Confirmation of Earth-Mass Planets Orbiting the Millisecond Pulsar PSR B1257+12

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The discovery of two Earth-mass planets orbiting an old (~10^9 years), rapidly spinning neutron star, the 6.2-millisecond radio pulsar PSR B1257+12, was announced in early 1992. It was soon pointed out that the approximately 3:2 ratio of the planets' orbital periods should lead to accurately predictable and possibly measurable gravitational perturbations of their orbits. The unambiguous detection of this effect, after 3 years of systematic timing observations of PSR B1257+12 with the 305-meter Arecibo radiotelescope, as well as the discovery of another, moon-mass object in orbit around the pulsar, constitutes irrefutable evidence that the first planetary system around a star other than the sun has been identified.

Searches for extrasolar planetary systems address the fundamental problem of the frequency of occurrence of planetary systems in the galaxy and the related question of our chances of finding extraterrestrial life. The steadily growing observational evidence for disks or rings of solid matter around stars at various evolutionary stages (*1*) has yet to culminate in a detection of planetary mass objects associated with stars other than the sun. Aside from possible astrophysical reasons, this unsatisfying situation is most certainly related to limited detection capabilities inherent in searches for extrasolar planets at optical wavelengths. For example, present techniques based on Doppler spectroscopy have long-term velocity resolution of about 10 m s^{-1}, which is not sufficient to detect sub-Jovian mass planets around sun-like stars (*2, 3*).

In the absence of discoveries of planetary systems around solar-type stars and with all the previous reports on planets around neutron stars either unconvincing or retracted (*4–6*), confirmation of the existence of two Earth-mass bodies orbiting the 6.2-ms pulsar PSR B1257+12 (*7*) has become particularly important. In this case, the evidence for planets is based on the observed periodic delays in the arrival times of pulsar pulses, which are interpreted as a signature of reflex motion of the neutron star induced by the presence of the two orbiting planets. Variations in the pulse arrival times due to each planet amount to about ± 1.5 ms, which corresponds to a ± 0.7 m s^{-1} amplitude variation in the pulsar's radial velocity. In addition, a much lower amplitude, microsecond-level effect due to planetary perturbations should be present in pulse timing residuals, if the above interpretation is correct (*8, 9*). A detection of this latter effect to obtain a final proof of reality of the pulsar planets has been the most important goal of the PSR B1257+12 timing program.

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Following the discovery of PSR B1257+12 (*10*), systematic pulse timing observations have been conducted with the Arecibo telescope since July 1990. The Princeton Mark III pulsar processor has been used to record the times of arrival (TOAs) of the pulsar pulses with a long-term accuracy of about 3 μs. Nearly simultaneous observations at 430 MHz and 1400 MHz have been made to monitor the influence of the interstellar propagation effects on TOA measurements. The analysis of the timing data has been carried out using the model fitting computer code TEMPO (*11*) modified to accommodate multiple orbit timing models. More details concerning the original survey, the following timing observations, and the analysis can be found in (*7, 10, 12, 13*) and references therein.

After the publication of the original account of planets around PSR B1257+12 (*7*), based on the first 16 months of timing measurements, two update analyses of the growing data set have been reported (*12, 13*). These results have shown that the measured TOAs continue to exhibit two highly stable, nearly sinusoidal periodicities at 66.6 and 98.2 days, in a precise agreement with the initial observations. An independent analysis of timing observations of PSR B1257+12 with the 140-foot telescope at Green Bank has led to exactly the same conclusion (*14*). The least-squares fit of the timing model consisting of the standard pulsar parameters and two Keplerian orbits to the observed TOAs is consistent, to within 0.1 percent, with the assumption that PSR B1257+12 is orbited by two bodies with minimum masses of 3.4 $M_\oplus$ and 2.8 $M_\oplus$ (*7, 12, 13*).

In addition to regular timing observations, an effort has been made to verify the idea that the observed periodicities could be generated by neutron star precession (*15*) rather than by planets. Since this effect is expected to lead to changes in the pulse profile morphology (*16–18*), the high time resolution profiles of PSR B1257+12 have been carefully examined in an attempt to detect any such low-level variations (*19, 20*). No pulse shape variations of any kind have been found at a level of ~1 percent.

