two separate gas delivery systems. As described in the 8 February *Nature*, the fuel cells consist of three thin layers laid down on a quartz surface: first a thin film of platinum that serves as one electrode; then a gas-permeable membrane less than 0.5 micrometer thick; and finally an outer film of platinum that serves as the second electrode and is porous enough to allow gases to pass through. When the cell is bathed in a hydrogen/oxygen mixture, hydrogen atoms donate electrons at the outer electrode while oxygen atoms accept them at the inner electrode.

No one who was familiar with fuel cells would have ever thought a cell could operate without separating the two gases, Dyer says. "Not in a million years." Even after having made the fuel cells, it took him a while to understand how the inner and outer electrodes differentiate between the two gases. It is a "new type of catalysis" that is "closer to biochemistry than classical chemistry," Dyer says, and it depends on the fact that the gases come in contact with the inner electrode only through the membrane but touch the outer electrode directly. Dyer declines to give more details, but says he is writing a second paper that will clear up the mystery.

However it works, the new battery promises a number of applications. Since it can be fabricated with standard semiconductor processing techniques, integrated circuit chips could be made with the power sources built in, thus eliminating the need for power leads and the heat they generate. Putting the chip in a hydrogen/oxygen atmosphere would automatically power up the built-in batteries.

The major limitation now on Dyer's fuel cells is their relatively low power levels—they put out only 1 to 5 milliwatts per square centimeter. However, the flexibility in their shape offers a way around this, he says. By making the cell in the form of an open spiral, for instance, it should be possible to get a high surface area, and thus high power, from a small volume. Still greater increases in surface area and power output might be achieved by roughening the electrode surfaces. It should be "relatively easy" to get power levels of 1 kilowatt per kilogram of fuel cell, Dyer predicts. This would allow 10-watt generators compact enough to be used with portable phones, he suggests, and larger batteries could be used to power small electric vehicles or as emergency generators.

The cells could also be used as gas sensors. If placed in a hydrogen atmosphere, for instance, a cell will respond to even tiny amounts of oxygen by generating a voltage across the electrodes. ■ ROBERT POOL

Packing Your *n*-Dimensional Marbles

Some of the questions mathematicians ask sound silly: How many pennies can you lay on a tabletop? How many marbles will fit into a semi-trailer?

At other times the same basic question takes a somewhat more serious form: How many digital signals can occupy a noisy channel?

A recent discovery by Noam Elkies, a mathematician at Harvard University, has given researchers new insight into the mathematical theory that encompasses all three questions. Elkies's result is an unexpected application of a branch of number theory to a geometric problem known as sphere packing.

Sphere packing, in mathematical parlance, is a problem of cramming an n-dimensional space with n-dimensional "spheres," all of the same size, with the least amount of empty space in between. Coins are a good example of two-dimensional "spheres" (circles); marbles are examples in three-dimensional space.

In the mathematical theory that underlies telecommunication, a digitized signal is encoded as the coordinates of the center of a higher dimensional sphere packed in a higher dimensional space. After transmission down a noisy channel, the signal may no longer be exactly at the center, but as long as it's still within the sphere, the receiver can restore it to the center and read the signal exactly.

The easiest way to keep the signal clear would be to space the spheres far apart. But that's wasteful. The phone company can satisfy its customers and stockholders simultaneously by finding efficient ways to pack the signals. Similar problems crop up in other places: data compression, antenna design, and x-ray tomography. What everyone wants is the best way to pack spheres into the space their problem occupies.

Surprisingly—given its wide applications—the problem has so far only been solved in the simplest case: that of two dimensions, where the pennies fill 90.69% of the tabletop. In three dimensions the obvious candidate is the face-centered cubic packing of spheres beloved of crystallographers and fruit vendors.

Now, nobody is betting against the face-centered cubic packing as the best possible solution, but so far no one has been able to give a rigorous proof that there's nothing better. And in the really formidable dimensions—above a thousand, say—mathematicians are at a loss. "We don't know a better way of packing spheres than just picking them at random until there's no room left," Elkies says.

Although new sphere packings are discovered all the time, most of them have come from standard techniques in the subject. Elkies's approach, on the other hand, is brand new. It is based on the theory of elliptic curves, a branch of number theory that is concerned with finding solutions of certain polynomial equations. Though a newcomer to sphere packing, Elkies is an expert on elliptic curves; 2 years ago he applied the same theory to solve a 200-year-old problem related to Fermat's Last Theorem.

Elkies's result uses the theory of elliptic curves to construct a regular array of points, called a lattice, to serve as the centers for his spheres. These lattices have been long familiar to number theorists, but no one had looked at their sphere-packing properties before. Not every elliptic curve has the right kind of lattice, and part of the trick was finding the ones that do.

"Theoretically it's quite exciting," says Andrew Odlyzko, head of the Mathematics of Communication and Computer Systems Department at Bell Laboratories in Murray Hill, New Jersey. "It's a new way of approaching a famous unsolved problem."

Elkies' new packings may or may not be the best possible—his work doesn't address that issue—but they do give improvements on the best methods known so far in a number of dimensions, the largest being 1024. In several other cases his approach agrees with the previous best known packings—notably in dimension 24, which has a surprisingly efficient packing known as the Leech lattice. (Some new computer modems make use of the Leech lattice sphere packing.)

Elkies's discovery has no immediate practical application, says Odlyzko, "but that might change." For one thing, theoreticians are toying with the idea of upgrading modems to work in higher dimensional spaces. But whether it has immediate applications or not, Elkies' new discovery certainly gives mathematicians more space to play around in. ■ BARRY CIPRA

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