Reports

A Wave Dynamical Interpretation of Saturn's Polar Hexagon

M. ALLISON, D. A. GODFREY, R. F. BEEBE

The hexagonal, pole-centered cloud feature in Saturn's northern atmosphere, as revealed in Voyager close-encounter imaging mosaics, may be interpreted as a stationary Rossby wave. The wave is embedded within a sharply peaked eastward jet (of 100 meters per second) and appears to be perturbed by at least one anticyclonic oval vortex immediately to the south. The effectively exact observational determination of the horizontal wave number and phase speed, applied to a simple model dispersion relation, suggests that the wave is vertically trapped and provides a diagnostic template for further modeling of the deep atmospheric stratification.

SATURN'S REMARKABLE HEXAGONAL cloud feature (Fig. 1), as revealed by Godfrey's (*1, 2*) analysis of map-projected, close-encounter Voyager images, is embedded within a sharply peaked eastward jet (of 100 m s^{-1}), centered at 76° north planetocentric latitude (*3*), but is itself stationary with respect to the frame of reference defined by the Saturn kilometric radio (SKR) period (*2, 4*). The latitudinal excursions of the feature appear to be confined to within ±2° latitude (or about 2000 km north and south) of the jet maximum. A large and visually prominent anticyclonic oval feature, some 6000 km in diameter, resides in the anticyclonic shear zone directly to the south of the jet, is also very nearly stationary, and appears to impinge on the flow. Godfrey's map of the streamlines associated with the motion in this region (*1*) suggests the presence of other anticyclones at the same latitude, generally centered in longitude between the hexagonal vertices. In his original study (*1*) Godfrey conjectured that the hexagonal feature might be directly coupled to the planet's magnetic field. Gierasch (*5*) has suggested that the morphology of the feature may be associated with Saturn's deep internal convection. Here we present a further account of our earlier proposal (*6*) that the hexagon is a stationary Rossby wave, forced by the interaction of the jet with one or more adjacent anticyclonic ovals immediately to the south and meridionally trapped by the strong relative vorticity gradient of the flow itself.

M. Allison, National Aeronautics and Space Administration/Goddard Institute for Space Studies, 2880 Broadway, New York, NY 10025.
D. A. Godfrey, National Optical Astronomy Observatories, 950 North Cherry Avenue, Tucson, AZ 85726.
R. F. Beebe, New Mexico State University, Post Office Box 4500, Las Cruces, NM 88003.

Rossby waves have been most familiarly studied as the large-scale, low-frequency pattern of oscillations in Earth's atmosphere and ocean (*7, 8*). As revealed by Voyager spacecraft observations, Jupiter's equatorial plumes (*9*) and the Saturn "ribbon" pattern (*10*) have also been tentatively identified as Rossby waves. The essential distinguishing feature of these planetary-scale wave modes is the westward phase drift of their streamlines (with respect to the zonal mean motion) in oscillatory response to the restoring latitudinal gradient of the ambient "potential vorticity" (*11*). In the most familiar case the meridional restoring force for the wave oscillation is directly related to the gradient of the so-called planetary vorticity, given as $\beta = 2\Omega\cos(\lambda)/a$, where Ω and a are, respectively, the planetary rotation frequency and radius and λ is the latitude. In general, however, the gradient of the relative vorticity of the mean zonal flow (given by its negative meridional curvature) can also contribute importantly to the Rossby wave motion (*12*).

