Research Article

The Inner Core Translational Triplet and the Density Near Earth's Center

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Four long records from superconducting gravimeters yield evidence of the triplet of translational oscillations of the solid inner core about its central position. Calculations of core oscillation modes allow identification of the three translational resonances at periods of 3.5820 ± 0.0008, 3.7677 ± 0.0006, and 4.015 ± 0.001 hours by their rotational splitting. Each resonance is defined by approximately 20 successive spectral estimates. A new Earth model brings the computed periods into agreement with observation. It has a central density of 12.960 grams per cubic centimeter, inner core radius of 1221.1 kilometers, and a density jump at the inner core boundary of 0.407 grams per cubic centimeter.

The Earth's solid inner core (*1*) is held in its central position in the outer fluid core mainly by gravitational forces. The weakness of the equilibrium allows it to undergo a pendulum motion, the period of which is extremely sensitive to core density structure and inner core radius. Geophysicists have searched for the motion's signature in gravimeter records since the suggestion of Slichter (*2*) that an 86-minute period seen in gravimeter spectra following the great Chilean earthquake of 22 May 1960 might be due to inner core translational oscillations. Slichter realized that such a short period would imply that the central density in the Earth is much greater than was acceptable, and he suggested a weak rigidity of the outer fluid core in an attempt to explain the discrepancy. Slichter also realized that Coriolis acceleration would split the oscillation into a triplet of periods, which he estimated were separated by 5 minutes each. Later studies (*3–5*) arrived at periods from 4 to 7 hours and rotational splitting as large as 0.4 to 0.5 hours. No identification of the oscillations was made, even though gravimeters were operated throughout most of the 1970s at the South Pole where both the diurnal and semidiurnal tidal contributions to vertical gravity are small (*6*).

In this article, I present evidence of a detection and identification of the triplet of inner core translational oscillations and arrive at a new density model of the inner and outer cores on the basis of the observed periods. Recent instrumental and theoretical advances have made the detection and identification possible. First, the development and deployment of low-noise gravimeters based on the levitation of a niobium ball by the Meissner effect in the field of a superconducting magnet (7) allows the measurement of changes in gravity down to ±2 nanogals (2×10^{-9} cm/s^2) (*8*). Second, new methods of calculating the long period modes of oscillation of the Earth's core (*9*), with the use of a variational technique implemented with local finite element support functions to overcome the convergence problems of conventional spherical harmonic expansions in the outer fluid core, allow definitive identification of the modes in the product spectrum of four long superconducting gravimeter records (*10*).

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Gravimeter observations and spectra. Following the development of superconducting gravimeters in the 1970s, a number of European laboratories acquired instruments with a view to operating them in an observatory environment to study long period tidal and other geodynamical signals. One of the earliest and longest records is that taken at Brussels (*11*) beginning 2 June 1982 and running to 14 October 1986. Much excitement in the core dynamics community was created by the suggestion (*12–14*) that core signals might be present in the spectrum of this record in the band between the diurnal and semidiurnal tides. This record has been augmented by a new series beginning 22 July 1987 and running to 30 December 1989. Another record was collected at the Bad Homburg observatory near Frankfurt from 22 March 1986 to 27 December 1988. A third installation near Strasbourg began operations in late 1987 and has produced a record running from 1 October 1987 to 12 March 1991.

