tions—computation can be a formidable problem, and a large branch of computer science has emerged to deal with it.

The authors of the present book have done a masterly—one might say heroic—job of organizing the vast literature on the subject into meaningful categories and setting forth the key ideas in each of them. The variety of mathematical concepts dealt with is suggested by a glance at section 1.3, "Mathematical preliminaries." This 55-page summary of material might be covered in two or three semester courses at the advanced undergraduate level. It constitutes an excellent review, but only a beginner who is "mathematically mature" will find it tractable. Subsequent chapters cover the basic construction, many of its generalizations, several computational algorithms, and various ways in which Voronoï diagrams are used to classify or model natural and even social patterns.

Although *Spatial Tessellations* was not written for the casual reader, one can learn a great deal by browsing through it. The book will be especially appreciated by those who already use Voronoï diagrams in their work, but even those who do not will be intrigued by the diversity and ingenuity of the applications. The book could easily have been twice as long as it is—in particular, much more could have been said about the use of Voronoï diagrams in pure mathematics. Still, this will be *the* resource for anyone interested in Voronoï diagrams, either casually or professionally; its over 400 references contain useful sources for mathematicians and scientists in almost every field.

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Chaos Theory

The General Problem of the Stability of Motion. A. M. LYAPUNOV. Taylor and Francis, Philadelphia, 1992. x, 270 pp., illus. $65. Translated by A. T. Fuller from a French translation (1907). Reprint of *International Journal of Control*, vol. 55, no. 3 (1992).

Nonlinearities in Action. Oscillations, Chaos, Order, Fractals. A. V. GAPONOV-GREKHOV and M. I. RABINOVICH. Springer-Verlag, New York, 1992. xii, 191 pp., illus. $59.

One hundred three years ago, the great French mathematician Henri Poincaré discovered chaos. In an effort to understand the dynamics of the *n*-body problem of celestial mechanics, he confronted the possibility that certain structures that we now call stable and unstable manifolds could meet at an angle rather than, as had commonly been assumed, match up exactly. When he admitted that such behavior might occur, it immediately became clear to him that the resulting dynamics would be much more complicated and unstable—chaotic, we now call it—than had ever been thought possible. Poincaré threw up his hands in defeat. How could anyone ever hope to understand the incredibly complex dynamics he was witnessing? Most Western dynamicists then abandoned ship too (Birkhoff and Julia were notable exceptions), choosing to disregard the irregular behavior they saw all too often in solutions of differential equations. Thus, in the West at least, the study of chaos languished until the mid-1960s, when the pioneering work of Lorenz and Smale revitalized the field.

The situation in Russia unfolded in quite a different fashion, as the two books under review attest. A span of 100 years separates these two books. While one is a classic and the other is a thoroughly modern treatment of nonlinear science, I think of them both as major historical contributions to the literature. *The General Problem of the Stability of Motion*, a welcome centennial translation by A. T. Fuller of A. M. Lyapunov's 1892 monograph, gives a wonderful view of how nonlinearity was handled 100 years ago. This is primarily a mathematics text that presents in full detail and in Lyapunov's own turn-of-the-century style his well-known first and second methods for proving the stability of certain nonlinear dynamical systems. Interestingly, the study of nonlinear differential equations has remained relatively unchanged since Lyapunov's time, at least until the nonlinear revolution of the past few decades.

The Lyapunov methods apply to nonlinear systems, but their main aim is to prove rigorously the stability of the system, usually near an equilibrium point. Having spawned such techniques and concepts as Lyapunov functions, Lyapunov exponents, and Lyapunov-Schmidt reduction, Lyapunov can be considered the father of nonlinear stability theory. Indeed, there are precious few mainstream techniques in the field that cannot be traced back to him in some form or other; the book under review is a wonderful historical testament to this fact.

While reading the text I was struck by a curious circumstance. Lyapunov admits that he owes a great deal to Poincaré. He even goes so far as to mention that while compiling his text he had received two "very interesting works by Poincaré." The first, he notes, seemed to be similar in spirit to his own work, but the second (*Les Méthodes Nouvelles de la Mécanique Céleste*) he had not yet had time to peruse. Too bad. Here he would have found the germ of Poincaré's ideas on nonlinear instability and chaos. One wonders what Lyapunov's major work would have looked like had he read a little bit more.

Nonlinearities in Action is a thoroughly modern treatment of nonlinearity and chaos as seen through the eyes of two physicists from the former Soviet Union, A. V. Gaponov-Grekhov and M. I. Rabinovich. Here we find a sometimes rambling but nevertheless readable account of the attack on Poincaré's problems, with particular emphasis on the contributions of Russian scientists and mathematicians. Nonlinear oscillation theory, the subject pioneered by Lyapunov, is treated in detail first, with emphasis placed on the important contributions of L. I. Mandelstam and A. A. Andronov. Next comes chaos. We learn that A. S. Alekseev "observed experimentally period doubling of oscillations and the birth of regimes with complex dynamics . . . the first unintentional contacts with a remarkable new phenomenon that later revolutionized our understanding of randomness—the first encounter with dynamical chaos." Strange attractors, solitons, bifurcations, and turbulence are all given their due—again, with particular stress on the Russian contributions. I found it strange, however, that some of the most celebrated contributions by Russians to nonlinear dynamics (the Kolmogorov-Arnol'd-Moser, or KAM, theory and Shilnikov's work on homoclinic orbits) received not one mention.

The book does contain a final chapter that, according to the authors, is a self-contained exploration of the beauties of nonlinear dynamics. Here we find 63 color plates illustrating all aspects of nonlinearity, including Julia sets, the Mandelbrot set, cellular automata, jet plumes, Belousov-Zhabotinsky (BZ) reactions, fractals—the usual cast of nonlinear characters. The accompanying text, however, is virtually unreadable and riddled with misstatements. One simply cannot present all of these ideas in 20 pages while doing justice to any of them.

It is apparent that both of these books present a view of nonlinearity that is uncommon in the West. From a historical point of view, then, they are important additions to the literature.

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