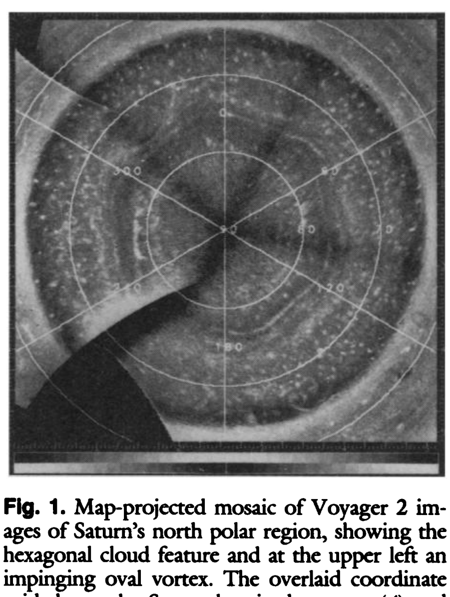

Fig. 1. Map-projected mosaic of Voyager 2 images of Saturn's north polar region, showing the hexagonal cloud feature and at the upper left an impinging oval vortex. The overlaid coordinate grid shows the Saturn longitude system (*4*) and the planetocentric latitude (*3*). The four dark spokes emanating from the center of the mosaic are the boundaries of the original images.

For the Saturn polar hexagon, the planetary vorticity gradient is negligible as compared to the mean meridional flow curvature. In order to model this in a simple way, we adopt a Gaussian profile representation of the jet, with

$$U = U_0 \exp(-by^2/2U_0) \qquad (1)$$

where $y \approx a(\lambda - \lambda_0)$ is the meridional distance from the coordinate origin λ_0, $U_0 = 100$ m s^{-1} is the peak velocity, and $b = 6 \times 10^{-11}$ m^{-1} s^{-1} is the peak latitudinal curvature. This model profile, displayed in Fig. 2 together with Godfrey's (*1*) cloud-tracked wind measurements, appears to be a good representation of the jet, within about ±3000 km of its core. The associated e-folding scale for its latitudinal variation, $L_e = (2U_0/b)^{1/2} \approx 1800$ km, is comparable to the apparent meridional span of the wave. The associated Rossby number Ro = $U/fL_e \approx 0.2$, implies a geostrophically balanced flow for which the relative vorticity is small as compared to the planetary vorticity $f = 2\Omega \sin\lambda_g$. The total variable part of the planetary plus relative vorticity, however, given by the "beta-plane" approximation as $\beta y - dU/dy$, is dominated by the mean flow contribution for all $|y| << [(2U_0/b)\ln(b/\beta)]^{1/2} \approx 3600$ km. The relative vorticity gradient of the jet, averaged over its meridional e-folding interval, is $\langle -d^2U/dy^2\rangle_e \approx 2.2 \times 10^{-11}$ m^{-1} s^{-1}, some 20 times the local β parameter. On the basis of these estimates, the planetary vorticity gradient will henceforth be neglected in our model analysis of the hexagonal wave pattern.

It is of some interest to consider the simple application of these estimates to the Rossby phase speed formula for barotropic waves harmonic in longitude and latitude (*13*), but with the β parameter replaced by the averaged relative vorticity gradient. For oscillations of sufficiently large meridional extent, the resulting dispersion relation is

$$c = U - \langle -d^2U/dy^2\rangle_e\,(r/n)^2 \qquad (2)$$

where c is the horizontal phase velocity, n is the zonal planetary wave number, and $r = a\cos\lambda_c \approx 1.4 \times 10^7$ m is the radius of the latitude circle. With $U = 100$ m s^{-1}, $\langle -d^2U/dy^2\rangle_e \approx 2.2 \times 10^{-11}$ m^{-1} s^{-1}, and $n \approx 6$, this implies that $c \approx 0$ as for a stationary wave pattern. This simple estimate can only be regarded as heuristic, however,

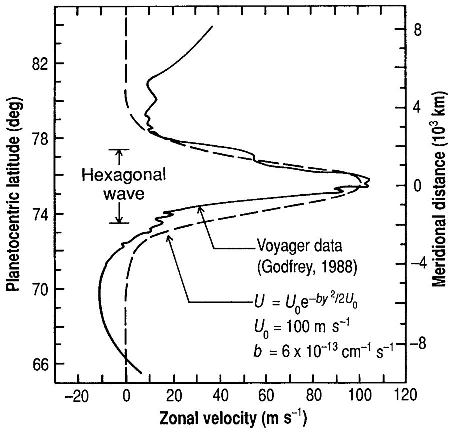

Fig. 2. Zonal velocity in the vicinity of the hexagonal wave feature (solid curve) as determined by Godfrey's cloud-tracked wind analysis of Voyager images (*1*) and the analytic Gaussian jet model for the latitudinal profile (dashed curve). The meridional distance from the core of the jet is indicated on the right ordinate.