Ideally, a worldwide distribution of instruments operated simultaneously is desired in order to use phase information and to average out systematic errors that might be present in the observations. A proposal for such a network has recently been endorsed by the international community (*15*). In the circumstances at hand, it was decided to calculate spectral density estimates of all four records and to form a product spectral density to bring out common features. In order to optimize the trade-off between resolution and the stability of individual spectral estimates, a common 12,000-hour Parzen window with a 75 percent overlap of record segments was used. The first 36,000 hours of the original Brussels record were used as record one, giving nine overlapping segments with 12.5 equivalent degrees of freedom (*16*). Record two consisted of the first 30,000 hourly values at Strasbourg, producing seven overlapping segments with 9.9 equivalent degrees of freedom. Record three was formed from the first 24,000 hours of the Bad Homburg data, giving five overlapping segments with 7.2 equivalent degrees of freedom. The fourth record was the first 21,000 hours of the new Brussels series, resulting in four overlapping segments with 5.9 equivalent degrees of freedom. Each of the individual spectral estimates for the four records is chi-square distributed, and the cumulative distribution function for the product of the four estimates is a triply infinite integral whose numerical evaluation allows the determination of confidence intervals (*17*) (Fig. 1).

Identification of the translational triplet. Regardless of the length and quality of the gravimeter observations, it is not possible to immediately associate the three resonances indicated by arrows in Fig. 1 with the translational triplet of the inner core. Other features of the spectrum are suggestive of resonances, and the extremely weak signal levels compared to noise discourage definitive conclusions.

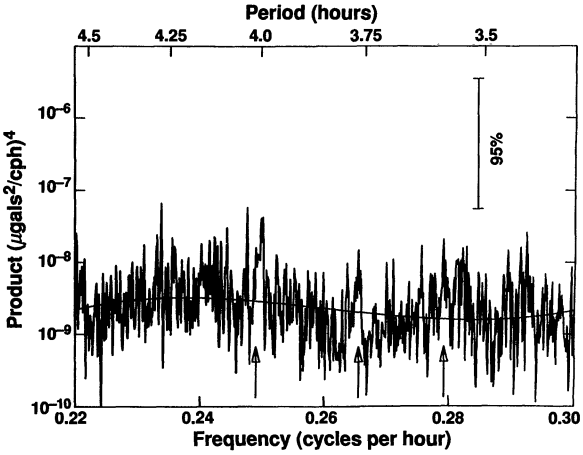

Fig. 1. Product spectrum of the four superconducting gravimeter records described in the text. A sinusoid is shown fitted across the whole spectrum to provide a reference noise level. The locations of the resonances identified by their rotational splitting as the triplet of inner core translational oscillations are shown by the arrows. Statistically, the spectrum represents the equivalent of 24.3 years of independent hourly samples. Vertical bar shows 95 percent confidence interval (CI).

Exploiting the expectation that all three oscillations should appear in the observations, I developed and applied stringent tests that claimed resonances must satisfy before they can be associated with the inner core translational triplet. These tests are based on the rotational splitting that the modes are known to have.

Rotational splitting arises from the effect of Coriolis acceleration, which scales according to the ratio of the period to half the length of the sidereal day. Thus, if the three periods are calculated for a given Earth model, their values will be offset for other Earth models, and for correctly identified observed periods, in nearly the same proportions as those calculated for the given Earth model.

Recent theoretical and numerical advances (*9*) allow the translational eigenperiods and displacement fields, as well as those for other long-period oscillations of the fluid outer core, to be accurately calculated. The forms of the displacement fields are found to vary little with Earth model (Fig. 2), but the translational eigenperiods are highly model-dependent.

For a widely accepted Earth model, CORE11 of Widmer *et al.* (*18*) (Fig. 2), the period of the retrograde equatorial mode (T_R) is 3.7195 hours, that of the axial mode (T_C) is 3.5056 hours, and that of the prograde equatorial mode (T_P) is 3.3432 hours. The period of the axial mode is offset by 0.02575 hours from T_M, the mean of T_R and T_P (T_M = 3.53135 hours), whereas T_R and T_P are offset from the mean by 0.18815 hours.