Another planet in the PSR B1257+12 system. The results summarized above provide compelling evidence that the observed periodic TOA variations are indeed due to the orbital motion of two Earth-mass planets around the pulsar. The possibility of another planet, in a 1-year orbit, was indicated by early analysis (*7*) but has been shown to be a consequence of the initially unrecognized ~200 km s^{-1} proper motion of the pulsar (*12, 13*). This effect, when not corrected for, contributes a sinusoidal variation with a 1-year period and with gradually increasing amplitude to the observed TOAs. Further analysis of the low-level timing residuals left over after fitting out the effects of the two planets has revealed the presence of a 25.3-day periodicity in the data. Initially, it was detected as a pronounced peak in the fluctuation spectrum of the post-fit residuals. Later, it was found directly in the data taken in April and July through August 1993 by measuring the TOAs every day over the intervals of time comparable to the expected period. This low-amplitude (±4 μs) periodicity indicates the presence of another low-mass body in orbit around PSR B1257+12. The post-fit timing residuals due to the three periodicities, labeled A, B, and C and folded modulo the respective periods from all available data, are shown in Fig. 1. The corresponding best fit three-planet timing model is shown in Table 1. No other strictly periodic modulations have been seen in the data at the present 3-μs timing precision level over the 3-year span of observations.

The detection of an inner planet A with about the mass of the moon suggests that continuing timing observations of PSR B1257+12 may reveal further objects belonging to the planetary system around the pulsar. This possibility is indicated by the presence of a significant second derivative of the pulsar period, $\ddot{P}$, in the timing model (Table 1). Although it cannot be ruled out that this effect is caused by long-term timing noise (*21*), the existence of three planets around the pulsar makes it quite plausible that the nonzero $\ddot{P}$ is due to changing acceleration induced by yet another, more distant planet with a much longer orbital period. In fact, the measured spindown rate, $\dot{P}$, of the pulsar (Table 1) can also be affected by acceleration, in addition to a calculable contribution from large proper motion (*22*). Continuing timing observations of PSR B1257+12 will help to resolve this intriguing problem.

Detection of planetary perturbations. Earlier detections of planets around pulsars have remained unconfirmed (*4, 6*) or have been retracted (*5*). In the case of PSR B1257+12, the evidence presented so far is difficult to refute, but it obviously arises indirectly from the analysis of measurements. PSR B1257+12 is about 400 pc away from the sun and a direct detection of any emissions from its planets will be very difficult, as evidenced by the recent unsuccessful infrared observations of the pulsar (*23–25*). However, it has been demonstrated that an approximately 3:2 orbital resonance between planets B and C leads to accurately predictable periodic perturbations of the two orbits whose effect on pulse arrival times may be measurable (*8, 9*). Detection of planetary perturbations would yield a unique proof that the timing behavior of the pulsar is a result of the orbital dynamics of two planet-sized bodies. Among the effects of the gravitational interaction between the two planets, the near-resonant periodic variation of the elements of their orbits should be easiest to detect. If the short-term, nonresonant fluctuations are averaged out and planetary masses are not too large, this variation can be adequately approximated by smooth, periodic oscillations of the orbital elements about their mean Keplerian values (*8*). The oscillation period is defined in terms of the mean angular velocities of the planets, n_1 and n_2, as $2\pi/(2n_1 - 3n_2) = 5.56$ years and the predicted maximum amplitude of the corresponding timing residuals (after fitting out the two noninteracting orbits B and C) is a function of planetary masses and the total time span of observations (*8, 9, 26*). A 3.5-year segment of resonant and nonresonant perturbations of orbital periods obtained from numerical integration of the equations of motion of planets B and C is shown in Fig. 2, A and B.

A purely phenomenological test for the presence of smooth variations of orbital elements of any origin, on the time scales comparable to the available data span, can be carried out by including the first-order time derivatives of orbital parameters in the process of fitting timing models to the observed TOAs. This approach is greatly simplified by the fact that changes in the orbital periods should be easiest to detect, because they result in the largest orbital phase deviations accumulated over time. Therefore, it is sufficient to restrict the test to fitting for the derivatives of the orbital periods alone. Since the perturbation-induced variations of the orbital periods over a 2- to 3-year interval can be adequately represented by their time derivatives (Fig. 2, A and B), the above procedure provides an efficient means to detect a dominant component of the perturbation effect, given a time-limited set of TOAs. For example, in the case of coplanar, high inclination orbits ($1/\sin i \approx 1$), the predicted approximate numerical values of these derivatives are $\dot{P}_{b1} \approx -14 \times 10^{-6}$ and $\dot{P}_{b2} \approx 26 \times 10^{-6}$ for planets B and C, respectively.

