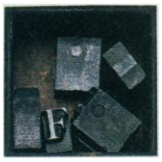
Herschel is drawn toward the more idealistic applications of photography in spectroscopy and photochemistry.

For those unfamiliar with the history of early photography, this book will provide a good background and chronicle of the seminal years. The lavish use of primary material and extensive annotations make this a useful source for those interested in further pursuit of the subject. Examples of Talbot's and Herschel's photographic experiments are reproduced in both duotone and full color and convey the variety and subtlety of tone found in the earliest paper photographs. For those already familiar with the early history this book may prove frustrating, in spite of the author's far-reaching research. There is not an integration of the source material adequate to provide a historian's insight into many of the questions that still surround Talbot's role in the discovery of photography. There are tantalizing bits of information given only passing mention, such as visual problems (a lack of stereopsis) that may have caused Talbot's trouble using a camera lucida—the impetus that led to his discovery of photogenic drawings. The postscript of the book only adds to this disappointment because in it Schaaf shows us that he does indeed have great insight into his subject that might have been applied more consistently throughout the book.

M. Susan Barger
Department of Materials Science and Engineering,
Johns Hopkins University,
Baltimore, MD 21208



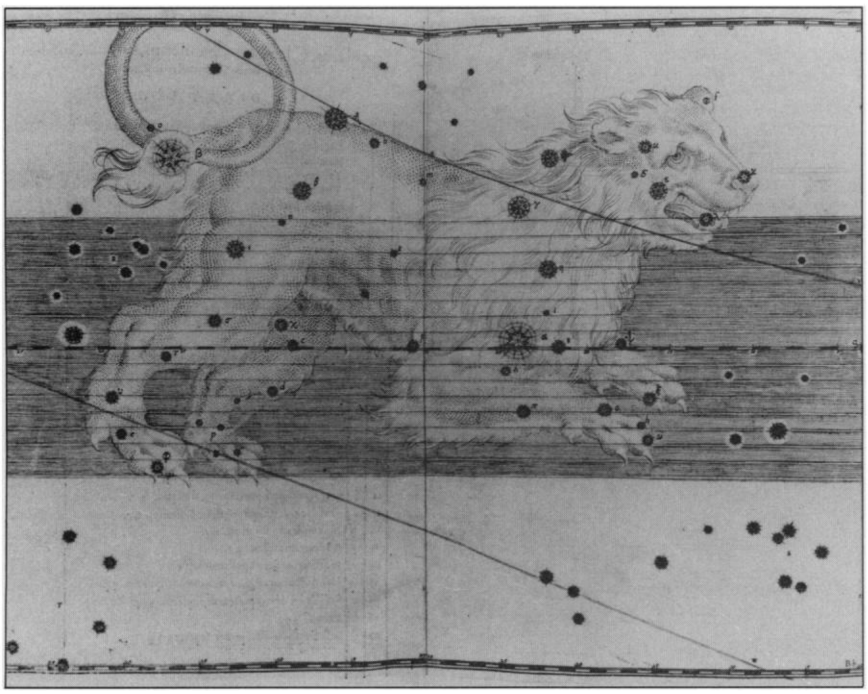
Mathematical Structures

Spatial Tessellations. Concepts and Applications of Voronoi Diagrams. ATSUYUKI OKABE, BARRY BOOTS, and KOKICHI SUGIHARA. Wiley, New York, 1992. xii, 532 pp., illus. $89.95. Wiley Series in Probability and Mathematical Statistics.

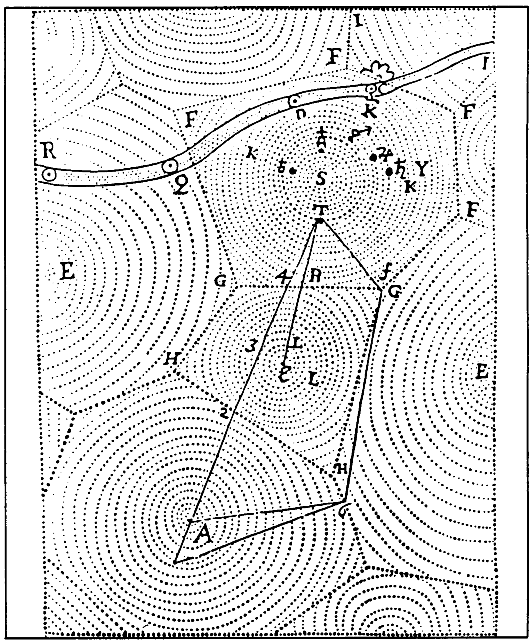
The year 1603 saw the publication of the magnificent *Uranometria* by Johann Bayer of Augsburg, a book of copper plate engravings of the constellations as described by Ptolemy. Within each constellation, the stars are labeled according to brightness. Bayer's pictures, one of which is shown at left below, are a fascinating mix of old and new astronomy, both efforts to introduce some coherence into that cultural Rorschach test, the night sky.

