Observations of Time Variation in the Sun's Rotation

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Observations of solar p-mode frequency splittings obtained at Big Bear Solar Observatory in 1986 and during 1988–90 reveal small (~1 percent) changes in the sun's subsurface angular velocity with solar cycle. An asymptotic inversion of the splitting data yields the latitude dependence of the rotation rate and shows that the largest changes in the angular velocity, ≈4 nanohertz, occurred between 1986 and the later years, at high (≈60°) solar latitudes. Earlier helioseismic observations suggest that solar cycle changes in the ratio of magnetic to turbulent pressure in the solar convection zone are large enough to account for the magnitude of the observed angular velocity variations but a detailed model of the phenomenon does not exist.

The detection of long-lived solar p-mode (acoustic) oscillations allowed direct observation of many properties of the sun's interior, including its rotation (*1, 2*). Several million oscillation modes, each characterized by a definite frequency and a unique vibrational pattern (eigenfunction) within the solar interior, are excited to amplitudes observable at the surface. The measured modes range in temporal frequency from about 1 to 10 mHz, that is, periods of a few minutes. The spatial variation of pressure (the pressure eigenfunction) of a given oscillation mode is the product of a radial function, R_{nl}, and a latitude- and longitude-dependent spherical harmonic function, Y_l^m. (As on Earth, latitude and longitude are defined with respect to the rotation axis.) The function R_{nl} describes a standing wave pattern in radius; the index n, the radial order of the mode, is essentially the number of zeroes in R_{nl} between the center and surface of the sun. The well-known spherical harmonic functions describe standing wave patterns in latitude, with $l - |m|$ zeroes, and traveling waves in longitude, with $|m|$ periods in a complete circle of latitude. The indices l and m are referred to as the degree and azimuthal order, and for a given l, m takes integer values in the range $-l$ to l. The sense of rotation of the traveling wave pattern depends on the sign of m, which is positive for mode patterns precessing in the direction of solar rotation.

Solar rotation produces an odd-m dependence (described by the odd-i terms in Eq. 1) in the frequency spectrum ν_{nlm} of the oscillation modes: The solar angular velocity advects the modal wave patterns in the direction of solar rotation, increasing the precession rate of corotating (positive m) modes while diminishing that of counterrotating (negative m) modes. The effect of rotation and other aspherical perturbations in the sun is quantified in terms of the mode frequency splitting, $\Delta\nu_{nlm} = \nu_{nlm} - \nu_{nl}$, where ν_{nl} is the average over m of ν_{nlm}. The advective precession rate of a mode is simply $\Delta\nu_{nlm}/m$ and is basically a weighted radial average of the interior angular velocity, where the weighting function (kernel) is different for different modes. The radial dependence of the weighting functions is sensitive to the ratio n/l; the kernels are all sharply peaked near the solar surface but have an inner cutoff radius that increases with l at fixed n. Thus the low l modes probe the interior most deeply. The angular (latitudinal) dependence of the weighting functions depends mainly on the ratio $|m|/l$; the kernels are sharply peaked about the solar equator for large $|m|/l$ but fairly uniform for small $|m|/l$. Because different modes sample different regions of the solar interior, one can in principle infer the radial and latitudinal variation of the interior angular velocity from a measurement of the odd-m dependence of the frequency splittings of many oscillation modes. This procedure is called an inversion.

Inversions of observed p-mode splittings have led to a picture in which the angular velocity within the convection zone (the outer 30 percent or so of the sun by radius) depends mainly on latitude (*3*). The inferred latitude dependence is roughly consistent with measurements of rotation obtained spectroscopically or from the motion of sunspots. The inversions of p-mode data run contrary to previous numerical simulations (*4*) of convection in a rotating sun, which had indicated that the angular velocity within the convection zone would show a cylindrical geometry, depending mainly on distance from the rotation axis. The mode-splitting inversions also show a relatively sharp transition, near the base of the convection zone, to an angular velocity pattern that at greater depths is less dependent on latitude. The implied velocity gradients at the bottom of the convection zone are probably important for the operation of the sun's magnetic dynamo, although details of the dynamo operation are uncertain.

