

At State Schools, Calculus Reform Goes Mainstream

was convened last fall at the request of NSF because project scientists hadn't yet secured a promised 50% of the funding from foreign contributors, says a House Appropriations Committee staffer. "[Congress] had extended the deadline three or four times, giving them maximum opportunity to get the contributions," he says. The panel was supposed to review the ability of the project to meet scientific goals within the budget constraints and to examine the possibility of building just one telescope for half the money.

But its conclusions focused instead on the mirror, which Houck says was seen as the project's potentially fatal flaw. "The decision [to adopt the meniscus design] traded a perceived short-term financial risk in the blank fabrication for a long-term technical risk to the telescope's performance," says the report. A safer bet, it adds, would have been the competing honeycomb design. "We conclude that it is essential that the project return to the honeycomb mirror concept," the panel says.

Panel leader Houck, along with a number of other astronomers, admits that either design carries a high risk. These mirrors must hold their shape to within 15 nanometers, he says, to achieve the promised resolution—three to four times better than the Keck's. Angel's honeycomb has its own drawbacks: Glass comprising different segments never gets mixed up as the mirror is cast, so the mirror-makers have a tough job making sure all the segments have the same heat expansion coefficient. A slight deviation would be catastrophic.

But the crucial difference, says Houck, is that a problem with Angel's design would show up early on, because early tests would detect a flaw. "It's something you would know in the shop," he says. A meniscus problem, on the other hand, probably wouldn't be apparent until the telescope was assembled on the mountain.

Randall himself agrees that the Angel design has a technical edge. He disagrees with the panel's criticism, though, because he says the project has a top-rate engineering staff that is developing a set of adjustable steel supports to make the mirrors wind-worthy.

Meanwhile, Boyce of AAS says he's worried that all the public bickering will conjure up the specter of the Hubble telescope disaster. "I have heard comparisons made to the Hubble Space Telescope problem where scientists did not ask enough questions during the process," he says in an editorial in the AAS newsletter. "The Gemini situation is different," he says, because in this case astronomers made the key choice themselves. Now that the last 10% of the needed foreign contributions have been sewn up, he and his colleagues don't want anything darkening the project's chances when Congress gets round to voting on new funding this May.

—Faye Flam

It started about 10 years ago with the tool shed tinkering of a few mathematical mechanics, moved on to a cottage industry with dozens of programs at schools around the country, and now it is beginning to enter the most challenging stage: mass production. No, it's not the latest development in computer technology, but something potentially just as important to the nation's high-tech future: a transformation in the way calculus is taught. The innovative teaching methods that characterize the calculus reform movement are taking over at some of the nation's big state universities, where the factory-style format in which calculus is commonly taught had looked a poor prospect for reform.

That's a major shift for the reform movement. Until recently, it had been concentrated at small colleges and elite institutions, where classes are generally smaller and re-

sources such as classroom computers easier to come by. But calculus reform, supported by the National Science Foundation (NSF) to the tune of more than \$14 million since 1988, has been gathering steam (*Science*, 28 February 1992, p. 1060), which has now helped carry the movement beyond the likes of Duke University and Harvard College to the University of Michigan, the University of Iowa, and the University of California, San Diego (UCSD). "It's almost impossible to keep up with the flurry of activity," says the University of Nebraska's James Leitzel, who heads an NSF-funded project at the Mathematical Association of America to assess the extent and impact of calculus reform efforts.

The move into the big state schools is also feeding on a new interest among academic mathematicians in teaching. Thanks in part to mounting evidence of U.S. students' poor

Turning Higher Mathematics Into Kids' Stuff

Mike Fellows could be found recently spray painting the grass at a local elementary school. Vandalism? No. Fellows, a theoretical computer scientist at the University of Victoria in Canada, was simply preparing a math lesson for a class of first-graders. Later in the day, kids could be seen stepping along Fellows' design, playing "sorting networks"—a game that was also a lesson in the workings of computing algorithms.

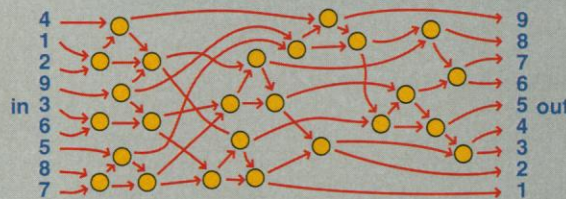
Fellows is in the vanguard of research mathematicians and computer scientists who are taking a serious—and at the same time playful—interest in how best to teach math to young kids. Some are running summer workshops for high school math teachers. Others, such as Fellows, are dealing directly with children in the elementary

schools. For many, the activities are a natural extension of the new concern with teaching that is pervading the mathematics community (see main text).

Of course, it doesn't take a Ph.D. to do a good job teaching long division or probability. But researchers have something crucial to offer school math classes, argues Judith Roitman of the University of Kansas. "We know the mathematics from the inside," she says. "It's the difference between a music critic and a musician." And recent philosophical shifts in the mathematics education community have provided an opening for researchers eager to lend their skills to school math teaching, Roitman adds.

In particular, a new set of standards developed by the National Council of Teachers of Mathematics stresses problem solving and open-ended exploration of mathematical patterns and relationships—activities that are much closer to researchers' experience than the rote arithmetic traditionally taught in elementary and high schools. "When school mathematics is no longer algorithmic, that gives us an entry," Roitman says.

