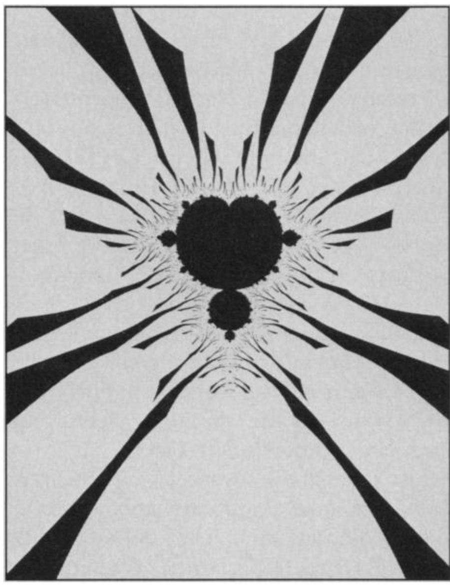
Fractal Fracas

The math community is in a flap over the question of whether fractals are just pretty pictures—or more substantial tools

THE MATHEMATICS COMMUNITY NORMALLY seems to be a genteel set of folks. They don't fuss over funding like physicists do about investing $8 billion in a supercollider, they don't fight about whose results are correct like meteorologists do over global warming, and they don't file lawsuits over who stole whose virus. When a dispute does arise, it's usually settled with pieces of chalk at 20 paces. But a tiff that has smoldered for nearly a year now—including an article that was apparently too hot for one journal to handle—shows that beneath that quiet exterior, mathematicians are willing and able to slug it out with the best of them.



International Business Machines

The subject of the fracas is fractals, those ubiquitous geometric objects that resemble clouds or trees or squashed bugs and contain patterns that repeat themselves at smaller and smaller scales. Partly because they make such pretty pictures and partly because they seem to pop up quite often in nature, fractals have become a hot topic both in the community and in the popular press. But some mathematicians are now saying that they're mostly hot air.

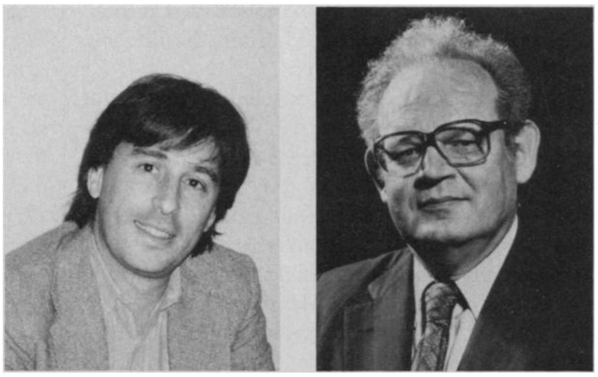


Image breaker, image maker. *Steven Krantz (left) and Benoit Mandelbrot disagree about the value of fractals, which Mandelbrot named and popularized in the 1980s.*

"This love affair with the fractal is disturbing to mathematicians like myself who see too many people believing that this stuff is serious mathematics," says mathematician Steven Krantz of Washington University in St. Louis. Krantz threw down the gauntlet last fall in an opinion piece published in *The Mathematical Intelligencer*, a journal read by most research mathematicians. "Fractal geometry," he wrote, "has not solved any problems. It is not even clear that it has created any new ones."

Even worse, Krantz charged, the intense publicity surrounding fractals has skewed perceptions of mathematics by policy-makers and the public, and this hits mathematicians where it hurts: the pocketbook. "In some circles," Krantz charged, "it is easier to obtain funding to buy hardware to generate pictures of fractals than to obtain funding to study algebraic geometry [a field that produces very deep and beautiful theorems, but that is rather inaccessible to the nonspecialist and has few practical applications]."

Originally, Krantz had submitted his essay to the *Bulletin of the American Mathematical Society* as a review of two recently published books on fractals. It was accepted in January 1989, but Krantz had made the mistake of sending prepublication copies of the review to a number of researchers, including Benoit Mandelbrot, the man who named and popularized fractals in the early 1980s. Mandelbrot, who has positions at both IBM and Yale, is best known for the Mandelbrot set, an infinitely complicated figure that reveals more and more detail as it is looked at with greater and greater magnification. When he received his courtesy copy of the review from Krantz, he was not amused.