Nonseismological measurements have provided evidence for small-amplitude time variations in the rotation rate. Spectroscopic measurements (*5*) reveal torsional oscillations with a period of ~22 years (the complete period of the solar magnetic cycle, as defined by field polarity reversals, or twice the time between successive maxima in the number of sunspots). The surface velocity changes associated with the torsional oscillations correspond to changes in rotation frequency of order 2 nHz between sunspot maximum and minimum, and the variations at different latitudes are phased so as to constitute a progressive wave moving from the poles to the equator. For comparison the average solar rotation rate is approximately 400 nHz. Near-equatorial (and presumably near-surface) velocity variations of order 3 nHz between solar minimum and maximum have been inferred from the motion of sunspots (*6*), and there have also been claims of angular velocity variations based on helioseismic data. One particular analysis using different helioseismic data sets suggests changes below the bottom of the convection zone of order 20 nHz (~5 percent of the angular velocity itself) during the period 1982 to 1988 (*7, 8*).

Time variations in the even-m dependence of the p-mode splittings (described by the even-i terms in Eq. 1) have been found (*9*). Even-m splittings can be produced, for example, by magnetic fields and by latitude-dependent temperatures because these perturbations, unlike rotation, have no directional effect on mode propagation. In (*9*) we used the odd-m splittings to put an upper limit on time variations in the near-equatorial rotation rate, but on closer inspection of the odd-m splittings we discovered systematic differences in the rotation rate between years of low and high solar activity.

Rotational splitting analysis of helioseismology data. Our data are series of Doppler images of the entire solar disk, taken 1 minute apart, with the dedicated helioseismology telescope at Big Bear Solar Observatory (BBSO). Data were acquired in 1986, near solar sunspot minimum, and 1988 to 1990 around solar maximum. Each year's observing season spans typically 5 to 6 months during the summer. The data constitute a long record of the space and time dependence of the surface (line-of-sight) velocity, from which the p-mode oscillation frequencies and splittings are derived with high precision. Within a given season the continuity of the data is broken mainly by nightly gaps and cloudy weather.

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For a more complete discussion of the data and their analysis, see (9) and references therein.

Figure 1 shows a power spectrum of measured solar velocity variations computed as a function of m and temporal frequency ν for fixed l. Each of the slanted hedgerows of peaks is produced by p-modes of just one radial order ($n = 15$); each horizontal trace contains the contribution of modes of the corresponding azimuthal order m. The solar rotation rate is basically given by the slant of the hedgerows. Ideally only the central row of peaks (of degree $l = 20$) would be present. Because of the aforementioned gaps and because only half of the solar surface is observed, the spectra contain spurious peaks, or sidelobes. The two major rows of peaks flanking the main row are produced by $l = 19$ (left) and $l = 21$ (right) modes. These spatial sidelobes are an artifact of incomplete sampling of the solar surface. The lesser rows are temporal sidelobes caused by the gaps and are separated by 11.6 μHz (Earth's rotation period) from the major rows. Each of the rows is further split (by solar rotation) into peaks corresponding to modes of azimuthal order differing slightly from the nominal value but of the same l. These additional spatial sidelobes are not separated widely enough to be seen in Fig. 1.

