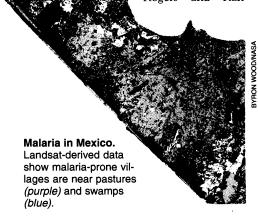
have rendered 10 million square kilometers (an area nearly 15 times the size of Texas) of the richest land in Africa off-limits to cattle raising.

Not all of that land is equally fly-infested, however. Tsetse flies breed best at certain humidity levels, and Rogers and Randolph used weather satellite data to zero in on areas with the right amount of moisture in the air. They did this by looking for vegetation that also flourishes at these humidity levels. The satellite records infrared wavelengths reflected by plant foliage and the visible red wavelengths the foliage absorbs, and a mathematical comparison of the wavelengths yields a characteristic signature for each type of plant life. Once

Rogers and Ran-



dolph found signatures of plants sharing tsetse humidity preferences, they knew they had identified risk areas for sleeping sickness.

This approach was most successful in Uganda, where the disease appears to be transmitted mainly by one species of tsetse fly. "We have correlates in real time between the satellite image in one month and human disease cases in the same or following month," says Rogers. In other areas where there are more than one species of tsetse fly occupying different habitats, the situation is more complicated, but Rogers says the Uganda result "suggests we can use satellites to produce direct risk maps."

Another way of figuring out just where insect vectors meet human victims is to analyze the continuously changing patterns of agricultural land use. Rice fields, for example, are a major breeding ground for malariacarrying Anopheles mosquitoes worldwide. But not all rice fields bear equal responsibility. A study of 104 California rice fields found that 15% of the fields accounted for 50% of the mosquito production. The researchers, led by entomologist Robert Washino of the University of California, Davis, and Byron Wood of NASA's Ames Research Center, discovered that they could pick the major mosquito-producing fields 2 months in advance by analyzing a single Landsat photo for factors such as early development

of the foliage canopy and the proximity of livestock to serve as blood meals.

While malaria isn't a worry in California, and the mosquitoes being studied were, in fact, disease free, the disease is a health problem in Mexico. In that country, a joint NASA-Mexican Ministry of Health team led by Mexico's Mario Rodriguez and NASA's Wood found they could use Landsat photos to pick out malaria-prone villages with 90% accuracy merely by analyzing the amount of pastureland in the vicinity, since flooded pastures are prime mosquito breeding sites. Jack Paris, a meteorology and remote sensing specialist in the biology department at California State University, Fresno, is working with the group to incorporate seasonal weather information to pinpoint the malaria-prone areas that have received enough rain to call for control efforts.

Satellite photos are not cheap, however, and their use may seem like "pie in the sky" for developing countries, says William Lyerly of the U.S. Agency for International Development. Landsat photos cost up to \$4,000 apiece, and then there is the expense of the data analysis, which varies depending on the project. For developing countries that spend as little as a dollar per person annually on health care, satellite epidemiology may seem prohibitively expensive.

But those considerations don't necessarily rule the approach out. A country might need only one or two photos a year to extract the necessary information, and when the cost is compared to the expense of transporting teams between remote ground sites, those satellite photos begin to look like a bargain, says University of Hawaii malaria expert Robert Desowitz. And there may be yet other ways of cutting costs. Paris points out that weather satellite information is available for free over the Internet computer network. In addition, participants at the May workshop discussed the possibility of groups of countries and agencies forming consortia to buy and process satellite data.

Despite the growing enthusiasm, disease prevention by satellite still has to prove itself for actual disease control. "We have established the potential of [the] technology," says Rudi Slooff, a scientist with the World Health Organization (WHO) who attended the meeting. "Now we need to show that this technology can work in an operational setting.... It will need to be demonstrated that with this information you can get your control methods in place faster, or more accurately targeted. It has to be measurable in terms of operational savings." And, of course, in terms of saving lives. Within a year, Slooff says, WHO would like to get several outbreak-prediction projects off the ground in disease-prone areas to learn just where the savings are.