"I applied pressure to the editor to have a response by me included," recalls Mandelbrot, who argued that the article was not a book review but an attack on fractals and on him in particular. But the book review editor, Edgar Lee Stout of the University of Washington, told Mandelbrot it was the *Bulletin*'s policy not to accept rebuttals of reviews. Instead, Stout asked Krantz to revise his article to tone down some of his statements, Krantz recalls. But after Krantz complied, Stout decided not to run even the diluted piece. (Stout told *Science* he would not comment on what he termed an "unpleasant" situation.)

Krantz appealed Stout's decision to the Council of the American Mathematical Society, which declined to overrule the editor, so Krantz took his piece to *The Mathematical Intelligencer*. That magazine ran the milder version of the review along with Mandelbrot's rebuttal and prefaced them both with a narrative describing the *Bulletin*'s wafflings.

Mandelbrot expresses irritation that the squabble ever made it into print. Mathematicians are always grousing about one thing or another in the privacy of the faculty lounge, he says, and that's where the fractal argument should have stayed. "It doesn't travel very well." Nonetheless, he is perfectly willing to cross swords with Krantz in public if that's what it takes.

To Krantz's charge that fractals are little more than "pretty pictures," Mandelbrot responds that studying fractals helps develop an intuition for certain mathematical problems that cannot be developed in any other way and that this insight leads to both new conjectures and new approaches to solving some profound mathematical theorems. His work and the work of other fractal geometers, Mandelbrot says, is in the tradition of geometrically minded mathematicians of the last century who drew pictures to gain insights. That approach stalled when the mathematics got too complicated for pencil and paper, but computers make it feasible once again.

But it's much more than looking at pictures, Mandelbrot says. "One picture is worthless. You make many pictures, you make many changes, you manipulate it like a real thing," and eventually the insight comes. "Pretty pictures in the appropriate minds," Mandelbrot says, "lead to pretty problems and entire new fields."

The mathematics community is divided on the issue. Some mathematicians, such as Alec Norton at the University of Texas in Austin, sympathize with Krantz's complaint. Norton says the emphasis on fractal patterns has misled students into thinking they can do mathematics merely by creating and looking at the computer-generated designs. Others, like Albert Marden at the University of Minnesota, defend Mandelbrot's approach, saying that "fractal-like situations come up all over science and mathematics."

Still others, such as Robert Devaney at Boston University, give the fight a split decision. Devaney, who works with high school teachers and students to develop new math curricula, says "high school kids love this stuff; they eat it up." Once they get interested in the pretty pictures, Devaney feels, it's easy to pull them into the mathematics behind the pictures.

And there is some real math there. Even Krantz acknowledges that some very important theorems are connected with fractals. But he is irritated by what he sees as a fascination with form over substance. "The fractal gurus spew data out on a computer, then see what they come up with. This is entirely counter to the scientific method, which in mathematics is called the proof. There are no proofs in fractal theory, just pretty pictures."

One thing's for sure: Krantz's article and Mandelbrot's rebuttal have stimulated the mathematical community to debate the value of fractals. "Everywhere I went, people were talking about the articles," says Sheldon Axler, editor of *The Intelligencer.*

Of the many mathematicians he has spoken to at conferences, Axler says a majority sided with Krantz, especially about the lack of mathematical content in fractal theory. "People are a little turned off by the hype. Where's the substance? Where's the theorems? Where's the beef?" Researchers also agree with Krantz in their frustration over having to compete with fractals for funds, Axler says, and some mathematicians have even tried working a mention of fractals into their grant applications. "It seems that if fractals are dabbled into grants, it's easier to get the money," Axler says.

Some mathematicians who have followed the feud over fractals suggest that it is as much a cultural conflict as anything else. "It's not traditional mathematics," says William Thurston at Princeton, and so "a lot of mathematicians are suspicious of fractals." And although the turf battles in mathematics may seem obscure to the outside world, they are very real to mathematicians.

"Mathematics is the most ferocious field in science," Mandelbrot says, "because there is no objective judgment of the value of things." Arguments can get "very bitter," he adds, "but it just stays in the commons and the lounges because no one outside the field knows what they're talking about."

