California Aftershock Model Uncertainties

P. A. Reasenberg and L. M. Jones (*1*) have estimated probabilities for the occurrence of large aftershocks in varying time intervals after a mainshock in California. These probabilities were calculated from a proposed "generic California" model of aftershock occurrence. The model has four parameters (a, b, c, and p), which are determined from an average of 62 previous aftershock sequences that had occurred throughout California from 1933 through 1987. Their plan is to use the a priori generic model as an initial estimate for any aftershock sequence, but then to update the model parameters (and the probabilities) as real-time data about the frequency and magnitudes of the aftershocks become available. In their report, however, tables are provided for the probabilities of hazardous aftershocks that are based on either the a priori estimates of the generic model (*1*, table 1) or on the final a posteriori values from an aftershock sequence (*1*, table 2); thus the utility of the update scheme is not clearly demonstrated. Moreover, because of inherent uncertainties probability estimates based on the generic model alone (*1*, table 1) are suspect.

The deviations in the parameters of the generic model (SD's of 18 to 33%) are seen in the histograms in figure 2 of their report. (The histograms contain about 45% more data values than the quoted 62 aftershock sequences.) Here, chi-squared tests were applied to the histograms of the a and p parameters, with the result that the null hypothesis of Gaussian distributions can be rejected at the $P = 0.05$ significance level ($\chi_a^2 = 40.8$, $P = 0.024$; $\chi_p^2 = 32.7$, $P = 0.036$). In fact, the values in the histogram of the a parameter spanning nearly ± 2 SD of the mean, produce a chi-squared statistic ($\chi^2 = 28.2$, $P = 0.059$) that does not formally reject the null hypothesis of a uniform distribution (5% significance level). The large uncertainties in these parameters can be shown to have a large effect on the estimated probabilities.

For example, consider estimating the probability of a large aftershock ($M \geq 5.5$) in the 24 hours immediately after a $M = 6.5$ mainshock in California. This would seem to be the time of the most value of the generic model, since Reasenberg and Jones have found that after about a day the model parameters are weighed more heavily by the real-time data from the aftershock sequence itself than by the a priori generic estimates. Allowing ± 1 SD in the two parameters that tested non-Gaussian (a and p), their equation 4 results in a spread of the estimated probability from 4 to 88%, compared with the 23.4% they tabulated from the median values of the generic model.

As another example, consider the probability of a large aftershock in the time interval 3 to 30 days after a mainshock. Uncertainties of ± 1 SD again in both a and p produce a spread of from 2 to 81%, compared with the tabulated value of 15.2%. According to Reasenberg and Jones, however, in this example the first 3 days of data after the mainshock can be used to update the parameters. This would presumably reduce the variance and thus decrease the spread in the above probability in accordance with the general scheme of going from table 1 to table 2 with real-time data. But in their report, no quantitative amount of variance reduction is given; thus no evaluation can be made of the reliability of the proposed update scheme in estimating probabilities for aftershocks.

In view of probable non-Gaussian statistics, the means of including the a priori generic averages into the update scheme is not readily apparent. In equation 5, Reasenberg and Jones suggest using a form of Bayes rule that assumes Gaussian statistics; this does not appear to be justified, and I believe alternative formulations or methods must be considered. A related question in non-Gaussian statistics is how close the mean value is to the most probable value of the data. As a worst-case illustration, consider rolling a die, that is, samples from a uniform distribution. An estimate, to any desired accuracy, of the mean value of the underlying stochastic process can be obtained by repeated rolls of the die. A histogram of the rolls provides constraints on the possible outcome of any roll of the die. But the next roll is unpredictable with any a priori model of the data. This illustration pertains to a discrete, limited process and obviously does not represent a continuous physical system, but the message is clear. In the aftershock model the a parameter is a measure of the production of aftershocks. The California average of a may therefore not be the best estimate (that is, the most probable) for describing aftershocks occurring in different tectonic settings of the state. Estimating model parameters from subsets of the data which focus on regional tectonics may actually prove more useful.