The timing residuals of PSR B1257+12 left over from the least-squares fit of a three-planet model without perturbations to the 3-year set of pulse arrival time data are shown in Fig. 2C, together with the

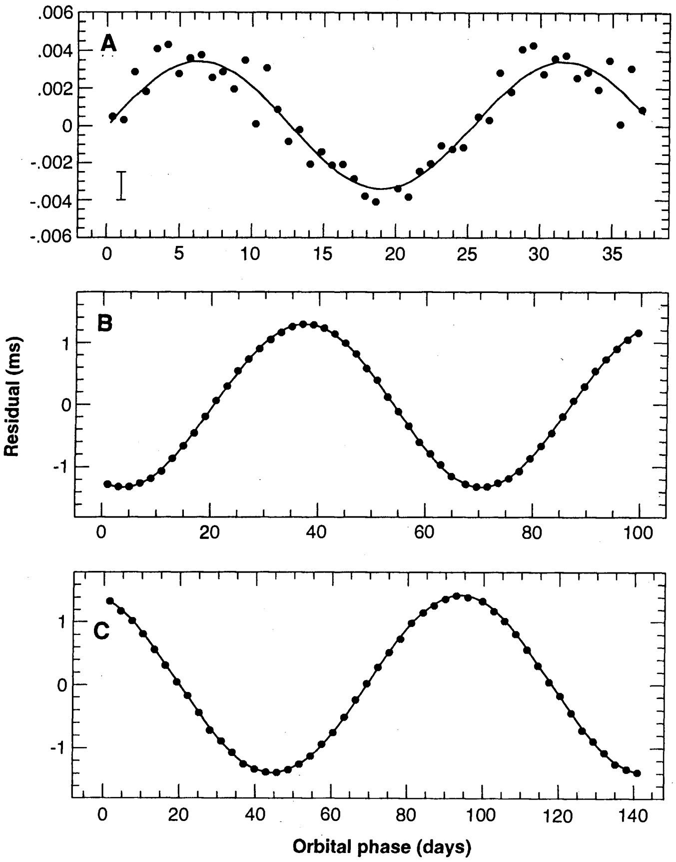

Fig. 1. The post-fit residuals of pulse arrival times from PSR B1257+12 folded modulo the orbital periods of 25.34 days (planet A), 66.54 days (planet B), and 98.22 days (planet C), over a 3-year span of timing observations. In each case, the arrival time variations due to the other two planets have been fitted out. For planet A, a 2σ uncertainty in residuals due to phase binning is indicated by the error bar. For planets B and C, the uncertainties are too small to be shown.

Table 1. Parameters of the PSR1257+12 planetary system

	Pulsar parameters		
Rotational period (s)			0.0062185319388187(2)*
Period derivative (s s^{-1})			$1.14334(6) \times 10^{-19}$
Second period derivative (s^{-1})			$4.5(9) \times 10^{-30}$
Right ascension, α_{1950}			12 h 57 m 33.12730(3) s
Declination, δ_{1950}			12° 57′ 06.406(1)″
Proper motion in α (marcs year^{-1})			46.4(6)
Proper motion in δ (marcs year^{-1})			−82.9(9)
Epoch			JD 2448788.9
Dispersion measure (pc cm^{-3})			10.186(1)
	Keplerian orbital parameters		
Planet	A	B	C
Semi-major axis (light ms)	0.0035(6)	1.3106(6)	1.4121(6)
Eccentricity	0.0	0.0182(9)	0.0264(9)
Epoch of periastron (JD)	2448754.3(7)	2448770.3(6)	2448784.4(6)
Orbital period (s)	2189645(4000)	5748713(90)	8486447(180)
Longitude of periastron (deg)	0.0	249(3)	106(2)
	Parameters of the planetary system		
Planet mass ($M_\oplus$)†	$0.015/\sin i_1$	$3.4/\sin i_2$	$2.8/\sin i_3$
Distance from the pulsar (AU)†	0.19	0.36	0.47
Orbital period (days)	25.34	66.54	98.22

*Figures in parentheses are 3σ statistical uncertainties in the least significant digit quoted. †Assumed pulsar mass, $M_{psr} = 1.4\ M_\odot$.