Contrast this now with a 1644 drawing of the heavens by Descartes (below right); the culture has changed entirely. Descartes's starry sky is divided into polygons, each with a star at its center (S is the star called Sun). The wavy path is the path of a comet; the concentric circles around the stars represent possible planetary orbits. Each star holds sway over a polygonal region of the sky or, more properly, a polyhedral region, since Descartes surely understood that the stars are distributed in space, not in a plane. The image is like a froth of soap bubbles.

The difference between these two drawings lies not only in Descartes's rejection of mythology in favor of Copernicanism; there is another important distinction. To give meaning to the star clusters called constellations, Bayer (like many modern scientists) tried to connect the dots. Since—unlike the connect-the-dot workbooks we played with as children—the stars do not come with numbers attached, this can be done in many different ways: the picture one draws is inherently subjective. Descartes did *not* connect the dots: he *isolated* them by assigning a region of the sky to each, according to a very



Copper plate engraving of the constellation Leo from Johann Bayer's *Uranometria* (Augsburg, 1603). [Rare Book Room, Smith College Library]



The disposition of matter in the solar system and its environs as represented by Descartes. S is the Sun; ϵ is a star; RQD represents the path of a comet; polygonal areas represent heavens. [From *Spatial Tessellations*] simple rule. Of course this rule is of human origin too, but once it is accepted its application is straightforward.

Today we find in contemporary journals of astronomy a new, improved version of Descartes's division of the heavens. This time the stars are on the boundaries of the regions instead of at their centers. In a paper recently published in the *Quarterly Journal of the Royal Astronomical Society* Vincent Icke and Rien van de Weygaert note that observations suggest "a Universe in which the galaxies are situated in walls (pancakes), denser filaments, and very dense nodes, forming a network which surrounds huge voids." In fact, the pancakes appear to be the walls, the filaments the edges, and the dense nodes the corners of polyhedral cells whose centers are empty. The problem is to explain why there are large empty regions and why the regions have such shapes.

Icke and van de Weygaert argue that this distribution of matter is to be expected: "Suppose that some cosmic process produces a collection of regions where the density is slightly less than average. . . . These regions are the seeds of voids, because underdense patches become expansion centres, from which matter flows away until it encounters similar material flowing out of an adjacent void. The matter must collect on planes that perpendicularly bisect the axes connecting the expansion centers." This leads to a picture much like Descartes's, but with the stars on the edges and at the corners instead of at the centers of the cells.

This contemporary application of a very old construction is one of hundreds of intriguing examples described in *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*. The construction is used throughout the natural sciences and even the social sciences today; it has been discovered independently in many different contexts, and so has many different names. Today it is usually called a Voronoï diagram, after Georgy Fedoseevich Voronoï, a Russian mathematician who lived from 1868 to 1909. Though Voronoï did not discover the construction, he was the first to study it systematically and to exploit many of its interesting properties. But Voronoï certainly never envisioned the wide range of applications it is finding today. The authors of *Spatial Tessellations* note,

> On initial consideration the following problems, which concern a variety of phenomena at disparate scales, would appear to have little in common: an astronomer studying the structure of the Universe; an archaeologist attempting to identify the parts of a region under the influence of different neolithic clans; a meteorologist estimating precipitation at a gauge which has failed to operate; an urban planner locating public schools in a city; a physicist studying the behavior of liquid argon; a physiologist examining capillary supply to muscle tissue. However, these problems . . . can be resolved by approaches developed from a single concept which forms the subject of this book.

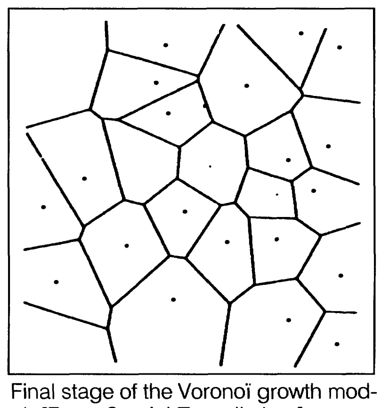
This is not to say that the very similar diagrams by means of which these varied ideas are presented have much if anything to do with one another at a deeper causal level. But two themes recur again and again, sometimes separately and sometimes together. One is a wish or need on our part to classify patterns, be they patterns of numbers, of atoms, of clusters of bookstores near railroad stations in Toshima, Japan, or of crime in Milwaukee, Wisconsin. The other is a general similarity among certain equilibrium structures, the resultants of systems of checks and balances, forces and counterforces. The Voronoï construction is by definition a balancing act, and that is one reason why it arises in so many different contexts.

