Fractional Charges in an Interacting Electron System

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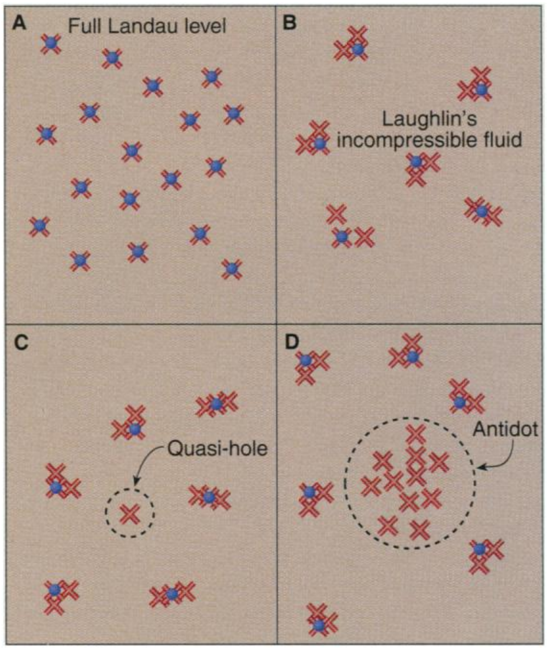
Systems in nature with many interacting constituents can exhibit properties that would be unrealizable if the constituents acted independently. In condensed matter physics, examples of this maxim abound when electrons in metals or semiconductors are strongly correlated. An important source of such examples is the fractional quantum Hall regime, where electrons are confined to a layer and a strong perpendicular magnetic field is oriented perpendicular to the layer. At particular ratios of the electron density to the magnetic field strength, the electrons organize themselves into strongly correlated (*1*) uniform electron density states. According to theory (*1*), the elementary excitations of these states have a quantized charge that is, paradoxically, a fraction of the quantized charge of the electrons of which the system is composed. The experimental work of Goldman and Su that appears in this issue of *Science* (*2*) reports on a direct measurement of these fractional charges.

When no magnetic field is present, the quantum states of a single electron have a definite momentum **p** and kinetic energy $\varepsilon(\mathbf{p}) = |\mathbf{p}|^2/2m$. When many electrons are present, it is still a good approximation, for many purposes, to ignore the electron-electron interactions that tend to correlate the motions of different electrons. The ground state of a many-electron system is then formed by placing electrons in the individual-particle states with the lowest available energy. Because electrons are fermions, the many-electron state must respect the Pauli exclusion principle, which forbids having more than one electron in a given individual-particle state. This gives rise to a ground state in which individual-particle states with momentum inside a sphere in momentum space (the Fermi sphere) are occupied and those outside the Fermi sphere are empty. Electron-electron interactions often have no essential importance, in part because correlations can be introduced only by mixing into the ground-state electronic configurations that have a larger kinetic energy than the ground state. Many-electron states of this type are called Fermi liquids.

All this is changed in the strong-magnetic field environment in which the fractional quantum Hall effect occurs. Classical charged particles confined to a plane and moving in a magnetic field are deflected by the Lorentz force. Electrons move in circular cyclotron orbits with angular frequency $\omega_c \equiv eB/m$, where e is the magnitude of the electron charge, B is the magnetic field strength, and m is the electron mass. When the cyclotron motion is treated quantum mechanically, it turns out that kinetic energy values of an electron in a magnetic field are quantized, $\varepsilon_n = \hbar\omega_c(n + 1/2)$, where n is an integer and $\hbar$ is Planck's constant divided by 2π. The kinetic energy of a cyclotron orbit is independent of the location of the center of the orbit, and correspondingly, there are many allowed individual-particle states with a given quantized kinetic energy. The number of available individual-particle states with a given kinetic energy is proportional to the magnetic field strength. The fractional quantum Hall effect occurs when the number of electrons is smaller than the number of available individual-particle states with the minimum allowed kinetic energy. There are then many configurations of the electrons with identical kinetic energies, and these can be freely mixed together to optimize electronic correlations.



