower rungs on the infamous distance ladder and thus could have a common systematic error. From our perspective, the most troublesome measurements are those based on type II (core collapse) supernovae; they "jump" the distance ladder—and thus do not share common systematic errors with the previously mentioned methods—and still give a value consistent with 80 ± 10 km s^{-1} Mpc^{-1} (*35*).

This leaves two lines of defense for our hypothesis: (i) common systematic error in the empirically based determinations and an error in the type II supernovae determination; or (ii) current measurements have yet to reach sufficient distances to sample the Hubble flow (or are still influenced by Malmquist bias). Both possibilities have some merit (*36*), but neither offers an easy resolution.

With regard to the first, because this is not the place to debate the intricacies of the distance scale, we simply remind the reader that since Hubble's time, the distance scale has changed by about a factor of 10, and so the issue may well not be settled yet. With regard to the second, we note that the only truly global measurements of H_0 (that is, measurements at redshifts $z \gtrsim 0.1$ to 0.2), namely those using the Sunyaev-Zel'dovich effect in galaxy clusters and time-delay measurements in the images produced by the gravitational lensing of a variable quasar, favor a value of H_0 systematically lower than is obtained from the more local measurements. Although these techniques are new and their results can only be considered preliminary, we note that a recent study of A2218 using the Interferometric Ryle Telescope places H_0 in the range 20 to 75 km s^{-1} Mpc^{-1} (*37*) and that modeling of the double quasar Q0957+561 gives $H_0 = 37 \pm 14$, if one adopts the time delay of Press *et al.* (*38*) and the velocity dispersion of Rhee (*39*).

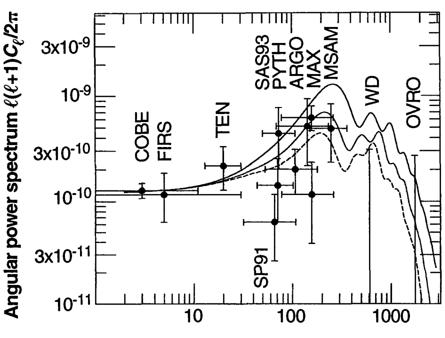

Fig. 4. Angular power spectrum for standard CDM with a Hubble constant of 30, 50, and 75 km s^{-1} Mpc^{-1} (top, middle, and bottom curves, respectively), with $\Omega_B h^2 = 0.015$. [Calculations provided by N. Sugiyama; compilation of anisotropy experiments supplied by Scott and White (*32*).] The CBR temperature anisotropy predicted for a given experiment depends on its filter function; very roughly, for an experiment that measures the temperature difference on angular scale θ, $\delta T/T \sim \sqrt{\ell(\ell+1)\langle|C_\ell|\rangle/2\pi}$ for $\ell \sim 200°/\theta$. The horizontal error bars represent the half-peak points of the window functions; the precise location of the vertical error bars is spectrum-dependent, and indicative values only are shown. Error bars are $\pm 1\sigma$; upper limits are 2σ.

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