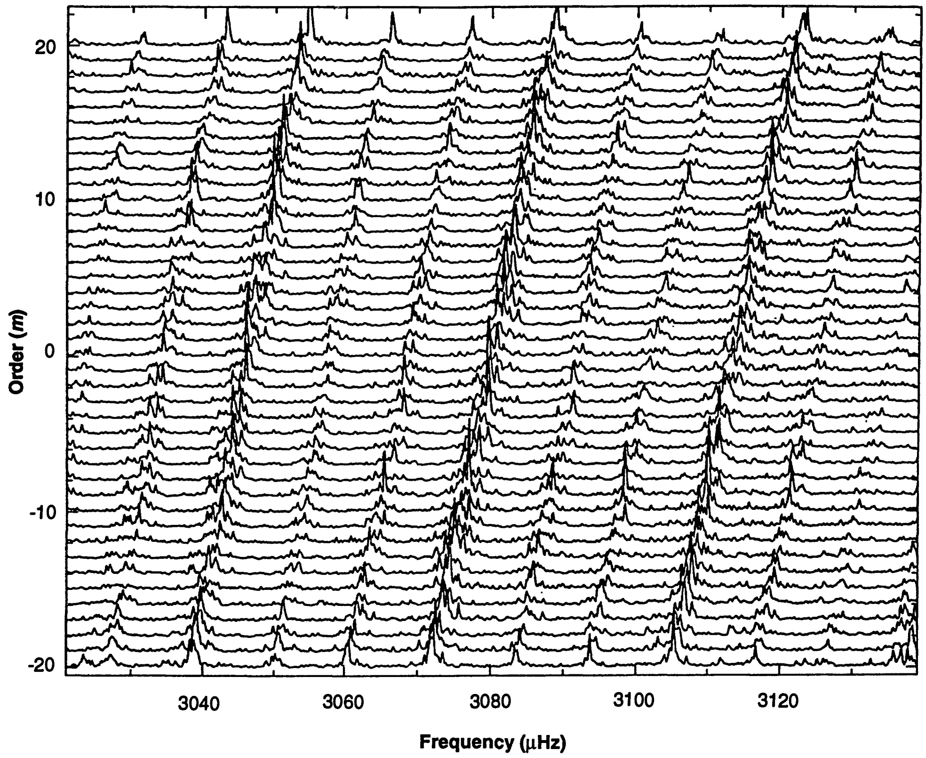

Fig. 1. Power in solar surface velocity oscillations from the 1986 BBSO Doppler data as a function of azimuthal order m and temporal frequency for $l = 20$. Each of the major rows of peaks is produced by solar p-modes of $n = 15$. The central row corresponds to modes of $l = 20$, while the additional rows are artifacts of the day-night observing cycle and the invisibility of half of the solar surface. The slanting of the rows is the odd-m splitting effect of solar rotation.

It is convenient to express p-mode splittings in the parameterized form

$$\Delta\nu_{nlm} = L\sum_{i=1}^{N} a_i(nl)P_i(m/L) \qquad (1)$$

where P_i is the Legendre polynomial of order i and $L^2 = l(l + 1)$. To obtain the splitting coefficients $a_i(nl)$ we performed a nonlinear least squares fit of the observed power spectra to a model containing these and other parameters, such as multiplet frequency ν_{nl} and mode line width. The model takes into account the above-mentioned sidelobes. Another useful expansion is

$$\Delta\nu_{nlm} = m\sum_{i=0}^{N-1} b_i(nl)P_i(m/L) \qquad (2)$$

The measured a values were transformed into b values using a recurrence relation for Legendre polynomials. From our earlier discussion and from the fact that Legendre polynomials of odd-index i are odd functions of their argument, it can be seen that solar rotation affects the odd-index a_i or, equivalently, the even-index b_i.

Our long-range program has been to obtain, for each summer's data, the splitting coefficients a_{nl} for as many p-mode multiplets (n,l combinations) as possible within the frequency range $1 \le \nu_{nl} \le 4$ mHz. We performed both "low-N" and "high-N" analyses, which correspond to truncating the expansion of Eq. 1 at $N = 6$ and $N = 12$, respectively (9). The low-N analysis has been carried out for multiplets in the range $5 \le l \le 140$. The high-N analysis, which previously covered the range $20 \le l \le 60$ used in the even-index splitting analysis (9), has been redone and extended through $l = 140$. The results below are based on the updated high-N analysis, except as noted.

To explore the possibility of a time-varying rotation rate we computed, as a function of heliographic latitude, θ, the quantity

$$\overline{\Omega}(\theta) = \sum_{j=0}^{5} b_{2j}P_{2j}(\sin\theta)/P_{2j}(0) \qquad (3)$$

where b_{2j} is a weighted average of $b_{2j}(nl)$ over all available p-mode multiplets (except as noted). The function $\overline{\Omega}(\theta)$ is a radial average of the latitude dependence of the sun's angular velocity $\Omega(r,\theta)$. By the expedient of averaging over many multiplets we gain in signal to noise at the expense of depth information. We estimate that the p-modes observed in the BBSO data set are collectively most sensitive to the outermost 5 percent of the solar radius. Equation 3 describes angular velocity profiles that are symmetric in θ. North-south asymmetries in angular velocity do not affect mode splittings and so would not be detected in our analysis even if they exist. The mathematical justification for our inversion method can be found in earlier papers (9, 10). The key approximation is that the degree l of a typical observed mode be much larger than the minimum truncation index N needed to make Eq. 1 a good approximation to the splitting. Because a typical mode in our analysis has $l \approx 80$ whereas $N = 12$, the asymptotic approximation appears to be valid. A more detailed study of the limitations of the asymptotic approach is beyond the scope of the present research article.