simulated effect of neglecting orbital period derivatives in the fitting process. Clearly, the measurements made in 1993 exhibit variations that are in a qualitative agreement with the theoretical prediction. As the effect of nonzero orbital period derivatives is very small, a more quantitative assessment of its importance is difficult to achieve by means of a direct least-squares χ^2 minimization technique. A more reliable method is to map the $\Delta\chi^2 = \chi^2 - \chi_0^2$ surface around its global minimum, χ_0^2, by holding $\dot{P}_{b1}$ and $\dot{P}_{b2}$ at fixed values and fitting for all other model parameters. The result of applying this approach to data is shown in Fig. 2D in the form of a contour map of confidence levels for both period derivatives. The nonzero orbital period derivatives are highly significant and their signs and magnitudes are consistent with the predictions of the theory of planetary perturbations, assuming two Earth-mass planets in approximately coplanar, high-inclination orbits around the pulsar.

The above result fully justifies the implementation of a model-dependent approach to study the dynamics of the PSR B1257+12 planetary system and to test the gravitational perturbations between planets B and C in more detail. In fact, the test involving the derivatives of orbital periods has been becoming increasingly inaccurate, because of the changing signs of $\dot{P}_{b1}$ and $\dot{P}_{b2}$ (Fig. 2, A and B). Furthermore, the $\dot{P}_{b1} - \dot{P}_{b2}$ test indicates that the effect of perturbations on timing residuals is small (Fig. 2C), which implies that the short-term and long-term perturbations are of comparable magnitude (*26–28*) (see also Fig. 2, A and B) and should be modeled more accurately. Finally, a comparison of the full model of planetary perturbations with the timing data is necessary to derive reliable constraints on the dynamical parameters of the PSR B1257+12 system.

A detailed model of mutual gravitational perturbations of planets B and C near a 3:2 resonance and their effect on pulse timing residuals has been discussed in (*8, 9*) and (*26–28*). A method of solving the equations of planetary motion by means of the Burlisch and Stoer algorithm (*29*) with initial conditions specified by the pulse timing model of PSR B1257+12 has been considered in (*26*) and (*30*). Briefly, the presence of perturbations will manifest itself in the form of oscillating timing residuals, if the least-squares fit of a timing model to data assumes fixed-parameter, noninteracting orbits. These oscillations are characterized by time scales of the order of orbital periods of the planets and by amplitudes that depend on the planet-to-pulsar mass ratios. For a single planet, this mass ratio is expressed in terms of the orbital inclination as $m/(m + M_{psr}) \approx m/M_{psr} = v_{psr}/(an \sin i)$. Here m, M_{psr}, v_{psr}, a, and n are the planet mass, the pulsar mass, the observationally determined maximum line-of-sight velocity of the pulsar relative to the system center of mass, the semimajor axis of the planet's orbit, and the mean orbital angular velocity, respectively. From the Keplerian relationship $a = (GM_{psr}/n^2)^{1/3}$, with observed quantities given by the timing model of Table 1, and with the planetary masses $m_{1,2}$, expressed in terms of the Earth mass and M_{psr} in units of 1.4 $M_\odot$, one obtains:

$$\frac{m_1}{M_{psr}} = \frac{3.4}{\sin i_1 \, M_{psr}^{1/3}} \qquad (1)$$

$$\frac{m_2}{M_{psr}} = \frac{2.8}{\sin i_2 \, M_{psr}^{1/3}} \qquad (2)$$

for planets B, C and orbital inclinations i_1, i_2, respectively. Since the effect of a mutual inclination of the two orbital planes would have to be quite large to become detectable (*28*), it is reasonable to assume coplanar orbits. Consequently, with $i_1 = i_2 = i$, the perturbation amplitude becomes a function of one variable $Q(\sin i, M_{psr}) = (\sin i)^{-1} M_{psr}^{-1/3}$, which is the only unknown parameter governing the effect of planetary perturbations on pulse arrival times.