Fellows has used that entry to introduce children to some of the concepts of computer science. His sorting networks game, for example, demonstrates how a machine can take an unordered set of numbers and, by making a relatively small number of pairwise comparisons, put them in order. Children carrying numbered cards enter as "input" at



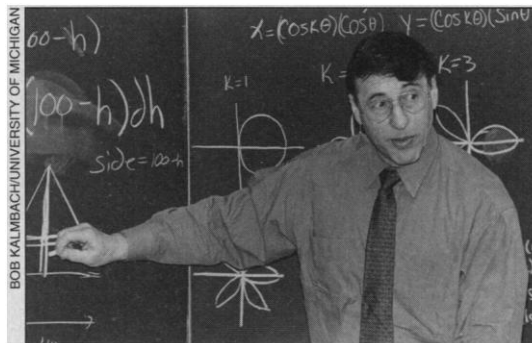
SOURCE: FELLOWS ILLUSTRATIONS: L. OWENS

performance in mathematics, says Jerry Bona, chair of the math department at Pennsylvania State University, "the culture really has changed." Some mathematicians are getting involved all the way down to grade-school math education (see sidebar). But even at the college level, the days when teaching was a perfunctory part of the job for research mathematicians are gone, or at least going. At Penn State, Bona recently pushed through a decision to phase out large lectures in favor of smaller, individually taught classes. Throughout academia, he adds, more and more tenure and promotion cases are being decided on the basis of candidates' teaching as well as their publication record.

The main target of the reformers is traditional calculus teaching, which can be caricatured as students copying into notebooks what a lecturer is copying from a textbook onto a blackboard. One popular tack in existing efforts, which range from Duke University's Project CALC to the Harvard-based "core calculus consortium," is the cooperative learning format, in which students work together in small groups. Another common theme is a concern with cognitive psychology, which has some reformers bandying ref-

erences to Piagetian-style constructivism. "We didn't know the word [constructivism] when we started, but it's what we saw happening as we watched what was going on in the classroom," says Duke's David Smith, who leads Project CALC.

State schools, where cost control rather



Ardent reformer. Michigan's Morton Brown.

than cognitive psychology has tended to reign in mathematics teaching, are now adapting approaches developed at other schools or trying their own homegrown strategies. The University of Michigan, where calculus was already taught in small sections, plans to switch

all of its calculus classes to a cooperative learning format by 1994, using materials from the Harvard program. Currently, 10 out of 85 sections, with a total of about 300 students, are taught in the new style. Once complete, the change-over will make Michigan the first large state university—for that matter, one of the first schools of any size—to go whole hog with calculus reform. "Our program is moving along at a rapid pace," boasts Morton Brown, who heads the effort.

Keith Stroyan at the University of Iowa is not far behind, nor is Al Shenk at UCSD. Stroyan heads a collaborative effort among Iowa, the University of Wisconsin, La Crosse, Brigham Young University, and the University of Northern Colorado to develop a new calculus course based on the computer algebra and graphics system *Mathematica*, which enables students to explore realistic applications of the concepts they learn.

Current plans for next fall, Stroyan says, call for about 380 students—roughly 40% of the total calculus enrollment at Iowa—to take the computer calculus course. Shenk, meanwhile, is trying structured, "exploratory" worksheets for use with graphing calculators to introduce new topics and emphasize the visual side of calculus. Even though calculus is taught in large lectures at UCSD, Shenk is also experimenting with cooperative, small group activities similar to those at Michigan.

If successful, these programs could provide models for calculus reform at other state universities, where reform efforts, if any, have so far been limited to special sections. At Michigan, for example, Brown claims the goal "was just to improve what we're doing [here]." But because Michigan supports a lot of young faculty in temporary positions, he adds, "we'll actually be training a lot of people to filter out to other places." As part of that training, the program includes an intensive, week-long orientation for new faculty and teaching assistants.

Although the reform efforts are catching on, traditional teaching still has a firm hold at many state schools. That's likely to be the case for the foreseeable future, says James Lightbourne, who heads the calculus reform program at NSF. One thing slowing the progress of reform, he says, is lack of a clear blueprint: "The verdict's not in" on how best to implement reform at large institutions, he says. Another is simple institutional inertia, notes David Pengelley, who heads a calculus reform project at New Mexico State University in Las Cruces.

But now that calculus reform has secured a beachhead at the state schools, the reformers aren't about to get discouraged. Says Bona, speaking about university mathematics teaching generally: "We could do a lot more...a whole lot more."

—Barry Cipra

one end and move along lines drawn on the ground until each reaches a "comparator node" (see figure on opposite page). When two children meet at a node, they compare values; the child whose card has the smaller value takes the right-hand exit from the node, while the one with the larger value takes the left-hand road. If the network is properly drawn, then no matter how the kids enter as input, they exit as perfectly ordered output (truly a metaphor for the educational process).

Fellows is also at work on techniques for giving kids the same kind of visceral sense of other computer science concepts, including Boolean circuits and finite state automata. And with Neal Koblitz, of the University of Washington in Seattle, he has developed a system to introduce cryptographic ideas into the classroom. Their system, which they call Kid Krypto, has even produced research problems of its own. Among them: How well can a cryptographic protocol work if the directions are not properly followed?—a question that arose when Fellows saw kids accidentally turn a Boolean circuit upside down.

Fellows isn't alone in finding that research mathematics and school lessons can intersect in surprising ways. Paul Sally, a mathematician at the University of Chicago, is collecting problems that can intrigue people at all levels of mathematical sophistication and introducing them to elementary school teachers. One of his favorites is a number game on squares. Starting with a positive number at each of the four corners of a square, compute the differences of adjacent corners and write these values at the midpoints of the four sides (see figure at right). Drawing a square on the four midpoints and repeating the procedure leads, ultimately, to a square with zeros at all four midpoints.

While the rules are simple enough for a first-grader, Sally says, there are aspects to the problem that can keep a postdoc busy for hours. Says Sally, "Math is math, kindergarten through research."

—B.C.