In Fig. 2 we compare $\langle\overline{\Omega}(\theta)\rangle$, obtained by averaging the inversions of the 4 years of BBSO seismology data, with angular velocity profiles inferred from the motion of sunspots and from spectroscopic measurements (11, 12). The ~1 percent differences in the measured rotation rates have not been satisfactorily explained. Part of the discrepancy could be instrumental error, but one must keep in mind the depth sensitivities of the different measurements. [For a recent review of solar rotation measurements see (13).] The spectroscopic results tell us only about the photosphere. Because sunspots are detectable (as yet) only near the surface, their depth sensitivity is uncertain, though one might guess that they are as deep as they are broad and, therefore, that they (like the p-modes) are most sensitive within the outermost few percent of the sun's radius. The use of sunspots to trace rotation is further complicated by their uncertain dynamics.

Time variation in the solar angular velocity. The change in the inferred rotation rate versus latitude between 1986 and each of the three solar-maximum years 1988 to 1990 are shown in Fig. 3. The error bars in the inversions were obtained by standard propagation of the errors in the splitting coefficients $a_i(nl)$; those errors were determined from the scatter among the measured $a_i(nl)$ values for the different observation years. The dominant angular velocity variation seen in Fig. 3 is the statistically significant high-latitude increase between solar minimum and maximum, peaking at about 4 nHz amplitude at ~60° latitude. There also appear to be significant, though smaller, variations at low latitude. These data are the strongest seismological evidence to date for changes in the sun's near-surface rotation rate. We did not see significant changes in the equatorial rotation in our previous rotational splitting analysis (9) because of the poorer latitude resolution obtainable with only three splitting coefficients (we used, in effect, b_1, b_2, and b_4). To further test the statistical significance of the changes, we inverted the splitting data for groups of multiplets in different l and ν ranges and found general consistency in the time variations in the inferred $\overline{\Omega}(\theta)$; no clear ν- or l-dependent trends are seen in the inverted curves.

One cannot determine whether the variations shown in Fig. 3 originate in the sun without addressing the issue of systematic errors. Effects that differ from one observing season to the next, which are not taken into account in modeling the oscillation spectra, include image distortion, drift, and orientation error. Such data imperfections affect the relative strength of the spectral sidelobes and may therefore lead to systematic errors in the measured splitting coefficients. Although we cannot absolutely rule out systematic errors at the level of the solar rotation changes seen in Fig. 3, we note that such errors are extremely sensitive to sidelobe separation and mode line width. In the case of image drift or distortion the measurement bias is estimated to be strongest when the temporal sidelobe separation, 11.6 μHz, is close to the separation, $\partial\nu/\partial l \equiv \partial\nu_{nlm}/\partial l$, of modes of adjacent l. The inversion of the p-mode splittings, however, shows no sensitive dependence on $\partial\nu/\partial l$ (Fig. 4). Improper modeling of sidelobes in which the value of l is the same as the l-value of the mode of interest but the value of m is different from the m-value of that mode should produce a frequency bias that is sensitive to mode line width. At low l and ν the oscillation spectra are resolved into peaks of definite m; in this regime the frequency bias should be minimal. The inversion performed on multiplets of $l \le 40$ and $\nu \le 1.5$ mHz shows the same essential features seen in Fig. 3. We conclude that errors in modeling sidelobes are not chiefly responsible for the observed changes.

Is the latitude dependence of the rotation rate adequately characterized by the expansion in Eq. 3 in the first six even Legendre polynomials? We performed two numerical tests to help answer this question. First, we examined the latitude resolution of the inversion by approximating delta functions, centered at different latitudes, by the truncated polynomial expansion. In each case, the polynomial reconstruction of the delta function contains a dominant peak at the latitude of the delta function, with lesser peaks at other latitudes. This indicates that the angular velocity change seen at latitude ~60° (Fig. 3) is not simply the consequence of an inadequate representation of changes at low latitude. Second, we expanded modified versions of the observed rotation curves: For latitudes greater than 37°, the observed