A practical fitting of orbital perturbation models to the pulse timing data involves generation of perturbed orbits over a suitable range of values of parameter Q and selection of the best fit model by minimizing the global difference between the sets of predicted and observed pulse arrival times. The theoretical orbits must be generated numerically, which is why a direct least-squares minimization technique of the standard pulsar timing program TEMPO (*11*) cannot be applied. Instead, a more general approach based on the downhill simplex method (*31*), which provides an efficient algorithm for obtaining the absolute minimum of a function of several variables, has been implemented. With the standard pulsar timing parameters and the orbital elements of planet A held fixed at their best fit values derived from the timing model without perturbations (Table 1), the initial perturbed orbits of planets B and C were computed over the 3-year span of observed TOAs, for a given value of Q. The timing program TEMPO was used to calculate the predicted TOAs for the two orbits, compare them with the observed pulse arrival times, and generate a resultant value of $\chi^2(Q)$,

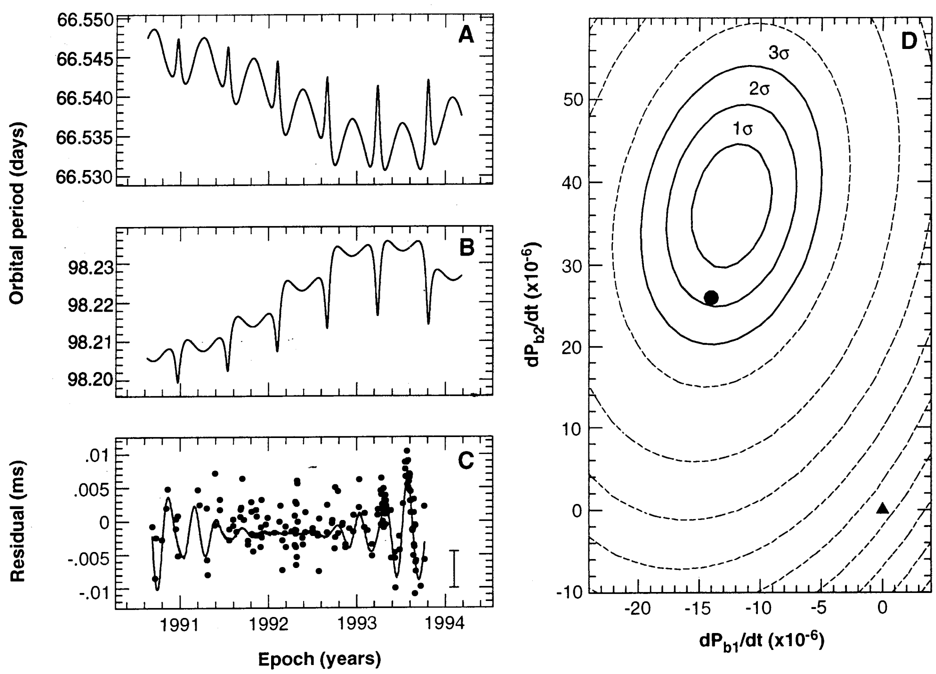

Fig. 2. (**A** and **B**) The predicted perturbations of orbital periods derived from numerical integration of equations of motion of planets B and C. (**C**) The timing residuals of PSR B1257+12 left over after the fit for the rotational and astrometric pulsar parameters and the Keplerian orbital parameters of the three planets without perturbations. A 2σ uncertainty of TOA (time of arrival) measurements is indicated by the error bar. The overplotted solid line shows the simulated effect of neglecting the time derivatives of orbital periods in the timing model. (**D**) Contours of $\Delta\chi^2 = \chi^2 - \chi_0^2$ in the vicinity of the global minimum, χ_0^2, displayed as a function of $\dot{P}_{b1}$ and $\dot{P}_{b2}$. The solid contours are drawn at the $\Delta\chi^2$ = 2.3, 6.1, and 11.7 levels delimiting the respective areas of 68.3 percent (1σ), 95.4 percent (2σ) and 99.73 percent (3σ) confidence for these parameters. The first dashed contour is at $\Delta\chi^2$ = 20 ($\approx$4σ) followed by contours drawn at intervals of 20. The filled circle denotes predicted values of $\dot{P}_{b1}$ and $\dot{P}_{b2}$ for 1/sin i = 1 and a 3-year data span. The filled triangle corresponds to a timing model without perturbations.

which was then minimized by varying the initial values of the two sets of orbital elements of planets B and C (five Keplerian elements per orbit) according to the downhill simplex algorithm. The result of applying this technique to the PSR B1257+12 timing data is shown in Fig. 3A. Compared to a timing model without planetary perturbations (Table 1), inclusion of this effect in the modeling process reduces the value of χ^2 for the global fit by nearly 3 percent, which is about 50 times the formal accuracy of the chosen minimization procedure. The presence of a well-defined minimum in the $\chi^2(Q)$ curve, at $Q = 1.025 \pm 0.033$, clearly confirms that planetary perturbations affect the observed pulse arrival times in the manner predicted by theory, as deduced earlier on the basis of the phenomenological approach (Fig. 2).