In addition to a and p, the other parameters (b and c), introduce even more uncertainty into the model. Therefore, the a priori generic model by itself appears to be unreliable in estimating probabilities of aftershocks because of poor constraints on some model parameters. Before the availability of real-time data, the generic model may have value as a predictive tool, but only in the broadest sense of assessing best- or worst-case scenarios for possible damaging aftershocks. To use it beyond its known time limitations, however, and without stating the important uncertainties, as in table 1 of Reasenberg and Jones, tends to give the apparent and misleading impression that aftershocks in California are reliably predictable.

PAUL A. RYDELEK
Observatorium Schiltach,
Heubach, 206,
7620 Wolfach,
Federal Republic of Germany

REFERENCES

1. P. A. Reasenberg and L. M. Jones, *Science* **243**, 1173 (1989).

1 September 1989; accepted 30 November 1989

Response: Rydelek criticizes our approach (*1*) to modeling the post-mainshock earthquake hazard, citing the existence of large uncertainty in the generic model results and alleging the unsuitability of our application of Bayes rule for the estimation of probabilities at times after the mainshock. His comments question the overall utility of our model for hazard assessment, and his main point concerns the uncertainty in the probabilities for earthquakes estimated for the generic model.

We first correct a mistake and amend terminology in our original report. Rydelek notes that the number of observations in our original figure 2 exceeds the stated number of earthquake sequences used in our formulation of the generic model. The stated number, 62, is correct, as are the parameter means, medians, and standard deviations. Unfortunately, the histograms shown in that figure were incorrect and do not represent those 62 sequences. The correct histograms are shown in Fig. 1. This error does not affect the results we originally reported.

We have refered to our probability estimates as Bayesian because they have the form of the posterior mean in the case that both the prior and sampling distributions are Gaussian. The relevant probability distributions are not Gaussian, so our estimates do not derive formally from Bayes rules. We will therefore refer to them here as the Reasenberg-Jones (RJ) estimates, while noting that their statistical informality does not diminish their effectiveness in meeting the needs for which they are designed. Our ad hoc algorithm uses only first and second moments of observed distributions in a simple fashion to produce accurate reflections of our best current knowledge of the behavior of aftershock sequences. We do not believe that a more statistically formal alternative method would yield appreciably different results.

Rydelek is correct in stating that the variance in the a priori model parameters for California earthquake seqences (in our original figure 2) is the major source of uncertainty in the interval probabilities calculated for the generic model. We stated in our report that the generic model provides a useful starting point for estimating post-mainshock hazard but that departures from the generic behavior can be expected in any particular sequence. By definition, a generic model of a process—one based solely on the central tendencies of a priori distributions of the parameters representing the process—provides information about the expected value and standard deviation of a future observation. Our generic model provides estimates of the probability of earthquakes after a mainshock. We did not include in our original report an analysis of the uncertainties in the generic model probabilities. While we stated that these uncertainties decrease rapidly with time after the mainshock because of the inclusion of observations from the current earthquake sequence, we did not demonstrate this behavior. We now more fully explore this aspect.

We investigated the uncertainties in probabilities estimated from both the generic model and from an ongoing earthquake sequence at selected times after the mainshock by conducting a series of experiments employing a Monte Carlo technique. The RJ probability estimates are given by linear combination of a component estimated from the current aftershock sequence and a component reflecting the central tendency of past sequences. In the first experiment, we investigated the effect of a priori variability in the parameter distribution by randomly sampling such variability, rather than by taking a central value as the starting point. We examine Rydelek's example of estimating, at the time of the mainshock, the probability of aftershocks $M \geq 5.5$ in the 1-day interval immediately after a mainshock with magnitude $M_m = 6.5$. Five hundred sets of values for the model parameters, a, b, and p (2) were drawn at random from the empirical distributions for 62 California earthquake sequences (Fig. 1). From the resulting distribution of probabilities $P(M_1 = M_m - 1, M_2 = \infty, S = 0.01, T - S = 1)$ (3), we determined the quantile points corresponding to median and ±1 SD (Table 1). As Rydelek points out, the uncertainty in this probability is substantial: the ±1 SD range about the generic value (0.234) is 0.070 to 0.590. For the case of a larger mainshock in the 7-day interval immediately after a mainshock, $P(M_1 = M_m, M_2 = \infty, S = 0.01, T - S = 7)$. The ±1 SD range about the generic value (0.049) is 0.015 to 0.145 (Table 1).