Views of four many-electron states. In these schematic illustrations, red crosses represent zeros of the many-electron wave functions as a function of one coordinate, and blue dots represent the positions of the other electrons. (**A**) A full Landau-level state in which the number of zeroes equals the number of electrons. (**B**) An incompressible fluid state in which three zeroes are bound close to the positions of each electron. (**C**) A state with a single zero, that is, a single quasi-hole, that is bound to a maximum in the disorder potential rather than to other electrons. In this single–quasi-hole state, zeroes will occur at the quasi-hole site as a function of every electron coordinate. (**D**) A state with many quasi-holes bound inside an antidot as studied experimentally by Goldman and Su (*2*).

When the kinetic energy is minimized, it turns out that many-electron wave functions (Ψ) have the following property, which is useful (*3*) for thinking about correlated many-electron states: The area over which the electron system is spread is proportional to the number of zeroes of the many-body wave function as a function of any individual-particle coordinate (the constant of proportionality is the area A_0 that encloses one quantum of magnetic flux $\Phi_0 = hc/e$ from the perpendicular field; here, h is the Planck constant and c is the speed of light). The figure schematically illustrates the locations of the zeroes of Ψ as a function of one electron position with the positions of all other electrons held fixed. In this point of view, the Pauli exclusion principle requires that no two electrons be at the same position, so one zero of the wave functions must occur at the location of each of the other electrons. The minimum area consistent with the minimum kinetic energy for a system consisting of N electrons is therefore $(N - 1)A_0$. As the electron density decreases, the average number of zeroes of Ψ per electron increases. The simplest of the strongly correlated states discussed above occurs when the number of zeroes per electron is three; in the ground state, three zeroes are bound close to the positions of the other electrons, and the electron density is $(3A_0)^{-1}$. These states are called incompressible because, for a given number of electrons, their area is fixed. The energy of this state is low because the quantum mechanical probability density $|\Psi|^2$ becomes very small when any two electrons are close together. It is now possible to understand the origin of the fractionally charged excitations of this state. When the area per electron is close to but larger than $3A_0$, then Ψ will have additional zeroes. The extra zeroes will tend to be bound to maxima of any random disorder potential. These extra zeroes are the fractionally charged excitations. Each extra zero will increase the area of the electron system

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by A_0; because the electron density is $(3A_0)^{-1}$, it follows that the magnitude of the charge of these elementary excitations of the system is $e/3$.

The report by Goldman and Su (2) describes an experiment in which an "antidot" is artificially created in the interior of an incompressible fluid of precisely this type. The antidot is created by forming a maximum in the electrostatic potential seen by the electrons. In the experiment, the antidot is surrounded by an incompressible fluid with density $(3A_0)^{-1}$. The tunneling conductance measured in the experiment has peaks when states that differ through the transfer of charge from the antidot to the outer edge of the incompressible fluid are nearly degenerate. The experiments are performed at low temperatures where only very low energy states of the electron system are relevant. According to the above discussion, we should expect the peaks to correspond to degeneracy between states in which the number of quasi-holes N_{qh}, that is, the number of zeroes of Ψ, bound inside the antidot differs by one. When the magnetic field is increased, A_0 decreases and level crossings occur in which the number of quasi-holes in the ground state increases in order to keep the area of the antidot $A_{anti} = N_{qh}A_0$ as close as possible to the value dictated by electrostatics. The separation in magnetic field between level crossings, signaled in these experiments by tunneling conductance peaks, constitutes an experimental measurement of A_{anti}. The charge associated with an individual quasi-hole can then be determined from the separation between the tunneling conductance peaks that occur as a function of the voltage between the electron system and a back gate. Provided they are sufficiently remote, the electron system and the back gate together act like a parallel-plate capacitor system, and the rate of change of the charge on the antidot with the backgate voltage is known. The experiment shows charge transfer, flagged by tunneling conductance peaks, occurring in discrete $e/3$ units.