Relevance to other planetary systems. The complete success of the planetary perturbation theory (*8, 9, 26, 30*) in explaining the low-level, nonrandom fluctuations in the pulse arrival times from PSR B1257+12 confirms that the original interpretation of the timing data in terms of the presence of at least two planet-mass bodies around the pulsar was correct (*7, 12, 13*). The results described here constitute a final proof that the first extrasolar planetary system has been unambiguously identified.

Although the planetary perturbation modeling does not allow a unique determination of all system parameters, useful constraints on the masses of the pulsar and its planetary companions can be established. With the aid of Eqs. 1 and 2, the result of the perturbation model fitting (Fig. 3A) can be presented in the form of a pulsar mass-normalized planetary mass diagram (Fig. 3B). The uncertainty of the determination of a χ^2 minimum in Fig. 3A defines a range of allowable masses in this diagram. It is further constrained by the highest possible orbital inclination ($i = 90°$) and by the maximum neutron star mass (taken to be $2M_{\odot}$). Within these limits, the minimum pulsar mass is $\sim 1.2\ M_{\odot}$ and the range of possible pulsar masses for orbital inclinations close to $i = 90°$ includes both the typical theoretical value ($\sim 1.3\ M_{\odot}$) (*32*) and the observed average neutron star mass ($1.35 \pm 0.27\ M_{\odot}$) (*33*). Furthermore, the masses of planets B and C must be similar to their respective "canonical," Earth-like values of $3.4\ M_{\oplus}$ and $2.8\ M_{\oplus}$ and the orbital inclinations are unlikely to be less than 60° for any reasonable choice of neutron star mass. No additional information concerning the orbit of planet A can be extracted from this analysis, because its effect on the mutual perturbations of planets B and C is entirely negligible. However, if the orbits of all three planets are approximately coplanar, planet A must be a very low-mass, moon-like object (Table 1).

The discovery of pulsar planets is obviously relevant to searches for planets around solar-type stars. To assess its full significance a basic understanding of the formation mechanisms of neutron star planetary systems is needed, but with only one such system identified so far, the problem remains too poorly constrained to allow a detailed theoretical modeling (*34, 35*). Nevertheless, the observed characteristics of the planets and PSR B1257+12 itself, when confronted with current ideas concerning planetary formation (*36–38*) and the origin and evolution of millisecond pulsars (*39*), indicate that the planets are likely to have evolved in a circumpulsar disk of matter created from the remains of the pulsar's binary stellar companion. Most of the scenarios for planet formation around a millisecond pulsar concern themselves with possible ways to transform a fraction of the companion's mass into a protoplanetary disk, on the understanding planets would subsequently form in a manner similar to that envisioned for the origin of the solar system (Fig. 4) (*34, 35*). Consequently, planets around solar-type stars and pulsars may differ in their physical and chemical characteristics, but the fundamental features of the dynamics of their parent planetary systems should be comparable. In fact, the confirmed detection of planets around PSR B1257+12 provides direct observational evidence that planetary systems with dynamical characteristics not unlike those of the inner solar system do exist.