Below I fit resonances to the candidate spectral features indicated by arrows in Fig. 1 and obtain the observed values $T_R = 4.015 \pm 0.001$ hours, $T_C = 3.7677 \pm 0.0006$ hours, and $T_P = 3.5820 \pm 0.0008$ hours. The offset of T_C from the mean of T_R and T_P, T_M = 3.7985 hours, is 0.0308 hours. If the offsets follow the expected proportionality to those for Earth model CORE11, we would then expect to find the observed retrograde resonance at 4.0235 hours and the prograde resonance at 3.5735 hours. The relative errors in the forecasts of the locations of the observed equatorial mode resonances are 0.21 and 0.24 percent, respectively. Although more generally based, this test is numerically equivalent to the known quadratic dependence of the eigenfrequencies on azimuthal number given by second-order perturbation theory (*5*).

The small variation of the displacement field forms shown in Fig. 2, over Earth models with widely different eigenperiods, permits the construction of an even more stringent test of whether the observed resonances can be associated with the translational triplet. Under general conditions (*9*, *10*), the period T of a core oscillation obeys the equation

$$\left(\frac{T}{T_0}\right)^2 + 2\frac{g}{T_S}\left(\frac{T}{T_0}\right)T_0 - 1 = 0 \qquad (1)$$

where T_0 is the period neglecting rotation, T_S is the length of a sidereal day, and g is a dimensionless form factor representing a weighted average of the product of the meridional and azimuthal components of the displacement fields, compared to a weighted average of the squared magnitude of the full displacement vector. The form factor obeys $|g| \leq 1$ and for a particular mode is nearly invariant with the Earth model.

On the basis of the form factors of CORE11, three curves (branches of hyperbolas) can be constructed along which all possible translational periods must lie (Fig. 3). For example, the locations of the periods for an older model 1066A (*19*) are accurately predicted (Table 1). Forecast locations for the observed periods with this identification test are 4.0166, 3.7687, and 3.5813 hours. The