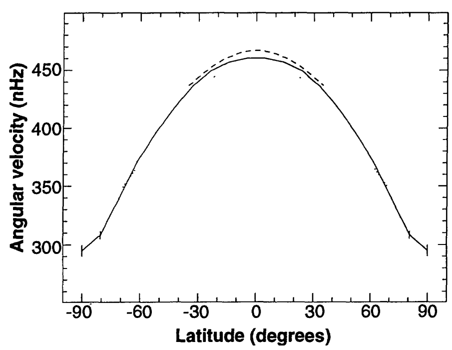

Fig. 2. Latitude dependence of the (sidereal) solar angular velocity. The solid curve was obtained by inverting the splitting coefficients a_i, averaged over all multiplets observed in the BBSO helioseismology data and over all years of data. The values, in nanohertz, of the averaged odd-index coefficients a_1 through a_{11} are, respectively, 442.38 ± 0.06, 22.04 ± 0.07, −3.98 ± 0.09, 0.58 ± 0.11, −0.73 ± 0.13, and 0.29 ± 0.15. The dashed curve shows the all-sunspot rotation rate obtained from analysis of the Mount Wilson white-light images between 1921 and 1982 (*11*). The dotted curve shows rotation deduced from Mount Wilson Doppler measurements from 1967 to 1982 (*12*). Error bars in this and other figures are single standard deviations of the quantities plotted.

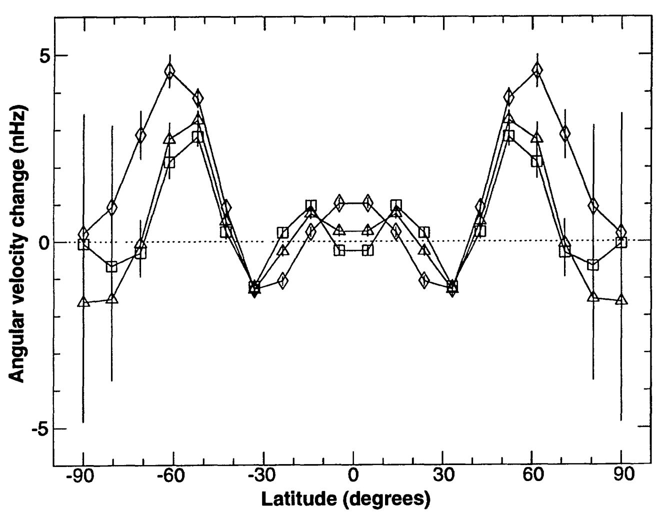

Fig. 3. Inferred differences in the sun's angular velocity between 1986 and 1988 to 1990 versus solar latitude from p-mode splitting inversions, as described in the text. The years 1988, 1989, and 1990 are represented by squares, triangles, and diamonds, respectively. The formal significance of the changes is fairly high, even between the later years of data. Note that the inferred angular velocities at different latitudes are statistically correlated, partly because the inversions are based on a six-parameter fit.

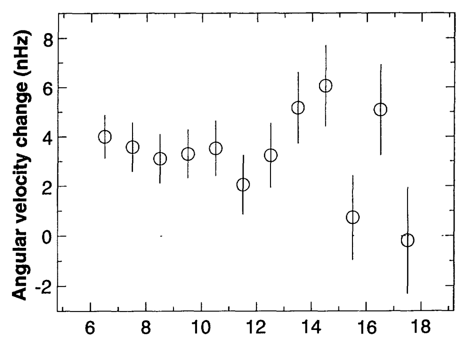

Fig. 4. The difference at latitude 60°, inferred from inversions of p-mode frequency splittings, between the 1986 angular velocity and the average angular velocity for 1988 to 1990 versus mode separation, $\partial\nu/\partial l$. The lack of strong dependence on mode separation, particularly near $\partial\nu/\partial l \approx 11.6$ μHz, makes it unlikely that the inferred changes in rotation rate are dominated by errors arising from improper modeling of spectral sidelobes.

rotation rate changes were set equal to their values at 37°; the truncated polynomial expansions of the modified curves do not show a high-latitude feature, again indicating that the high-latitude feature is not an artifact. By similarly removing the low-latitude variations, we find that they are not an artifact of the high-latitude variations. Thus we have indications, but not rigorous proof, that the basic features of the rotation curves are not artifacts of imperfect modeling.