■ ROBERT POOL

Seeing Proteins in 4D

It doesn't take special 4D glasses, just state-of-the-art NMR spectroscopy, to bring new protein structures into view

A "QUANTUM JUMP" in nuclear magnetic resonance (NMR) spectroscopy, achieved by researchers at the National Institutes of Health, could open a new window on complex protein structures. Lewis Kay, Marius Clore, Ad Bax, and Angela Gronenborn at NIH's Laboratory of Chemical Physics report on page 411 of this issue of *Science* that they have literally added an extra dimension to NMR, going from the current three dimensions to four. Their technique, which Bax says "nobody really believed could be done," will make it possible to apply NMR spectroscopy to the structural analysis of much larger proteins than before.

"It's an exciting advance," says Stephen Fesik, a chemist at Abbott Laboratories in Abbott, Illinois, who helped develop 3D NMR. To date, NMR structural determinations have been done mostly on proteins with molecular weights below 10,000, with the largest being under 20,000, Fesik notes. But many proteins are much bigger than that, and the new technique should help bring them into range.

This is good news for scientists who want to learn how protein chains fold and twist in space. That knowledge is needed to understand how enzymes and other proteins work, and it may also aid drug design.

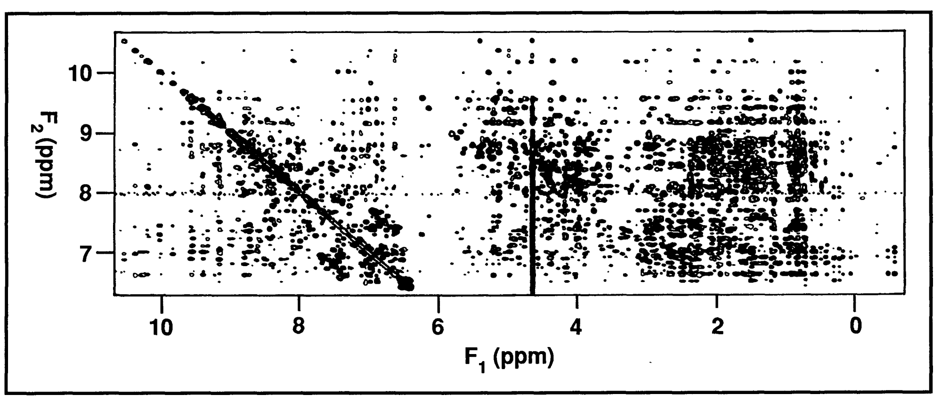
A majority of protein structures now are determined by x-ray crystallography, which can analyze much larger proteins than NMR, but crystallography has several limitations. The major one is that it depends on getting good quality crystals, which is always difficult, and sometimes impossible, for proteins. But NMR spectroscopy works on proteins in solution. It can also reveal details, such as how a protein moves over time, that are invisible to crystallography.

NMR spectroscopy analyzes a molecule by studying its magnetic structure. The nucleus of each hydrogen atom in a molecule, as well as the nuclei of some of the molecule's other atoms, act like tiny magnets, setting up their own magnetic fields and influencing the fields of nearby atoms. By perturbing these fields in various ways and watching how they respond, researchers can get a tremendous amount of data—so much, in fact, that it's hard to sort it all out.

That's where the multiple dimensions come in. The 1D, 2D, 3D, and 4D NMR experiments don't imply a physical image of a molecule in one, two, three, and four dimensions. Instead, they refer to how the data are collected and displayed. If the data were printed words, the different dimensions would correspond to a line of text, a page, a book, and a multi-volume book set.

A 1D NMR experiment, which gives a single "line" of data, is straightforward. First, a powerful magnet aligns the nuclear spins of the atoms in the sample so that they are all pointing the same direction. Then the sample is bombarded with radiofrequency radiation which has the effect of turning all these tiny nuclear magnets on their sides, where they begin to precess, or rotate around the axis of the applied magnetic field. The precessing nuclei generate their own magnetic fields which are detected by a magnetic coil and analyzed.

This allows scientists to get information about the sample because each nucleus precesses at a slightly different frequency, called its resonance frequency, which depends on its immediate surroundings. A proton (hydrogen nucleus) bonded to a carbon atom



National Institutes of Health

2D is not enough. *For interleukin-1β, the 2D spectrum is impossible to interpret.*