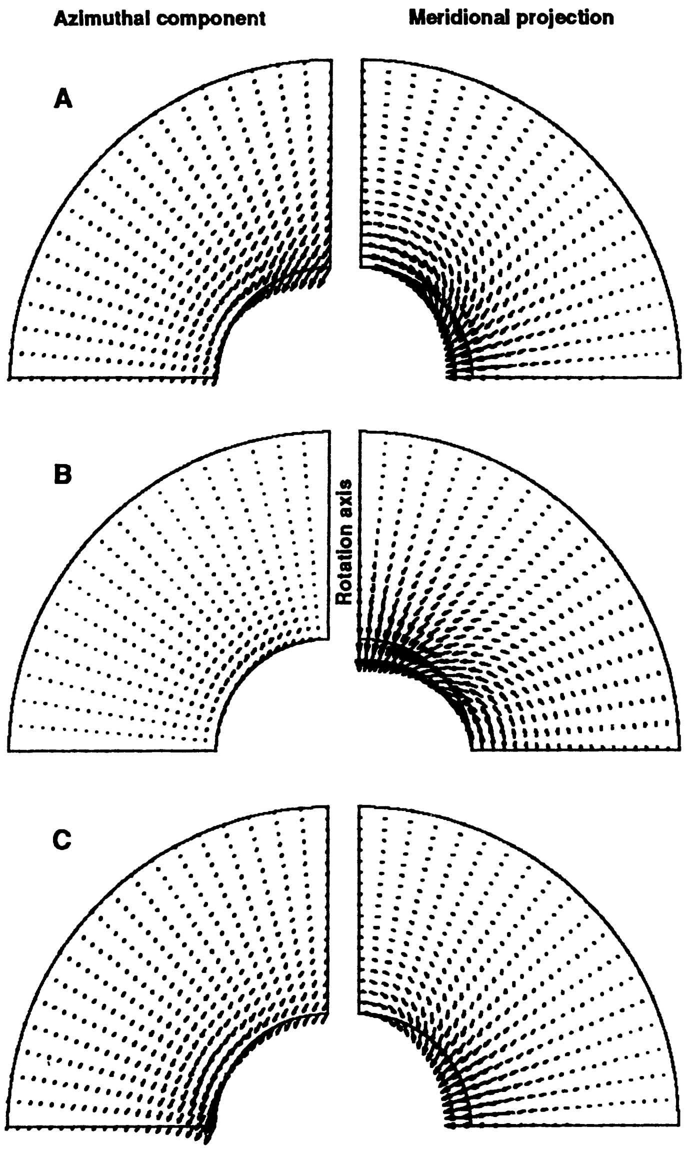

Fig. 2. Displacement vector fields in the fluid outer core of the three translational modes of the solid inner core. The azimuthal components are shown in perspective. (**A**) Retrograde equatorial mode. (**B**) Axial mode. (**C**) Prograde equatorial mode. In each case the azimuthal component leads the meridional component by 90°.

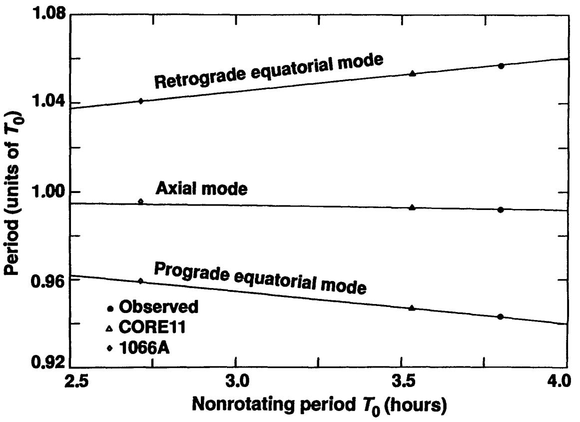

Fig. 3. Rotational splitting of translational triplet periods as a function of the period in the absence of rotation. The curves are branches of hyperbolas and are generated with the parameters of Earth model CORE11. Computed periods for Earth model 1066A and the observed periods are both accurately forecast. Relative errors in predicting locations of the observed resonances are 0.040, 0.027, and 0.020 percent.

Table 1. Parameters of Eq. 1 for Earth models CORE11, 1066A, and the observed resonances (in hours).

Model	T_R	T_C	T_P	T_0	g_R	g_C	g_P
CORE11	3.7195	3.5056	3.3432	3.5314	−0.3520	0.0496	−0.3713
1066A	2.8247	2.7023	2.6035	2.7141	−0.3524	0.0384	−0.3670
Observed	4.0150	3.7677	3.5820	3.7985	−0.3495	0.0513	−0.3700

respective relative errors are 0.040, 0.027, and 0.020 percent. Allowing for T_0 as an adjustable parameter, the probability of an equally good or better fit to the observed periods being obtained by chance is about one in 6×10^6.

Recovery of periods, amplitudes, and quality factors. On the assumption that each resonance is produced by a damped harmonic oscillator excited by approximately white noise, the power spectral density near the resonant frequency f_0 is given by

$$\frac{A^2}{1 + 4[(f - f_0)/\Delta f]^2} \tag{2}$$

where A is the peak amplitude and Δf is the width along the frequency axis between the half-power points of the resonance. The quality factor Q is determined by the ratio $f_0/\Delta f$. Total power for a resonance with the form of Eq. 2 is found from its integral to be $\pi A^2 \Delta f/2$, and the root-mean-square amplitude of the signal is the square root of total power. Comparison of an observed resonant frequency f_0 with f_u, the undamped, numerical value computed from an Earth model, requires the dispersion correction $f_u = f_0\ (1 + 1/8Q^2)$. The correction turns out to be just negligible.