Angular velocity changes and magnetic activity. If the changes we see are intrinsic to the sun, then the insensitivity of the inversions to either mode frequency or degree l is an important clue to their origin. The strong frequency dependence of the even-index splitting variations (9) was interpreted as evidence for physical changes close to the photosphere because the depth of the outer reflection point of the observed p-modes, typically a few thousand kilometers below the photosphere, decreases with increasing mode frequency, allowing the higher frequency modes to "tunnel" more effectively into the photosphere. The near-photospheric changes are ascribed to variations in the number of magnetic tubes, whose influence on oscillation frequencies is most strongly felt within a few pressure scale heights below the photosphere (*14*). The virtual independence of the rotation changes on ν and l, on the other hand, is consistent with variations in the angular velocity occurring over a substantial volume of the solar interior, perhaps the entire convection zone.

What could cause the solar rotation rate to vary? Turbulent transport of angular momentum certainly plays an important role in establishing the observed time-averaged differential rotation of the convection zone. It follows that if the efficiency of convective transport were somehow modified, a change in the differential rotation could ensue. Magnetic fields can obviously modify turbulent flows and are known to vary in overall strength, at and above the photosphere, from solar minimum to maximum. Angular momentum can also be redistributed within the sun's interior because a magnetized fluid can support shear stresses. The observed frequency dependence of the p-mode frequency variations was used to argue that the ratio of the mean magnetic pressure of tubes to the turbulent pressure is roughly independent of depth, to a depth of at least several thousand kilometers below the photosphere (*14*). The inferred solar cycle change in the ratio of magnetic to turbulent pressure is approximately 1 percent. That this ratio applies, to order of magnitude, to the entire convection zone is suggested by models of the sunspot cycle that place the magnetic dynamo at the base of the convection zone. In a diffusion approximation the efficiency of convective transport is proportional to the typical convective velocity and to its characteristic scale (the mixing length). The observed level of change in the rotation rate might therefore be understood if the convective velocity were to vary by ~1 percent, that is, if the change in the turbulent pressure were comparable to the estimated change in magnetic pressure. One would expect any angular velocity changes induced by changes in magnetic pressure to repeat with an 11-year (rather than 22-year) periodicity. The large change at high latitude, where there is little sunspot activity, is somewhat surprising and certainly poses a challenge for theory. Of course, the latitude dependence of solar activity itself may depend on depth. Another possible explanation for the odd splitting variations involves changes in large-scale convective flows (*15*). It has been shown that the nonaxisymmetric flows in the convection zone can produce odd-*m* frequency splittings. Although the estimated splittings produced by these flows do not seem large enough to dominate the observed time variation, they merit further study.

Other observations of rotation changes. The latitude and time dependence of other reported variations in the sun's rotation rate seem to conflict with one another and with the current BBSO measurements. The changes in the near equatorial angular velocity seen in the BBSO data (9) are significantly smaller than those deduced from sunspot measurements (6). Furthermore, the dominant pattern seen in the p-mode data (Fig. 3) does not seem to migrate toward the equator, contrary to the torsional oscillation picture (5). Different methods sample different radius ranges, however, and sunspots may not be ideal tracers of rotation. In addition, the sunspot measurements show large variations from one sunspot cycle to the next. The BBSO p-mode measurements span less than a quarter of a complete solar cycle. For all these reasons it would be hasty to conclude that the different observations of rotation changes are completely inconsistent. In (7) an apparent (inverse) correlation between sunspot number and the near-equatorial angular velocity at 0.4 solar radii was taken as tentative evidence of a magnetic torsional oscillation beneath the convection zone. This interpretation rested in part on earlier mode splitting analyses of the BBSO data from 1986 and 1988 for $l < 20$ and would have required that the equatorial rotation rate continue to decrease through the most recent solar maximum because it was conceived as an explanation for the 22-year sunspot cycle. However, the combined years of BBSO data do not show such a trend, and, in fact no statistically significant variations are seen in the rotational splittings of modes of $l < 20$ (the relevant comparison was made by averaging the sum $a_1 + a_3 + a_5$, from the most recent low-N analysis, over different ranges of l). We anticipate that long-duration helioseismology observations planned for later in this decade—from the Global Oscillation Network Group (*16*) and the Michelson Doppler Imager instrument aboard the Solar Heliospheric Observatory spacecraft (*17*)—will further illuminate the question of solar cycle–solar rotation variations.

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