–Marcia Barinaga

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MATHEMATICS

Fermat's Last Theorem Finally Yields

There was a moment of stunned silence in a lecture hall at the Isaac Newton Institute in Cambridge, England, last week. Then a burst of applause. The audience, which included many of the world's leading number theorists, knew they'd witnessed a historic event. Andrew Wiles, a mathematician from Princeton University, had just solved mathematics' most famous problem: proving Fermat's Last Theorem, a deceptively simple conjecture that has challenged, irritated, and daunted mathematicians for three and a half centuries since it was sketched out by the French mathematician Pierre Fermat in 1637.

Many have tried to prove Fermat right (or wrong) in saying that the equation $x^n + y^n = z^n$ has no solutions in positive integers when the exponent n is greater than 2. All proofs until now, however, have failed to hold up, so it would be natural to expect that Wiles' result would be met with skepticism. Not so. "The logic of his argument is utterly compelling," says Ken Ribet of the University of California, Berkeley. "It really is marvelous," adds Barry Mazur of Harvard University. Neither Mazur nor any of the other experts contacted by Science had gone over the text of the proof. He explains, though, that "the conceptual outline [in Wiles' talks] makes it very believable." Perhaps most important, it emerges from the very heart of number theory, drawing on an array of advances made over the past 30 years. Concludes Mazur, "It simply has the ring of truth."

Fermat's Last Theorem has fascinated mathematicians and nonmathematicians alike because it is so simple to state and understand, yet so hard to prove. Adding to the mystique of the problem is its history. Fermat wrote his famous assertion in the margin of a book, adding the tantalizing comment, "I have discovered a truly remarkable proof, which this margin is too small to contain." The problem is called Fermat's "last" theorem because it is the only one of Fermat's many assertions that mathematicians had been unable—until now—to either prove or disprove.

Fermat did find room (elsewhere) to write down a proof for the case n=4. Nearly 100 years later, the Swiss mathematician Leonhard Euler found a proof for n=3. And in the 1840s, Ernst Eduard Kummer initiated the study of what's now called algebraic number theory, which enabled him to prove Fermat's Last Theorem for a large number of exponents. By late last year, researchers using a small arsenal of computers had extended Kummer's approach to include all exponents up to 4 million (*Science*, 12 February, p. 895).

But a general proof remained elusive. Most number theorists had abandoned the problem as a quixotic quest—moreover, one with little payoff beyond the theorem itself. Unlike many other famous unsolved problems in mathematics, Fermat's Last Theorem has no particularly important consequences. Mathematically its main contribution has been the theories such as Kummer's—and now Wiles'—that mathematicians have created in their attempts to prove it.

There was a brief flurry of excitement in 1988, when Yoichi Miyaoka of the Tokyo Metropolitan University announced a proof (*Science*, 18 March 1988, p. 1373). Miyaoka's claim proved short-lived, however. Experts quickly identified some questionable points in the proof—and eventually an unfixable flaw (*Science*, 25 March 1988, p. 1481, and 3 June 1988, p. 1275). Aside from Miyaoka's attempt, which had serious mathematical content, direct attacks on Fermat's Last Theorem have been largely the province of amateur mathematicians and mathematical cranks.

The experts who heard Wiles lecture point out that his proof might also have a

mistake or two in it, but they don't see much chance of a fatal flaw that would derail the result. "Even if there's a small mistake, it's likely to be fixable," says Karl Rubin of Ohio State University. Indeed, the mathematicians who attended Wiles' lectures are eager to help him correct any errors or clarify any questionable points in the proof, according to Ribet, because it builds on work by so many other researchers. "He uses just about all the tools that have been

developed in algebraic-arithmetic geometry over the past 15 years," Ribet explains. Adds Rubin: "Everyone here [at the conference] feels a personal pride that our fields were successful in solving this problem."