Since we expect the three European observatories to be exposed to nearly the same average signal for each of the three resonances because of their geographic proximity, the fourth root of the product spectrum can be taken as an estimate of the overall power spectral density (Fig. 4). The resonance near 4 hours is complicated by the S_6 solar heating tide resulting from the nonlinear response of the atmosphere to the daily cycle in solar insolation. This response has a peak period of exactly 4 hours. To allow for this complication, two different resonances with the form of Eq. 2 were assumed for this case (Fig. 4C). For the isolated resonances illustrated in Fig. 4, A and B, the 20 spectral estimates closest to the peak frequency were used in the least squares fit, whereas in the case of the double resonance shown fitted in Fig. 4C, the nearest 30 spectral estimates were used. In all three cases, peak frequencies were initially picked from the plots and amplitudes and Q's were recovered from the fitted resonances (Table 2).

Model for the inner and outer cores. I chose Earth model CORE11 (*18*) as a starting model because the three translational periods for it are quite close to the observed values. Small adjustments to the density and its gradient in the inner and outer cores and to the inner core radius of this model suffice to bring the numerically computed translational periods into agreement with observation. The density adjustment in the inner core is expressed as

$$\rho = \rho_0 \left[1 - \alpha_1 - \beta_1 \frac{r}{a} \right] \tag{3}$$

and as

$$\rho = \rho_0 \left[1 + \alpha_2 + \beta_2 \frac{(b - r)}{(b - a)} \right] \tag{4}$$

in the outer core, where ρ_0 is the CORE11 density, a is the inner core radius, b is the outer core radius and α_1, β_1, α_2, and β_2 are fractional adjusting coefficients. In addition, a decrease in inner core radius of γ is incorporated. Density values were either extrapolated or interpolated to the new inner core radius after the adjustments of Eqs. 3 and 4 were made. Because the velocity structure of CORE11 is considered to be well determined, the same fractional adjustments

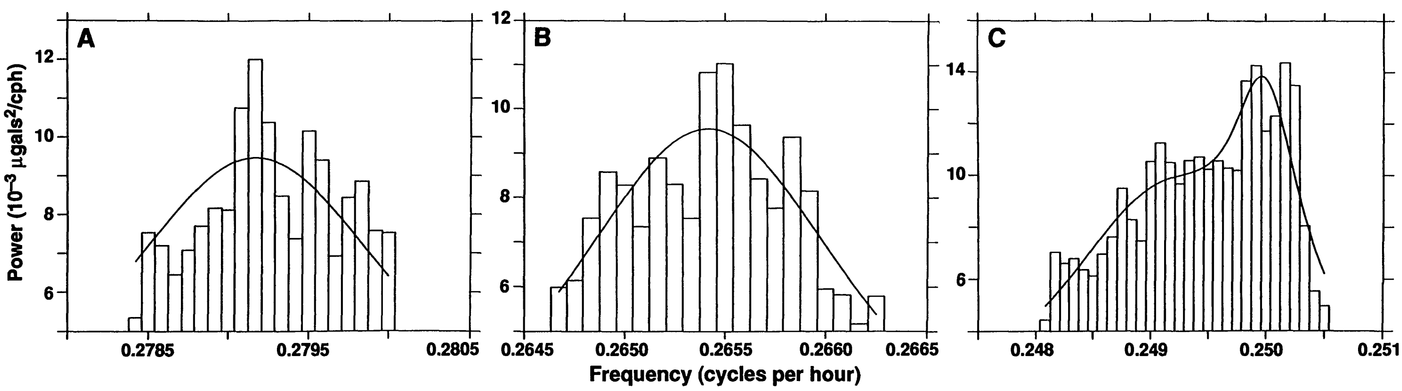

Fig. 4. Fitted resonance (**A**) for the prograde equatorial inner core translational oscillation (T_P), (**B**) the axial translational oscillation of the inner core (T_C), and (**C**) the retrograde equatorial translational oscillation of the inner core (T_R) and for the S_6 solar heating tide. Recovered parameters and uncertainties are given in Table 2.

Table 2. Recovered parameters from least squares fits of the resonances illustrated in Fig. 4, A, B, and C. The stated uncertainties are the formal standard errors of the linearized fitting procedure; cph, cycles per hour; rms, root mean square.