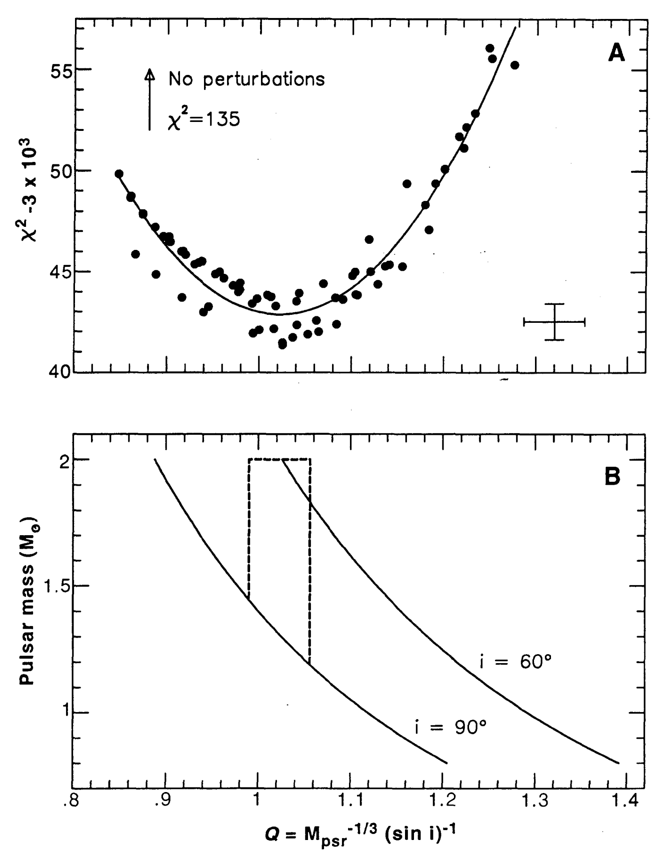

Fig. 3. (**A**) A fit of the numerical model of the mutual gravitational perturbations of planets B and C to pulse arrival time data for PSR B1257+12. Filled circles denote the best fits of perturbation models for fixed values of the normalized planet masses, parameterized in terms of $Q(\sin i, M_{psr}) = (\sin i)^{-1} M_{psr}^{-1/3}$. The solid line represents the best parabolic fit to the $\chi^2(Q)$ data. A 2σ uncertainty of the minimum value of Q obtained from this fit is indicated by the horizontal error bar. The vertical bar characterizes the accuracy of the downhill simplex χ^2 minimization procedure. (**B**) Constraints on the pulsar mass, the masses of planets B and C and the common orbital inclination resulting from the best fit perturbation model shown in (A). The dashed lines delimit the area containing the most likely combinations of these parameters, assuming $2\ M_{\odot}$ for the maximum neutron star mass.

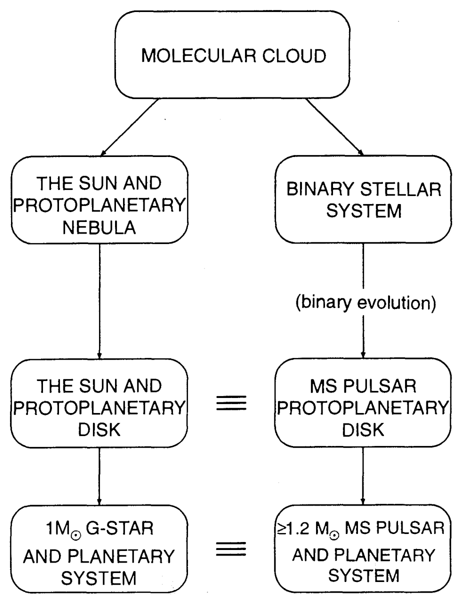

Fig. 4. A schematic comparison of possible similarities and differences in the evolutionary paths of the solar system and the planets around PSR B1257+12.

The precision of pulse timing measurements discussed in this article implies that, aside from furnishing the proof that planet-mass objects outside the solar system exist and are detectable, PSR B1257+12 can be used as a highly accurate probe of planetary dynamics. Positive identification of planetary perturbations and the detection of the moon-mass planet A involved measurements and analysis of pulse arrival times at microsecond precision level, which is equivalent to radial velocity resolution of the order of 1 mm s^{-1} (a factor of 10^4 better than the typical 10 m s^{-1} resolution achieved in modern single-line Doppler spectroscopy). Detection and dynamical studies of terrestrial-mass planets will remain beyond the reach of optical astrometry and Doppler spectroscopy in the foreseeable future (*40–42*), but will be quite feasible with the pulse timing method. Continuing studies of the PSR B1257+12 system, using the pulse timing analysis as a tool, will undoubtedly help guide further efforts to detect extrasolar planets.

The detection of neutron star planets dramatically emphasizes the value of enriching the strategies of planetary searches with nonstandard approaches. At present, among more than 20 known millisecond pulsars, there are five solitary objects that have apparently managed to dispose of their binary stellar companions. If a missing stellar companion to the pulsar indicates a possibility of "leftover" planets around it, PSR B1257+12 is the only confirmed case. Clearly, the pulsar planet formation is not 100 percent efficient, but the available statistics are too small to reliably constrain this efficiency. In addition, there is a large body of over 550 younger, "slow" pulsars, some of which may have retained planets of their parent stars (*43*). Extensive timing measurements (*6, 44*) and more direct observations (*25*) of many of these objects have been in progress, but no definitive detection of planets has been reported so far. In the years to come, the ongoing pulsar surveys and the followup timing programs will help in determining the significance of pulsar planetary systems as a class of astrophysical objects and their relationship to still hypothetical planets around more familiar kinds of stars.

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