This experiment confirms a basic prediction arising from Laughlin's theoretical work (1). However, it now seems that the creation of fractional charges from whole ones is one of the simpler tricks that electrons can perform in the fractional Hall regime. When the electron density is sufficiently different from $(3A_0)^{-1}$, the fractionally charged quasi-particles become dense, and the picture described above fails. Incredibly, recent experimental (4) and theoretical work (5) appears to show that when the density is $(2A_0)^{-1}$, the electrons organize themselves into a Fermi liquid state with a phenomenology similar to that of noninteracting electrons. However, because of the kinetic energy quantization, this exotic Fermi liquid must be composed of particles with an energy due entirely to electron-electron interactions. Nevertheless, these particles appear to interact relatively weakly with each other. The method by which this artifice is achieved is not yet fully understood. The intricacy of quantum interacting electrons in the fractional Hall regime continues to surprise.

References

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Express Yourself or Die: Peptides, MHC Molecules, and NK Cells

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Conventional wisdom has it that the lymphocytes defend the body from infectious agents and aberrant cells by recognizing the "foreign," detecting nonself proteins in the body fluids or on surfaces of cells. However, natural killer (NK) cells can also operate by the opposite strategy: They recognize and eliminate cells because critical "self" proteins are absent from the cell surface (1). NK cells refrain from killing when the target cells express self class I molecules of the major histocompatibility complex (MHC). When these molecules are absent or expressed only in reduced amounts, the NK cells proceed with their attack and can thereby reject tumor, virus-infected, and transplanted cells.

But what is the exact nature of the magic password that NK cells search for when they assess a cell? Is it a protein or carbohydrate part of the polymorphic MHC class I molecule itself? Or is it the 8– to 11–amino acid peptide carried by this molecule from the cytosol of the cell to the surface, where it is displayed as a quality control sample to another lymphocyte, the T cell? At least part of the answer to these questions emerges in two new studies. Malnati *et al.*, in this issue of *Science* (2), show that one (but not every) peptide that binds to the empty human leukocyte antigen (HLA)–B27 class I molecules on a mutant cell can confer protection against a human NK cell clone. Notably, the peptide that protects corresponds in sequence to a peptide from class I molecules of normal cells. The phenomenon is, as immunologists would like it to be, highly specific: Protection is seen only with certain NK clones. In the murine system, Correa and Raulet (3) demonstrate that most if not all peptides provide binding to the class I molecule H-$2D^d$ and can protect against a subset of NK cells.

NK cells exist as preactivated killer cells in the blood and in many organs without need for prior vaccination (4). They can be identified by their characteristic profile of cell surface molecules–Fc receptors for immunoglobulin, several adhesion molecules, and one or more C-type lectin-like receptors. The NK cell does not have a single rearranged *receptor magnificus*, such as that of the T cell or B cell immunoglobulin, but rather appear to operate by several different recognition strategies. Loss of class I molecules is sufficient to induce NK sensitivity (1, 5), even of normal cells. Also, human NK clones kill allogeneic but not autologous cells—because of the absence of self rather than the presence of nonself molecules on the surface of the target cell (6).

There are two general models for how NK cells might detect missing self molecules in these situations (1). The "effector inhibition" model postulates that NK cells are initially triggered to kill by broadly distributed molecules on most cells, but that the lytic program will be canceled by negative signals from receptors upon their recognition of specific MHC class I alleles on the target cell. The alternative "target interference" or "masking" model (1, 7) postulates that triggering receptors on NK cells recognize target structures that can be masked or otherwise interfered with by class I molecules of the target cell.

Restoration of the pathway that presents MHC class I peptides by transfection of defective mutant cells also restores their ability to resist the attack from NK cells (8). These experiments, however, do not directly test the role of the peptide. The elegant system used by Malnati *et al.* assesses the contribution of peptide by relying on three critical components: NK clones of different HLA specificity, a panel of genetic variants of a class I molecule called HLA-

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