Wiles' starting point was a theorem Ribet proved in 1986, which nailed down an idea proposed by Gerhard Frey of the University of Saarland in Saarbrücken, Germany (*Science*, 27 March 1987, p. 1572). Frey had proposed that an important unsolved problem in the theory of elliptic curves might provide a route to proving Fermat's Last Theorem. The theory of elliptic curves studies another class of equations that Fermat had a lot to say about. (An elliptic curve is the set of solutions to an equation that equates a quadratic polynomial in one variable with a cubic polynomial in another, such as $y^2=x^3 +$ 1.) The open problem, known as the Taniyama-Weil conjecture, asserts that every elliptic curve has an associated analytic function with very special properties. Frey pointed out that any counterexample to Fermat's Last Theorem —any case disproving it—would make it possible to construct an elliptic curve that violates the Taniyama-Weil conjecture, provided that another, somewhat more technical conjecture were true. Ribet proved the technical conjecture,

Wiles' proof "simply has

-Barry Mazur

the ring of truth."

thereby establishing the link between Fermat's Last Theorem and the Taniyama-Weil conjecture.

Ribet's proof raised the stakes in the effort to prove Fermat's Last Theorem. For while

Fermat's Last Theorem has little practical importance, the Taniyama-Weil conjecture has been of keen interest to number theorists since the mid-1950s, because, if true, it would provide a powerful tool for studying the number-theoretic properties of elliptic curves, which themselves are fundamental in many parts of number theory.

Much like Fermat's Last Theorem itself, the conjecture has been proved for many individual elliptic curves, and there's a com-



Challenge and reply. Fermat (right) posed the problem; Wiles, wielding modern mathematical tools, has cracked it.

putational way of verifying it for any given curve. But in order to prove Fermat's Last Theorem, someone would have to prove that the Taniyama-Weil conjecture holds for an infinite class of elliptic curves.

That's what Wiles did last week. The proof, which he has written up in a 200-page paper, most definitely would not fit in the margin of Fermat's book. Wiles, who turned 40 in April, had been quietly working on it since Ribet and Frey opened the door in 1986.

He was equally quiet when he arrived at the Newton Institute to speak at a conference on number theory, but rumors of a breakthrough were starting to fly among the other participants—in part because Wiles, who normally doesn't ask to give lectures, had asked to give not just one, but three hour-long talks. John Coates of Cambridge University, who was Wiles' thesis adviser at Cambridge in the mid-1970s, scheduled him on Monday, Tuesday, and Wednesday, June 21-23.

Wiles' audience could see from the beginning where his research might be heading. Starting from recent work of Matthias Flach at the University of Heidelberg and using an arsenal of techniques developed by many other theorists, Wiles established the Taniyama-Weil conjecture for a limited subclass of

what number theorists call the semi-stable elliptic curves. That was a milestone in itself, but to prove Fermat's Last Theorem, he would have to prove that the conjecture holds for all semi-

stable elliptic curves, a class that includes all the curves needed for Fermat's Last Theorem. And during his first two lectures, Wiles said nothing about how far he had gotten. (He had privately consulted with a handful of close colleagues at the conference, who were also keeping quiet.)

"The excitement was increasing each day," says Rubin. Finally, on Wednesday, Wiles unveiled what Ribet calls "the endgame" and Mazur refers to simply as "quite a piece of magic." He showed how to parlav his result for the limited subclass of semi-stable curves into a proof of the Taniyama-Weil conjecture for the entire class. There was little need for Wiles to remind his audience of the implications, but he modestly noted that Fermat's Last Theorem was a corollary of his main result. After a moment of silence, the room erupted in applause for the historic announcement. In a bit of borrowed British understatement, Rubin (who is American) recalls, "It was very hard for the later lecturers" that day.

While Wiles' proof may close the book on Fermat's Last Theorem, it hardly brings an end to research in this branch of number theory, Mazur and others note. For one thing, the full extent of the Taniyama-Weil conjecture remains open, and a vast array of other problems in the theory of elliptic curves and related subjects is waiting to be solved.

But even though mathematicians have plenty of work ahead of them in related topics—and in scrutinizing the full text of Wiles' manuscript—the 350-year-old challenge may be missed. Asked to reflect on the significance of Fermat's Last Theorem, Ron Graham, adjunct director of mathematical sciences at AT&T Bell Laboratories and president of the American Mathematical Society, quoted the Danish scientist-turnedpoet Piet Hein: Problems worthy of attack/ prove their worth by hitting back. Added Graham, "This one fought back for quite a while." –Barry Cipra