Our second experiment was designed to evaluate the uncertainty in estimates of P at selected times after the mainshock. We generated an ensemble of 500 synthetic earthquake sequences with parameter values equal to the generic model. These sequences included aftershocks with magnitudes $M \geq M_m - 4$, corresponding, in the case of a $M_m = 6.5$ mainshock, to complete aftershock observation for $M \geq 2.5$ (4). At selected times after the mainshock we estimated the parameters for each synthetic sequence with a maximum likelihood (ML) method. We then computed the RJ estimates using randomly sampled values from the 62 empirical sequences used in this study. This procedure isolates the uncertainty in our estimates due to the inherent variability of past sequences. We determined from the resulting distribution of $P(M_1 = M_m - 1, M_2 = \infty, S, T - S$

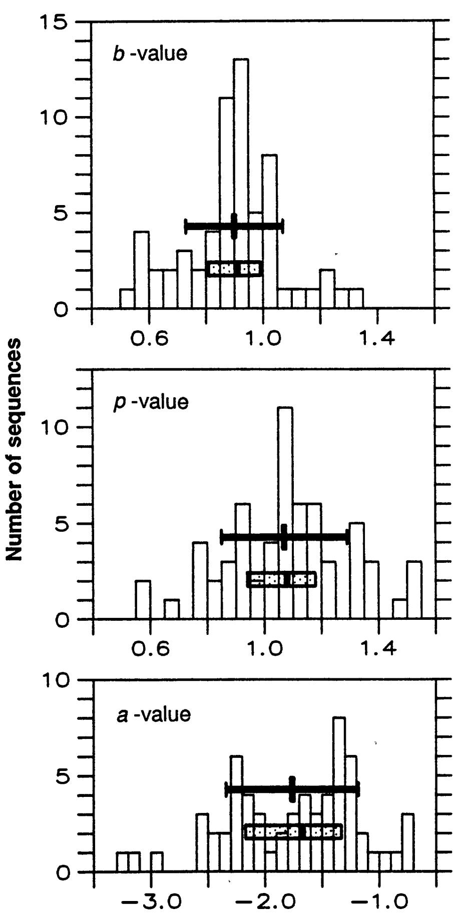

Fig. 1. Distributions of parameters (b, p, and a) determined for aftershock sequences after 62 ($M \geq 5.0$) mainshocks in California from 1933 to 1987. Solid bar indicates mean ± 1 SD. Shaded bar indicates median (central line) and upper and lower quartiles (end points) of distribution.

Table 1. Interval probabilities, $P(M_1, M_2, S, T)$, for strong aftershocks or larger mainshocks ($M_1 = M_m - 1$, $M_2 = \infty$), and for larger aftershocks only ($M_1 = M_m$, $M_2 = \infty$), estimated at the time of the mainshock (generic model, $S = 0.01$) and at selected times (S, in days) after the mainshock ($S = 0.25$, 0.5, and so forth). Generic model (GM) values are compared with results of the Monte Carlo (MC) experiment in which standard errors (±1 SD) of the model are estimated. Time intervals are described by S (interval start time, in days, after the mainshock) and $(T - S)$ (duration, in days).