The idea behind the Voronoï diagram—in its most elementary form—is so simple that it could be taught to students in the seventh grade. All one needs is a ruler and compass. (Or, if those tools are too old-fashioned, the construction can be demonstrated through an interactive computer program.) Given a set of points scattered in the plane, say, we want to assign to each of them the region of the plane that "belongs" to it. For example, if we have only two points X and Y, then, since the locus of points equidistant from two points is the perpendicular bisector of the line segment that joins them, X gets all the space on its side of the bisector, Y gets all the space on its side, and they share the bisector itself.

If we have more than two points, the picture looks more complicated but the construction is not. We choose one point to be X and another to be Y and carry out the construction above. Then, for the same X, we choose another point to be Y and carry out the construction again. After we do this for all the points of the set we find that X is enclosed by a polygon. That is the Voronoï cell of X, the region of the plane that "belongs" to X.

If we have a great many points and we construct the Voronoï cell for each of them, we get a network of cells as shown at right called a Voronoï diagram. The points we started with are at the centers of the polygons. The edges of the polygons are shared by the points at the centers of the adjacent cells because the edge is equidistant from both. The corners of the polygons belong to at least three edges. This means that they are equidistant from at least three of our points, which thus lie on a circle about the corner. Thus the vertices of the polygons as well as the polygon centers are excellent positions for objects to occupy in an equilibrium structure or some other "location optimization" arrangement. For example, in a crystal composed of several different atoms or ions, say *A*, *B*, and *C*, the points labeled A may occupy the centers of Voronoï cells while those labeled *B* and *C* take up positions at the vertices.

The Voronoï construction can be generalized and made more complicated in a variety of ways. In some applications in which the points may come in two or more varieties, it is useful to weight them so that they get proportionately different shares of the pie (this very common variant seems to be the one that Descartes used). In some applications, such as data compression, the Voronoï cells are given and the problem is to determine the cell to which a given data point should be assigned. In other applications, the points



Final stage of the Voronoï growth model. [From *Spatial Tessellations*]

are allowed to move; as they do so, the Voronoï diagram changes. An amusing example, discussed in the book, is drawn from a 1971 paper by W. D. Hamilton called "Geometry for the selfish herd." Cows graze peacefully in a field, chomping on the grass in their Voronoï cells. But there is a lion lurking in the field that will attack the closest cow. Thus the grassy Voronoï region is also a "domain of danger." On the assumption that the lion is equally likely to appear at any spot in the field, the magnitude of the danger is proportional to the area of the cow's Voronoï region. The aware cow will move to a position that reduces that area, even though it may entail reducing her food supply. One by one the cows converge to a herd, not to protect each other but to protect themselves. In other applications one considers Voronoï cells of objects that are not points. This kind of problem arises, for example, in robotics. In these cases—indeed, in almost all applications—computation can be a formidable problem, and a large branch of computer science has emerged to deal with it.

The authors of the present book have done a masterly—one might say heroic—job of organizing the vast literature on the subject into meaningful categories and setting forth the key ideas in each of them. The variety of mathematical concepts dealt with is suggested by a glance at section 1.3, "Mathematical preliminaries." This 55-page summary of material might be covered in two or three semester courses at the advanced undergraduate level. It constitutes an excellent review, but only a beginner who is "mathematically mature" will find it tractable. Subsequent chapters cover the basic construction, many of its generalizations, several computational algorithms, and various ways in which Voronoï diagrams are used to classify or model natural and even social patterns.

Although *Spatial Tessellations* was not written for the casual reader, one can learn a great deal by browsing through it. The book will be especially appreciated by those who already use Voronoï diagrams in their work, but even those who do not will be intrigued by the diversity and ingenuity of the applications. The book could easily have been twice as long as it is—in particular, much more could have been said about the use of Voronoï diagrams in pure mathematics. Still, this will be *the* resource for anyone interested in Voronoï diagrams, either casually or professionally; its over 400 references contain useful sources for mathematicians and scientists in almost every field.

Marjorie Senechal
Department of Mathematics,
Smith College,
Northampton, MA 01063

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Chaos Theory

The General Problem of the Stability of Motion. A. M. LYAPUNOV. Taylor and Francis, Philadelphia, 1992. x, 270 pp., illus. $65. Translated by A. T. Fuller from a French translation (1907). Reprint of *International Journal of Control*, vol. 55, no. 3 (1992).