owing to its neglect of the observed meridional confinement of the wave perturbation as well as its (as yet undetermined) vertical structure.

Although the vertical structure of the hexagon cannot be observed directly, it may be usefully constrained by applying the effectively exact observational determination of the horizontal scale and frequency of the wave motion to the total potential vorticity balance. It is this prospect that makes the wave interpretation of the hexagonal feature an especially important subject for model analysis.

For small Rossby number motion, the dynamics may be prescribed in terms of a horizontal stream function ψ (defined so that the eastward and northward velocities are given, respectively, as $u \equiv -\partial\psi/\partial y$ and $v \equiv \partial\psi/\partial x$, with x and y denoting, respectively, the eastward and northward coordinates). The quasi-geostrophic potential vorticity equation for the motion in (log-pressure) height coordinates (*13*), where again we are neglecting the planetary vorticity gradient (as well as heating and dissipation), is given as

$$\frac{d}{dt}\left[\nabla_h^2\psi + f_0^2\left(\frac{\partial}{\partial\hat{z}} - 1\right)\frac{1}{\Gamma}\frac{\partial\psi}{\partial\hat{z}}\right] = 0 \quad (3)$$

Here $d/dt \equiv (\partial/\partial t + \partial\psi/\partial x \cdot \partial/\partial y - \partial\psi/\partial y \cdot \partial/\partial x)$ is the total material time derivative associated with the geostrophic motion, ∇_h^2 is the horizontal Laplacian operator, f_0 is the local Coriolis parameter, and Γ is the static stability parameter (positive for stable stratification) in the notation of Haltiner and Williams (*13*). This form of the governing equation presumes that vertical contrasts in potential temperature are at least comparable to the horizontal contrasts over the characteristic scales of the motion, as discussed by Charney (*14*). If the deep Saturn atmosphere were pervasively very weakly stratified, perhaps as a result of its internal convection, then, as suggested by Gierasch (*15*), the vorticity field might be decoupled from the thermodynamics and a very different dynamical regime would prevail. A reviewer has pointed out that in this limit stationary pertubations are zonally symmetric. The further analysis of the very weakly stratified system in application to the Jovian meteorology is an important problem for continued study. In our view, however, the observed longitudinal structure of the Saturn hexagon is itself an indication of the importance of deep stratification, at least on the local scales of motion. In order to diagnose the stratification in the simplest possible way, we will assume that the wave has a separable vertical and horizontal structure and neglect the vertical shear of the base-state flow (*16*). We may then consider a "wave-mean flow" decomposition of the total geostrophic stream function of the form

$$\psi = \exp[inx/r - by^2/2U_0]\, G(\hat{z}) - \int_0^y U\, dy \quad (4)$$

where $i \equiv \sqrt{-1}$, r is the radius of a latitude circle, and G is the vertical structure variable. The first term in Eq. 4 represents a stationary, meridionally trapped perturbation superimposed on the mean flow contribution represented by the second term. (We implicitly assume, consistent with the wave dynamical interpretation, that the stationary oval "obstacle" impinging on the jet enforces a stationary perturbation as observed.) If we substitute Eq. 4 into Eq. 3, assuming a mean zonal flow U given by Eq. 1, the meridional structure is seen to be identically satisfied by the assumed form for the wave perturbation (*17*), provided the vertical structure is related to the zonal wave number as

$$\left(\frac{d}{d\hat{z}} - 1\right)\frac{1}{\Gamma}\frac{dG}{d\hat{z}} - \left(\frac{n}{rf_0}\right)^2 G = 0 \quad (5)$$