Model or interval	S							
	0.01	0.25	0.5	1	3	7	15	30
Earthquakes with $M \geq M_m - 1$								
$(T - S) = 1$								
GM	0.234	0.119	0.083	0.052	0.021	0.009	0.004	0.002
MC result	0.240	0.115	0.080	0.051	0.021	0.009	0.004	0.002
−1 SD	0.070	0.070	0.054	0.034	0.015	0.007	0.003	0.001
+1 SD	0.590	0.190	0.124	0.075	0.029	0.012	0.006	0.003
$(T - S) = 7$								
GM	0.338	0.227	0.186	0.144	0.083	0.046	0.025	0.013
MC result	0.340	0.220	0.185	0.145	0.082	0.047	0.025	0.013
−1 SD	0.100	0.130	0.115	0.095	0.056	0.033	0.019	0.001
+1 SD	0.710	0.350	0.270	0.205	0.114	0.063	0.033	0.017
Earthquakes with $M \geq M_m$								
$(T - S) = 1$								
GM	0.032	0.015	0.011	0.007	0.003	0.001	0.001	0.000
MC result	0.045	0.015	0.011	0.007	0.003	0.001	0.001	0.000
−1 SD	0.010	0.008	0.006	0.004	0.002	0.001	0.000	0.000
+1 SD	0.120	0.028	0.018	0.011	0.004	0.002	0.001	0.000
$(T - S) = 7$								
GM	0.049	0.031	0.025	0.019	0.011	0.006	0.003	0.002
MC result	0.060	0.030	0.024	0.018	0.011	0.006	0.003	0.002
−1 SD	0.015	0.016	0.014	0.011	0.007	0.004	0.002	0.001
+1 SD	0.145	0.060	0.041	0.031	0.016	0.009	0.005	0.002

= 1) quantile points at the selected times, S, corresponding to the expected probability and ± 1 SD (Table 1). The standard error in P rapidly decreases with increasing time after the mainshock due to the inclusion of current data. For example, at $S = 1$ day after the mainshock, the ± 1 SD range about the generic 1-day interval probability (0.052) is 0.034 to 0.075 (Table 1).

Rydelek suggests estimating parameters from subsets of the a priori data corresponding to particular tectonic regions. While this approach has potential merit, it was not very successful for the California data. Parameter estimates for subsets of the data corresponding to the strike-slip regime of central California, the compressional regime of southwestern California and the strike-slip and extensional regime of eastern California do not differ significantly from each other, with one exception. The a value for sequences in eastern California is significantly higher than in central or southwestern California, which indicates a higher productivity of aftershocks there. In future applications of our method to other areas, however, a search for regional or tectonic subsets of earthquake sequences that significantly differ in some parameter values could provide an improvement over the single generic model approach.

PAUL A. REASENBERG
MARK V. MATTHEWS
U.S. Geological Survey,
345 Middlefield Road,
Menlo Park, CA 94025

REFERENCES AND NOTES

1. P. A. Reasenberg and L. M. Jones, *Science* **243**, 1173 (1989).
2. Model parameters a, b and p, defined in (*1*), describe the total number, magnitude distribution and time distribution of the aftershocks, respectively.
3. As defined in (*1*), M_1 and M_2 are, respectively, the lower and upper limits of a magnitude range, and S and T are, respectively, the lower and upper limits of a time interval, for which P is computed.
4. In practice, observation of earthquakes within the central and southern California U.S. Geological Survey networks is complete above approximately magnitude 1.5.

21 November 1989; accepted 30 November 1989

California Aftershock Hazard Forecasts

The first practical application for our model for real-time probabilistic hazard assessment (*1*) was provided by the 6 March 1989 *M*4.7 Obsidian Butte earthquake sequence in the northern Brawley Seismic Zone at the southern end of the Salton Sea, California (Fig. 1). The earthquake sequence was initially very active and included a relatively high proportion of large-magnitude aftershocks ($a = -0.5$, $b = 0.6$). As a result, the model-estimated probability for a larger ($M \geq 4.7$) earthquake during the first week in the sequence was relatively high—on the order of 0.30. Scientists familiar with the Brawley Seismic Zone generally felt that this estimate was reasonable. We did find, however, that other factors, in addition to those considered in the model, also warranted consideration.

