of the cytoplasmic domain of the sea urchin bindin receptor doesn't resemble other known receptors means there are few clues as to exactly what the hypothetical signal transduction pathway might be. But it should be possible to prove that the receptor is transmitting a signal by expressing it in eggs from related species, such as starfish, to see if the modified eggs can be activated by *S. purpuratus* sperm or bindin. And if this activation could then be prevented by mutating the receptor's cytoplasmic domain, the case would be closed. Foltz and Lennarz have already begun these experiments, collaborating independently with developmental biologist Laurinda Jaffe of the University of Connecticut. So far their efforts haven't met with success. But if 15 years spent struggling to get this far have taught Lennarz anything, it's the value of persistence. "We haven't given up," he promises.

–Peter Aldhous

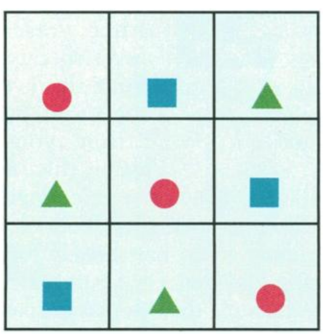
MATHEMATICS

If You're Stumped, Try Something Harder

In mathematics, it's often hard to tell the difference between an "easy" problem and a "hard" one. Some of the simplest-sounding questions turn out to be the most challenging to solve. And to get the answers, researchers may have to work through a thicket of seemingly much harder theoretical problems.

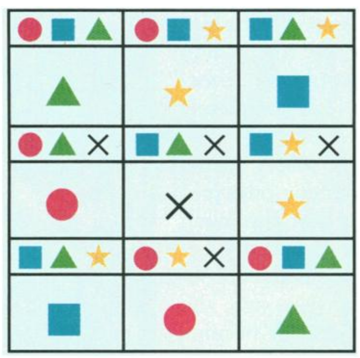
Something like that happened recently in the mathematical subspecialty known as combinatorial design—an area with implications for computer algorithms, scheduling, and experimental design. Jeannette Janssen, a graduate student at Lehigh University, has solved a variation of a "simple" combinatorial problem that has stumped mathematicians since it was first posed in the late 1970s. To do so, she had to turn to more abstruse-sounding theoretical developments that arose from earlier attacks on the problem. While her new result doesn't completely solve the original problem, notes Herb Wilf, a combinatorialist at the University of Pennsylvania, "it moves the problem much closer to a resolution than anyone had expected."

The problem Janssen tackled has to do with what mathematicians call Latin squares. A Latin square is an n-by-n arrangement of n objects in which each object appears exactly once in each row and each column (see below). Even though the definition sounds very restrictive, it's easy to produce a Latin square of any size.



These patterns have intrigued scholars dating back to the Swiss mathematician Leonhard Euler in the 18th century. And in 1977, Jeff Dinitz, now at the University of Vermont, prompted a new surge of interest when he changed the rules. He asked what would happen if a different set of objects was available at each position in a Latin square and the criterion was changed to say that no object can appear more than once in any row or column. In particular, Dinitz wondered whether it would always be possible to form such a "generalized Latin square" having n rows and columns as long as each entry had n objects to choose from. Although that sounds like an even simpler puzzle than the classical version, it may not be: If the sets of choices overlap, there's no obvious rule for avoiding repetition (see below).

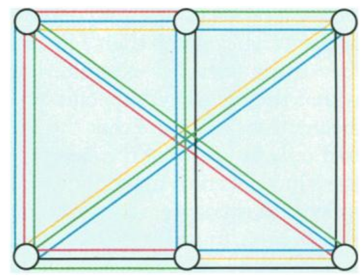
Dinitz's problem is just one case of a broader meta-problem: Given a bunch of constraints on what you're allowed to do (such as



not scheduling the same person or piece of machinery to be in two places at the same time), how can you tell if what you're trying to accomplish is even feasible? And the deceptive simplicity of Dinitz's problem has inspired a lot of related work. "A good question will motivate people to work in an area," says Dinitz, who describes his problem as "easy to state and easy to see, but very hard to prove."

Hard enough that mathematicians had been unable, until recently, to solve the problem for squares larger than n=3. Then, in 1991, Noga Alon and Michael Tarsi at Tel Aviv University in Israel proved that you can always find a generalized Latin square for n=4 and n=6 also. They did so in a time-honored way: by proving a much deeper, more abstract theorem first.

Alon and Tarsi's main theorem is about "coloring" the edges of graphs so that no two edges with the same color meet at a vertex (the edges of a graph are the lines or curves that connect pairs of vertices). Alon and Tarsi were able to find a condition under which arbitrary graphs, with an arbitrary list of allowed colors assigned to each edge, are guaranteed to have a "legal" coloring. That result



had immediate implications for Dinitz's problem, because the combinatorics of n-by-n Latin squares translate readily into graph-coloring terms: Think of n vertices, representing the rows in a square, connected to another n "column" vertices by n^2 edges corresponding to the entries in the square (see above).

Alon and Tarsi showed that the condition of their theorem is satisfied by the Dinitz-problem graph when n=4 and n=6, thereby solving Dinitz's problem in those two cases. However, the condition isn't satisfied for n=5—nor for any other odd value of n. Alon and Tarsi's theorem, it seemed, could only apply to Dinitz's problem for even values of n.

Enter Janssen. His inspired idea was to apply Alon and Tarsi's theorem to Latin *rectangles*. She proved that for generalized Latin rectangles of any size, with n rows and fewer than n columns, it's enough to have n choices for each entry. That's a big advance over what had been proved before, according to Jeff Kahn at Rutgers University. The previous best result had been that n choices sufficed for rectangles having no more than two-sevenths as many columns as rows.

Moreover, Janssen's rectangle result comes extremely close to solving the original Dinitz problem: It implies that you can always build a generalized n-by-n square if you have $n+1$ choices for each entry. The trick is to think of the n-by-n square as part of an $n+1$-by-n rectangle; since $n+1$ objects per entry suffice for the rectangle, they'll do for the smaller square as well.

Whether Janssen's breakthrough presages a complete solution to Dinitz's problem is impossible to predict. However, Kahn thinks it may well lead to the solution of other long-standing problems in graph coloring—and that bodes well for the simple-seeming problem that started it all. Not only can a good question motivate work in an area, but so can a good solution.

–Barry Cipra