With the introduction of the transformation

$$\chi \equiv e^{-\hat{z}/2}\frac{g}{\Gamma}\frac{dG}{d\hat{z}} \quad (6)$$

the vertical structure equation assumes the simple canonical form

$$\frac{d^2\chi}{d\hat{z}^2} + \left(\frac{\Gamma}{gh} - \frac{1}{4}\right)\chi = 0 \quad (7)$$

where in this case consistency with Eq. 5 requires that the vertical eigenvalue is given as

$$gh = -(rf_0/n)^2 \quad (8)$$

Here g is the gravitational acceleration and h is the so-called equivalent depth. With $g \approx 12$ m s^{-2}, as appropriate for Saturn's polar region (*18*), $f_0 \approx 3.2 \times 10^{-4}$ s^{-1}, $r \approx 1.4 \times 10^7$ m, and, taking the observed planetary wave number $n \equiv 6$, we infer that $h \approx -46$ km. By inspection of Eqs. 6 and 7, it is apparent that vertical propagation requires that $0 < h < 4\Gamma/g$, whereas a negative equivalent depth necessarily implies that the wave is vertically trapped within a region of positive static stability.

In Earth's atmosphere, stationary, vertically trapped wave motion can be forced either by heating or by flow over surface topography (*19*). Lindzen (*20*) has shown that Rossby waves with negative equivalent depths can exist as free oscillations in the mid-latitude terrestrial atmosphere in conjunction with flow over a level, rigid lower boundary. The vertical structure of the Saturn hexagon may be forced at deep levels below the cloud tops by the planet's internal convection, consistent with the suggestion by Gierasch (*5*). The diagnostically inferred value for the equivalent depth poses an important problem for the modeling of the vertical stratification and diabatic heating of the Saturn atmosphere. This will require a further consideration of the processes that might provide the appropriate stable stratification assumed by Eq. 3. One possibility is that the large-scale motion field is stabilized by the vertical redistribution of latent heat associated with the Saturn ammonia clouds, just below the 1-bar pressure level, or perhaps the much deeper water clouds, somewhere in the vicinity of 10 to 20 bars, according to simple estimates given by Gierasch and Conrath (*21*). Del Genio and McGrattan (*22*) have performed a preliminary numerical calculation of the moist stabilization process by the adaptation of a cumulus parameterization for a terrestrial general circulation model to atmospheric conditions on Jupiter. Their results suggest that for a water abundance comparable to that of a solar mixture, stable potential temperature contrasts as large as a few degrees Kelvin may be supported over a depth of a scale height or less. Although the actual abundance of condensables is not well constrained, the Saturn dynamics might be supported by a mixing ratio of heavy elements relative to hydrogen that is larger than that for the sun, as previously considered by Gierasch (*15*).

Without an independent account of the forcing or lower boundary condition that maintains the inferred vertical eigenvalue, which requires further study, we are at present unable to explain the selection of the observed planetary wave number 6 for the Saturn hexagon. It is tantalizing to note, however, that the zonal wave number for the stationary feature is very nearly the same as the meridional wave number of the alternating high-latitude jet streams, estimated as 2π divided by the peak-to-peak horizontal spacing of the eastward jets. Godfrey's (*1*) cloud-tracked wind measurements show peak eastward velocities at 76° and 61° north planetocentric latitude, corresponding to a meridional length scale for the jets of $L_J \approx 2.3 \times 10^6$ m $\approx r/6$. This coincidence may suggest that planetary wave number 6 is selected as the natural response to the same forcing that gives rise to the alternating zonal wind pattern. The inverse model analysis of the vertical structure problem for the inferred negative equivalent depth may therefore offer an important characterization of the planet's general circulation.