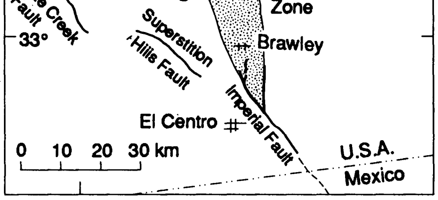

Fig. 1. Aftershock zone (black area at south end of the Salton Sea) of the 1989 Obsidian Butte earthquake sequence. The Brawley Seismic Zone (shaded area) is the site of numerous earthquake swarms in the cross over region between the San Andreas and Imperial faults.

One factor was the proximity (18 km) of the Obsidian Butte earthquakes to the intersection of the Brawley Seismic Zone and the San Andreas fault and the possibility that a great ($M \approx 8$) earthquake might be triggered by the Obsidian Butte sequence. The concensus was that the distance to the San Andreas fault was too great to warrant an upward revision of the model probability estimate for a great earthquake.

Another factor was that the Brawley Seismic Zone may not be capable of producing very large earthquakes because it is composed of numerous small faults, rather than a continuous long fault. If we assume that the largest possible earthquake in the Brawley Seismic Zone is *M*6.2 (the magnitude of the largest known historic event), then the model-estimated probability of a $M \geq 4.7$ earthquake decreases from 0.30 to 0.26.

The U.S. Geological Survey used the model to issue frequent public forecasts during the 17 October 1989 Loma Prieta earthquake sequence of probabilities of strong aftershocks within a day, a week, and 2 months. While this earthquake produced fewer aftershocks than expected for a generic *M*7.1 earthquake, the final model parameters determined for it ($a = -1.67$, $b = 0.75$, $p = 1.19$) all differ by less than 1 SD from their respective generic values (*2*, figure 1). We reported 24 hours after the earthquake that the chance of a $M \geq 5$ aftershock in the next day was 0.13 (none occurred). One week later that probability had decreased to 0.05, while the probability of a $M \geq 5$ aftershock over the next 2 months was 0.50 (none occurred). Forecasts were made first daily, and then less frequently, through 30 November 1989. These were issued to federal, state, and regional government agencies and were widely reported by Bay Area printed and electronic media. Public demand for and interest in aftershock forecasts was greatest immediately after the earthquake and remained high for about 2 weeks, decreasing as the felt aftershocks subsided.

Some local and regional government agencies requested model results particular to their needs during the first week of the sequence. The Port of Oakland requested estimates of probabilities for strong aftershocks in order to decide whether and when to reoccupy a damaged structure. The San Francisco Fire Department requested probabilities of strong shaking in the Marina and China Basin districts to guide decisions about equipment deployment and staffing levels in these damaged areas. Within the U.S. Geological Survey, scientists coordinating the regional deployment of strong motion portable seismographs frequently consulted model results in planning their experiment design and field strategy.

Our experience with the Obsidian Butte sequence and the Loma Prieta sequence has shown that the model can provide important information for real-time hazard assessment for earthquake sequences. Sensible real-time assessment of the seismic hazard during future earthquake sequences in California should also take into account relevant regional factors, including proximity to stressed fault segments, fault complications or gaps, and possible regional limitation of the maximum possible earthquake size.

In the Loma Prieta sequence, we found that regularly released short-term forecasts of expected aftershock activity were useful in meeting the high public demand for earthquake hazard information after a strong earthquake. We also saw that the press and public can easily misunderstand a probabilistic forecast; such public statements should be simple, clear, and consistent. Overall, however, we feel that our use of model probabilities to forecast the continuing