Nonlinearities in Action. Oscillations, Chaos, Order, Fractals. A. V. GAPONOV-GREKHOV and M. I. RABINOVICH. Springer-Verlag, New York, 1992. xii, 191 pp., illus. $59.

One hundred three years ago, the great French mathematician Henri Poincaré discovered chaos. In an effort to understand the dynamics of the *n*-body problem of celestial mechanics, he confronted the possibility that certain structures that we now call stable and unstable manifolds could meet at an angle rather than, as had commonly been assumed, match up exactly. When he admitted that such behavior might occur, it immediately became clear to him that the resulting dynamics would be much more complicated and unstable—chaotic, we now call it—than had ever been thought possible. Poincaré threw up his hands in defeat. How could anyone ever hope to understand the incredibly complex dynamics he was witnessing? Most Western dynamicists then abandoned ship too (Birkhoff and Julia were notable exceptions), choosing to disregard the irregular behavior they saw all too often in solutions of differential equations. Thus, in the West at least, the study of chaos languished until the mid-1960s, when the pioneering work of Lorenz and Smale revitalized the field.

The situation in Russia unfolded in quite a different fashion, as the two books under review attest. A span of 100 years separates these two books. While one is a classic and the other is a thoroughly modern treatment of nonlinear science, I think of them both as major historical contributions to the literature. *The General Problem of the Stability of Motion*, a welcome centennial translation by A. T. Fuller of A. M. Lyapunov's 1892 monograph, gives a wonderful view of how nonlinearity was handled 100 years ago. This is primarily a mathematics text that presents in full detail and in Lyapunov's own turn-of-the-century style his well-known first and second methods for proving the stability of certain nonlinear dynamical systems. Interestingly, the study of nonlinear differential equations has remained relatively unchanged since Lyapunov's time, at least until the nonlinear revolution of the past few decades.

The Lyapunov methods apply to nonlinear systems, but their main aim is to prove rigorously the stability of the system, usually near an equilibrium point. Having spawned such techniques and concepts as Lyapunov functions, Lyapunov exponents, and Lyapunov-Schmidt reduction, Lyapunov can be considered the father of nonlinear stability theory. Indeed, there are precious few mainstream techniques in the field that cannot be traced back to him in some form or other; the book under review is a wonderful historical testament to this fact.

While reading the text I was struck by a curious circumstance. Lyapunov admits that he owes a great deal to Poincaré. He even goes so far as to mention that while compiling his text he had received two "very interesting works by Poincaré." The first, he notes, seemed to be similar in spirit to his own work, but the second (*Les Méthodes Nouvelles de la Mécanique Céleste*) he had not yet had time to peruse. Too bad. Here he would have found the germ of Poincaré's ideas on nonlinear instability and chaos. One wonders what Lyapunov's major work would have looked like had he read a little bit more.

Nonlinearities in Action is a thoroughly modern treatment of nonlinearity and chaos as seen through the eyes of two physicists from the former Soviet Union, A. V. Gaponov-Grekhov and M. I. Rabinovich. Here we find a sometimes rambling but nevertheless readable account of the attack on Poincaré's problems, with particular emphasis on the contributions of Russian scientists and mathematicians. Nonlinear oscillation theory, the subject pioneered by Lyapunov, is treated in detail first, with emphasis placed on the important contributions of L. I. Mandelstam and A. A. Andronov. Next comes chaos. We learn that A. S. Alekseev "observed experimentally period doubling of oscillations and the birth of regimes with complex dynamics . . . the first unintentional contacts with a remarkable new phenomenon that later revolutionized our understanding of randomness—the first encounter with dynamical chaos." Strange attractors, solitons, bifurcations, and turbulence are all given their due—again, with particular stress on the Russian contributions. I found it strange, however, that some of the most celebrated contributions by Russians to nonlinear dynamics (the Kolmogorov-Arnol'd-Moser, or KAM, theory and Shilnikov's work on homoclinic orbits) received not one mention.

The book does contain a final chapter that, according to the authors, is a self-contained exploration of the beauties of nonlinear dynamics. Here we find 63 color plates illustrating all aspects of nonlinearity, including Julia sets, the Mandelbrot set, cellular automata, jet plumes, Belousov-Zhabotinsky (BZ) reactions, fractals—the usual cast of nonlinear characters. The accompanying text, however, is virtually unreadable and riddled with misstatements. One simply cannot present all of these ideas in 20 pages while doing justice to any of them.

It is apparent that both of these books present a view of nonlinearity that is uncommon in the West. From a historical point of view, then, they are important additions to the literature.

Robert L. Devaney
Department of Mathematics,
Boston University,
Boston, MA 02215