Further observational constraints on the deep atmospheric structure and dynamics of Saturn may be provided by a longwave (~ 10-cm) radiometry experiment on the Cassini orbiter. In the meantime, further interpretative studies of synoptic-scale features in the Voyager imaging data may offer the only observational avenue.

REFERENCES AND NOTES

1. D. A. Godfrey, *Icarus* **76**, 335 (1988).
2. ——, *Bull. Am. Astron. Soc.* **20**, 880 (1988). In this preliminary further study, the comparison of a Voyager 1 image taken 9 months before the Voyager 2 images analyzed in (*1*) implies a drift velocity for the hexagon of only just over 10 cm s^{-1}, less than the error in the corresponding drift of the measured SKR signal (*4*).
3. For precise navigation on an oblate planet it is important to distinguish between the planetocentric latitude λ_c, measured with respect to the planetary center, and the planetographic latitude λ_g, measured with respect to the vector normal to the local horizontal (equipotential) surface. The two are related as $\lambda_g = \tan^{-1}[(1 - \epsilon)^{-2} \tan\lambda_c]$, where the planetary oblateness ϵ is the fractional difference between the equatorial and polar radii. To first order in the oblateness, the planetary radius varies with the planetocentric latitude as $a \approx a_e(1 - \epsilon \sin^2\lambda_c)$, where a_e is the equatorial value. For Saturn, $\epsilon \approx 0.098$ and $a_e = 60{,}270$ km. Although the observations are most directly navigated in the planetocentric system, the planetographic form is more directly related to the local normal projection of the planetary rotation, as required for dynamical analysis.
4. M. D. Desch and M. L. Kaiser, *Geophys. Res. Lett.* **8**, 253 (1981).
5. P. J. Gierasch, *Nature* **337**, 309 (1989).
6. M. Allison, D. A. Godfrey, R. F. Beebe, *Bull. Am. Astron. Soc.* **19**, 848 (1987).
7. J. Pedlosky, *Geophysical Fluid Dynamics* (Springer-Verlag, New York, ed. 2, 1987).
8. An excellent historical review of the observations, theory, and laboratory simulation of Rossby waves is given by G. W. Platzman, *Q. J. R. Meteorol. Soc.* **94**, 225 (1968).
9. M. Allison, *Icarus* **83**, 282 (1990).
10. L. A. Sromovsky, H. E. Revercomb, R. J. Krauss, V. E. Suomi, *J. Geophys. Res.* **88**, 8650 (1983).
11. The so-called potential vorticity is a conserved property of the motion, representing the balance of the total relative plus planetary vorticity in response to the vertical stretching of a stratified column of rotating fluid. The relative vorticity is just the curl of the horizontal motion field, whereas the planetary vorticity is given as $f = 2\Omega \sin\lambda_g$. The stretching contribution is related to the vertical motion and static stability (or potential temperature stratification). A complete account of the theory is given by Pedlosky (*7*).
12. P. D. Thompson, *J. Meteorol.* **5**, 166 (1948).
13. G. J. Haltiner and R. T. Williams, *Numerical Prediction and Dynamic Meteorology* (Wiley, New York, ed. 2, 1980).
14. J. G. Charney, in *Dynamic Meteorology*, P. Morel, Ed. (Reidel, New York, 1973), pp. 97–351.
15. P. J. Gierasch, *Icarus* **29**, 445 (1976).
16. According to the thermal wind shear analysis of Voyager infrared measurements by B. J. Conrath and J. A. Pirraglia [*Icarus* **53**, 286 (1983)], the vertical scale of variation of the cloud-top winds within the Saturn stratosphere amounts to about 6 pressure scale heights. The vertical shear below the cloud tops is essentially unconstrained by the available observations.
17. The given solution represents only the gravest wave mode satisfying the meridional structure prescribed by the adopted Gaussian jet profile. More generally, Eq. 3 assumes a form equivalent to the Schrödinger equation for the harmonic oscillator, with higher order solutions given by the Hermite polynomial functions. These higher order solutions have a broader meridional distribution, however, and probably exceed the latitudinal span of validity for the model representation of the ambient shear.
18. G. F. Lindal, D. N. Sweetnam, V. R. Eshleman, *Astron. J.* **90**, 1136 (1985).
19. I. M. Held, in *Large-Scale Dynamical Processes in the Atmosphere*, B. Hoskins and R. Pearce, Eds. (Academic Press, New York, 1983), p. 127–168.
20. R. S. Lindzen, *Q. J. R. Meteorol. Soc.* **94**, 402 (1968).
21. P. J. Gierasch and B. J. Conrath, in *Recent Advances in Planetary Meteorology*, G. Hunt, Ed. (Cambridge Univ. Press, Cambridge, 1985), pp. 121–146.
22. A. D. Del Genio and K. B. McGrattan, *Icarus*, in press.

