Euclid Unseated

Mathematical Visions. The Pursuit of Geometry in Victorian England. JOAN L. RICHARDS. Academic Press, San Diego, CA, 1988. xiv, 266 pp., illus. $34.95.

In describing his unpublished research on non-Euclidean geometry, Gauss took the view that its publication would bring on the "cries of the Boeotians." The founding of this area of mathematics was undertaken later by Nikolai Lobachevskii and Janos Bóylai, whose work in the mid-19th century resolved the problem of the independence of the parallel postulate of Euclid. This changed the status of Euclid's geometry from that of God-given and universal to membership in a family of geometries, all co-dependent as logical entities, and all candidates for the description of nature.

In fact, the "Boeotians," the philosophical descendants of Kant, were largely silent about the problem non-Euclidean geometry presented to their theory of knowledge. The silence in England was especially profound, and in spite of considerable later development of geometry on the Continent, word of Lobachevskii's and Bóylai's newly discovered universe fell on deaf ears. In order to understand the lack of response to such a fundamental change in the study of geometry, Richards sets about recreating the academic world in which these ideas would have found their life. Her careful research reveals a world in which mathematics held a special place for all educated persons, a place that non-Euclidean geometry threatened to upset.

To the mid-19th-century English the study of geometry was part of a liberal education, which sought the development of intellectual skills applicable to the challenges of life. Science and mathematics were among these skills, and Euclidean geometry was central to the vision of mathematics as a kind of universal science, understandable through logical reasoning and yet empirical as it described the space in which we live. Success in education was measured through examinations, the Tripos, and at the premier educational institutions of the time the Mathematical Tripos was based on Euclid and Newton. Students were expected to know the propositions by number and to be able to prove assertions in the manner of the *Elements* and the *Principia*.

With developments in geometry the view of mathematics so well served by the *Elements* became a much-discussed topic for the intellectual community, and reforms were sought that were consistent with the goals of liberal education. Complicating the discussion was the vindication of the empirical point of view surrounding Darwin's work, which increased the fervor for naturalistic scientific training. At this point projective geometry was becoming a major topic of research in England, and the program of unification of Euclidean and non-Euclidean geometry due to Klein made projective ideas exciting to the discussants on education. In particular, the methods of continuity and motions that seemed to allow passage from particular concrete cases to general theorems were consistent with the empirical views that guided educational philosophy.

The search for reforms involved researchers and educators at all levels, and Richards portrays the resulting turmoil in carefully chosen quotations from the representative participants. A figure at the cusp of these changes is Bertrand Russell, whose education in the older order prepared him for his earliest work on the foundation of geometry. His subsequent rejection of the foundations that underlay his training led to his later abstract logicist views. Richards examines Russell's early work to organize her portrait of the final decade of the 19th century and the time of transition. Mathematics and in particular geometry lost their preferred status as universal and descriptive, giving way to the power of analysis with its formalism. The truth supplied by mathematics no longer derived from its descriptive power but from its internal logic and rigor.

Richards's goal in this book is not simply a history of geometry in Victorian England. It is a history of ideas and a portrayal of a culture's pursuit of truth through education and research. She has enriched the mathematics by her recreation of the culture in which it was received and so has succeeded in giving the reader what history is capable of—real insight into what makes people think and act.

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The Idea of Symmetry

Felix Klein and Sophus Lie. Evolution of the Idea of Symmetry in the Nineteenth Century. I. M. YAGLOM. Birkhäuser Boston, Cambridge, MA, 1987. xii, 237 pp., illus. $40. Translated from the Russian by Sergei Sossinsky.

This study is best described by its subtitle, as a history of the concept of symmetry in 19th-century mathematics, rather than as a biography focused on the lives and work of its two chief protagonists, the Norwegian Sophus Lie (1842–1899) and the German Felix Klein (1849–1925). Based on the author's lectures at Yaroslavl University, the book retains a lively, informal flavor while giving an opinionated account of group theory, modern projective geometry, and the men who created them. Yaglom clearly has taken pains to address as broad an audience as possible, and the style and substance of his book make it well suited for the general reader who would like to learn something about the historical development of group theory and the role it plays in geometry, mechanics, and modern physics.

This sweeping saga begins with algebraic equation theory and the efforts of Lagrange, Ruffini, Abel, and Galois to penetrate the mysteries of quintic and higher-order equations. Lie and Klein step on stage briefly in chapter 2 as the "pupils" of Camille Jordan (the leading heir to Galois's legacy) during their brief visit to Paris in 1870. Yaglom then goes back to recount the emergence of projective geometry during the preceding 50 years, highlighting the work of Poncelet, Möbius, Steiner, Plücker, and Chasles. He then gives a reasonably complete narrative of contemporaneous developments in the field of non-Euclidean geometry, rightly chiding Gauss for his failure to promote the work of Bolyai and Lobachevsky, despite the fact that it was fully in accord with his own (unpublished) ideas. There follows a brief overview of work undertaken by Cayley, Hamilton, Grassmann, *et al.* on *n*-dimensional spaces and hypercomplex number systems, which serves as a preface to a well-written chapter on Lie groups and Lie algebras. Klein's "Erlangen Program" forms the subject of the penultimate chapter, and the book closes with biographical sketches of Klein and Lie, whose lives and careers were closely intertwined. The text of 137 pages is supplemented by 100 pages of notes that contain many interesting remarks as well as useful information; indeed, the most penetrating things Yaglom has to say generally appear in the notes rather than in his main text. These include allusions to the modern era in Lie group and Lie algebra theory inaugurated by Elie Cartan, the contributions of Hermann Weyl and the Bourbaki school, and numerous references to the work of leading Soviet mathematicians.

Considering the central importance of symmetry for pure mathematics and the burgeoning use of symmetry considerations in nearly every scientific discipline today, the pertinence of a work like this one surely requires no special comment. The author stands on firm ground when he asserts in the foreword that "of all the general scientific ideas which arose in the nineteenth century and were inherited by our century, none contributed so much to the intellectual atmosphere of our time as the idea of symmetry."