MATHEMATICS

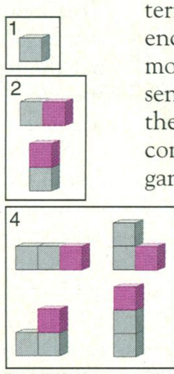
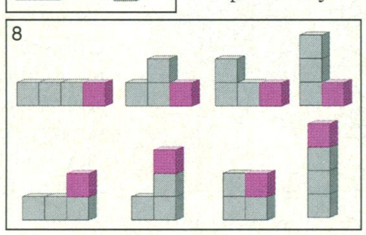
Mathematicians Get an On-Line Fingerprint File

When the police arrive at a crime scene, one of the first things they do is dust for fingerprints. Mathematicians have less experience with corpses and blunt instruments, but when faced with a new problem, they often try to lift a few prints themselves. They're on the lookout for what Ron Graham of the information sciences division at AT&T Bell Labs calls combinatorial fingerprints: characteristic sequences of numbers, such as 1, 2, 5, 14, 42, ..., generated by the problem. Now mathematicians, like the fingerprint experts who consult the FBI, have their own clearinghouse for suspicious prints: an on-line "encyclopedia of sequences" developed by Neil Sloane at Bell Labs.

"I think it's super," says Herb Wilf, a combinatorialist at the University of Pennsylvania. Since the encyclopedia went on line in late March, he says, "I've already made a number of requests to it, with very good results." When a new sequence matches one in the database, there's a good chance the problem is an old one in a new disguise. When it doesn't, the problem may be new. And a partial resemblance may hint at a deep connection between the new problem and an existing one.

ILLUSTRATIONS: K. SUTLIFF

These clues emerge because a sequence is a tip-off to the nature of the problem that generates it. For example, stacking blocks seemingly has little to do with powers of 2. But the sequence it gives rise to says otherwise. There's only one way to stack one block, two ways to stack two, four ways to stack three, and eight to stack four. The appearance of powers of 2 is a clue to a simple doubling process that comes into play each time you add a new block (see figure at right).

16 . . .

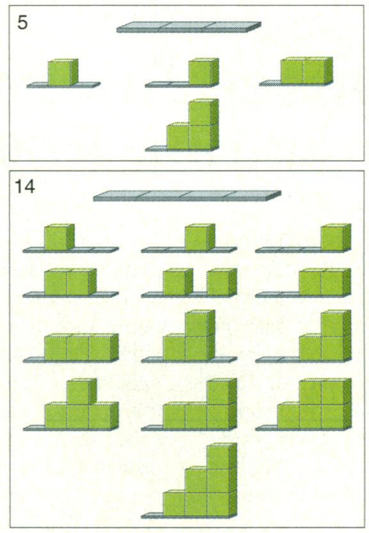
Doubling up. The possible stacking patterns for sets of blocks generates a sequence of powers of two.

Sloane has been fascinated by such sequences since the 1960s. "I like sequences myself," says Graham. "Neil likes them a *lot*." In 1973, Sloane published *A Handbook of Integer Sequences*, a book listing more than 2300 sequences, with references to the problems that generated them. Combinatorial theorists received it enthusiastically; as one fervent reader wrote, "There's the Old Testament, the New Testament, and the *Handbook of Integer Sequences*."

Since then, Sloane has been accumulating entries for a new version. When he finally got around to preparing the update, he had "about a cubic meter" of material to sort through, consisting of letters, preprints, reprints, books, and technical reports, all containing new sequences. With help from Simon Plouffe at the University of Quebec at Montreal, Sloane started putting the update together. "Once I'd got that far, it wasn't that much more work to put up this on-line version," he recalls.

Researchers can consult the encyclopedia through two on-line sequence analyzers. The first, whose e-mail address is sequences@research.att.com, accepts messages listing terms from the suspect sequence (the more terms the better). It returns matches from the encyclopedia's table, which currently has more than 5000 entries. For example, if you send the request "lookup 1 2 4 10 25 64 172," the program will suggest you're looking at a combinatorial problem that comes up in organic chemistry: the number of esters that can be made with n carbon atoms (n increases by 1 for each place in the sequence and, in this case, starts with $n = 2$). The program will also refer you to a book called *Chemical Applications of Graph Theory* and give you the next 16 terms, the largest being 6,011,920,720.

"What this table is," explains Sloane, "is an index to all of mathematics." If you find your sequence in this index, he adds, "it [may] draw your attention to the fact that somebody else has already solved [your] problem—or if not solved it, at least worked very hard on it." Or it may draw an unexpected connection between problems. Take that sequence 1, 2, 5, 14, 42, ... You might run across it in counting the number of ways you can stack blocks in n columns so that the leftmost column is empty and no column is more than one block taller than the column to its left (see figure at top). But the same sequence also occurs in counting the number of ways you can put parentheses around symbols in an algebraic product, in enumerating the different ways to cut a triangle, square, or pentagon into triangles, and in dozens of other counting problems.



42 . . .

Celebrity sighting. Stack blocks in 3, 4, 5,... columns with the leftmost column empty and no column more than one block higher than the column to its left, and the result is a sequence well-known in mathematics.

Not finding a sequence in the database, notes Wilf, "means you're probably into some new territory." But first you should let Sloane's second analyzer, superseeker (e-mail: superseeker@research.att.com), take a crack at the sequence.

Superseeker begins by looking for matches in the table, but if it doesn't find any, it doesn't stop there. Instead, it runs a battery of programs to see whether your sequence can be transformed into one on its list—for example, it checks to see whether the sequence made by taking the difference of consecutive terms of your sequence is familiar—or can be explained by simple mathematical formulas (if you give superseeker enough terms, it will recognize the sequence 3, 8, 17, 32, 57, 100 as numbers of the form $n^2 + 2^n$). As a last resort, "it goes through all the sequences in the table and looks at how 'close' they are to your sequence," Sloane explains. "If you've made a typing error, for example, then this might pick it up." Because superseeker can spend up to several minutes on a sequence, Sloane limits each user to one request per hour.

Wilf found out how slick Sloane's program is when he tried to trick it. "I sneaked up on it and handed it a sequence that I had generated myself out of a recurrence relation that had second-order polynomial coefficients," Wilf recalls. "Sure enough, it rediscovered the recurrence relation."

Sloane's own favorite sequence starts out 1, 11, 21, 1211, 111221, 312211, 13112221. The next term is 1113213211—but what comes after that? Young children just starting to read numbers often see the pattern right away. The rest of us can find the answer below—or try looking it up in the on-line encyclopedia of sequences.

–Barry Cipra

Solution to 1, 11, 21, 1211, 111221, 312211, 13112221, ...: To get the next term, read the last term out loud as „one one, one three, two ones, three twos, one one," then convert the words into digits, to get 1113213211. The three terms after that are thus 31131211131221, 13211311123113112211, and 11131221133112132113212221.