Oscillation	f_0 (cph)	A (10^2 μgals/cph$^{1/2}$)	Δf (10^3 cph)	Period (hours)	Q	rms signal (10^{-9} gal)
T_P	0.27917 ± 0.00006	9.7 ± 0.3	2.40 ± 0.45	3.5820 ± 0.0008	116 ± 35	6.0 ± 0.7
T_C	0.26542 ± 0.00004	9.8 ± 0.2	1.88 ± 0.21	3.7677 ± 0.0006	141 ± 16	5.3 ± 0.4
T_R	0.24907 ± 0.00006	9.2 ± 0.3	2.16 ± 0.35	4.015 ± 0.001	115 ± 19	5.4 ± 0.6
S_6*	0.25000*	9.4 ± 0.5	0.73 ± 0.13	4.0000	342 ± 63	3.2 ± 0.9

*Solar heating tide.

as given by Eqs. 3 and 4 were applied to the Lamé elastic constants to preserve compressional and shear velocities.

As well as matching the observed periods, the mass M and mean moment of inertia I of the inner and outer core system are preserved in the density adjustment. These are respectively

$$M = 4\pi \int_0^b r^2\rho(r)dr \qquad (5)$$

and

$$I = \frac{8}{3}\pi \int_0^b r^4\rho(r)dr. \qquad (6)$$

I made successive, locally linear adjustments by calculating partial derivatives of the five quantities M, I, and the three periods T_P, T_C, and T_R with respect to the five parameters defining the new Earth model, α_1, β_1, α_2, β_2, and γ, and then solving for the requisite adjustments necessary to preserve M and I and to match the observed periods. Several iterations of this procedure resulted in the new model 71BG (Table 3) for the inner and outer cores (Fig. 5). In comparison to CORE11, 71BG has an inner core density reduced by 0.45 percent, an inner core density gradient decreased by 0.04 percent, an inner core radius increased by 2.3 km, an outer core density decreased by 0.04 percent, and an outer core density gradient increased by 0.18 percent. The new model has translational periods for the inner core of 3.5826, 3.7675, and 4.0164 hours, all within the estimated errors in determining the observed periods. Fractional departures of M and I for 71BG compared to CORE11 are -3.2×10^{-8} and -3.7×10^{-8}, respectively.

Implications for structure and properties of the core. Core density structure and inner core radius are tightly constrained by the three observed translational periods. From the partial derivatives listed in Table 3, the changes in the periods per unit percent change in the density structure are of the order of 0.5 hours, whereas the changes in the periods per kilometer change in inner core radius are approximately 0.0007 hours. Even for the largest (0.001 hours) of the estimated formal standard errors of the least-squares recovery of the three periods given in Table 2, the corresponding relative uncertainty in the density is only 2×10^{-5}, and in inner core radius it is 1.4 km. Of course the accuracy of the resolution of core density and inner core radius is undoubtedly much smaller, because the values of the recovered parameters shown in Table 3 are not completely independent, and the assumed simple forms of the density adjustments given by Eqs. 3 and 4 are too restrictive. A full reinversion of the complete Earth model data set (*23*) incorporating the newly discovered periods is required.

Although from the confidence interval indicated in Fig. 1, individual excursions of the spectral estimates are well within values expected for white noise, each resonance is clearly defined by many independent spectral estimates. Identification of the resonances as the translational triplet of the inner core is based on their peak periods being in strict accordance with known Coriolis splitting. Although it is possible that some other oscillation in the Earth system, subject to the same Coriolis splitting, produces the observed gravimetric signature, the slight adjustment of the good recent Earth model CORE11 required to bring the numerically calculated periods into agreement with the observed values suggests that the modes are correctly identified. Nonetheless, the extreme weakness of the detected signals argues caution in this conclusion, and confirmation from superconducting gravimeter sites with broader global distribution is required.