11 September 1989; accepted 5 December 1989

The Strange Periodic Comet Machholz

DANIEL W. E. GREEN, HANS RICKMAN, ALAIN C. PORTER,* KAREN J. MEECH

The recently discovered periodic comet Machholz 1986 VIII (1986e) travels closer to the sun than any known planet and any known comet with an orbital period of less than 150 years, thus providing astronomers with a unique object for studying cometary evolution. The comet is spiraling steadily closer to the sun, from perihelion distance $q \simeq 0.9$ astronomical unit at about A.D. 700 to $q \simeq 0.13$ at present (orbital period, 5.25 years), to an expected $q \simeq 0.03$ by about 2450; should the comet survive such increasingly close perihelion passages, q will begin steadily to increase shortly thereafter. A review of observations made since discovery is presented, together with a discussion of numerical investigations of the comet's orbit over 4000 years and prospects for observing the upcoming return to perihelion in 1991.

A MOST UNUSUAL SHORT-PERIOD comet was discovered visually on 12 May 1986 by California amateur astronomer Donald Machholz (*1*), who was using a large pair of binoculars. Following the usual practice for a newly discovered comet, Marsden (*2*) computed parabolic orbital elements under the assumption that this object was traveling in a nearly parabolic, long-period (that is, orbital period >200 years) path around the sun, and he found that its orbit was also highly inclined ($i \sim 60°$) with respect to the ecliptic. Indeed, almost all comets discovered visually nowadays that have high-inclination orbits are long-period comets. As more observations became available, however, it soon became evident that this object has an orbital period (P) as short as 5.3 years, and hence it became known as periodic comet Machholz 1986 VIII = 1986e (hereafter, P/Machholz).

What emerged as so unusual is the combination of the comet's short orbital period and high inclination, and its small perihelion distance (q = 0.127 AU). No other known comet with $P < 150$ years goes closer to the sun than P/Encke (P = 3.3, q = 0.341). Of the ~150 known short-period ($P < 200$ years) comets, only five have orbital periods less than that of P/Machholz (two of the five being considered "lost"). Most of these short-period comets belong to the so-called Jupiter family, characterized by relatively close approaches to Jupiter over periods of a few hundred years or less that affect the comets' orbits. P/Machholz apparently has not had such perturbing close approaches to Jupiter in thousands of years, even though its aphelion distance of $Q \simeq 5.9$ AU is well outside that planet's orbital distance ($a_J \simeq 5.2$ AU). P/Machholz's high orbital inclination is responsible for keeping the comet at a safe distance from Jupiter. Only four of the known periodic comets have orbits inclined more steeply (with respect to Earth's orbit) than does P/Machholz. The

D. W. E. Green, Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138.
H. Rickman, Astronomiska Observatoriet, 75120 Uppsala, Sweden.
A. C. Porter, California Institute of Technology, Pasadena, CA 91109.
K. J. Meech, Institute for Astronomy, University of Hawaii, Honolulu, HI 96822.

*Present address: Kitt Peak National Observatory, Tucson, AZ 85726.