The simple physical arrangement of a sphere oscillating in a fluid,

Table 3. Parameter values defining core model 71BG relative to model CORE11. The three inner core translational periods of 71BG are T_P = 3.5826 hours (hr), T_C = 3.7675 hours, and T_R = 4.0164 hours. Their partial derivatives with respect to the Earth model parameters are also listed below. N^2 is the square of the dimensionless Väisälä angular frequency (measured against twice the diurnal angular frequency of the Earth's rotation). A positive value of N^2 indicates a stably stratified outer core; a negative value, an unstable stratification.

Parameter p	$\partial T_P/\partial p$ (hr/p)	$\partial T_C/\partial p$ (hr/p)	$\partial T_R/\partial p$ (hr/p)
α_1 = 0.44607%	−0.4890	0.5374	0.6134
β_1 = −0.04185%	−0.3645	0.4005	0.4571
α_2 = −0.04476%	−0.4781	0.5253	0.5996
β_2 = 0.17797%	−0.4780	0.5253	0.5995
γ = −2.312 km	8.3×10^{-4}	-7.4×10^{-4}	-6.5×10^{-4}
N^2	7.8×10^{-2}	-8.6×10^{-2}	-9.9×10^{-2}

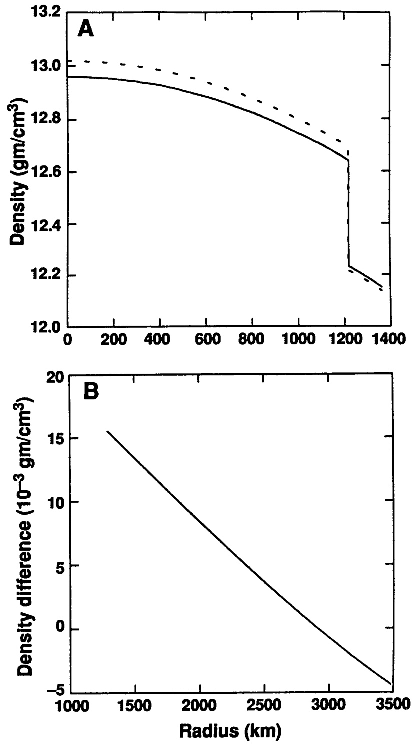

Fig. 5. (**A**) Density profiles for model CORE11 (dashed curve) and the adjusted model 71BG (solid curve) for the inner core and boundary region. (**B**) The excess density of 71BG over that for CORE11 in the outer core is shown as the density difference. The inner core density is reduced by 0.45 percent, and the density jump at the boundary is reduced from 0.475 to 0.407 g cm^{-3}.

presented by the inner core translational modes potentially provides an elegant method of directly measuring outer core viscosity. In particular, the axial mode seems little influenced by rotation and the presence of the outer core-mantle boundary (Figs. 2 and 3), and the classical formula (*20*) for the drag on a sphere oscillating with angular frequency σ in an unbounded viscous fluid gives the approximate kinematic viscosity

$$\nu \approx \frac{2}{9}\frac{\sigma a^2}{Q^2} \qquad (7)$$

For the Q of the axial mode, the kinematic viscosity is 7.7×10^7 cm^2/s. Similar values of this notoriously uncertain but geophysically important parameter are found from other direct measures of damping (*21*).

Ohmic dissipation arising from the presence of strong magnetic fields in the core is found to be slight at these short periods (*22*). Indeed, the skin depth for the translational modes is only about 0.1 km and the core behaves as a nearly perfect conductor with electromagnetic Q's in excess of 10^{10}. Strong magnetic fields might have a just detectable effect on the periods. Magnetic lines of force can be regarded as being under a tension $B^2/2\mu_0$ per unit area, where B is the magnetic field strength and μ_0 is the permeability of free space. Displacement against this tension in a field of 500 gauss would give a relative perturbation in the periods of about 0.07 percent. The intriguing question of the exact nature of the interaction with the in situ magnetic field deserves further investigation.

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