selhaus said. Here they have a built-in advantage because of their efforts on low-temperature superconductors in the same areas. The Japanese already have a working mag-lev train with conventional magnets and have been experimenting for 17 years with one that uses low-temperature superconducting magnets. They also have an ongoing program to develop an electric generator using low-temperature superconducting magnets, while the United States dropped its last such program in 1983.

U.S. researchers, on the other hand, generally believe it will be difficult, if not impossible, to develop applications dependent on very large magnetic fields. Instead, their research has been largely aimed at small-scale uses, such as superconducting electronics and microwave applications. The one exception, Dresselhaus said, is a superconducting energy storage system, and the push for this has come not from industry but from the Pentagon, which needs it for the Strategic Defense Initiative.

But even in the area of electronics, the Japanese may have a subtle advantage. Thin films of high-temperature superconductors, which will be needed to make electronic components, can be made in various ways. U.S. researchers are generally ahead of the Japanese in the laser ablation technique and electron beam evaporation, the panel said. But the Japanese are better at sputter deposition and chemical vapor deposition, and it is these two techniques that will probably be easiest to apply commercially.

The panel's most surprising discovery was the emphasis in Japan on organic superconductors. Japan has some 100 researchers studying these carbon-based materials, while a small group at Argonne National Laboratory provides the only major effort in the United States. Organic superconductors are scientifically interesting, but few U.S. researchers believe they will have significant commercial applications. However, the Japanese work has some of the panel members wondering if the Japanese know something they don't. "I was so worried about this," Dynes said, "that we now have a couple of researchers at Bell Labs working on it."

Dresselhaus said the fundamental advantage Japan has over the United States is a leadership that can assign projects and coordinate between different groups. In response, the panel recommended increasing the number of consortia and collaborations between industry, government, and university labs in the United States. It is particularly important, the panel added, that industry be included in the collaborations early because eventually industry must make the decision whether to commercialize a product. ■ ROBERT POOL

How the Grinch Stole Mathematics

Herb Wilf has been having fun making mathematics less fun for his colleagues. Wilf, who is a professor of mathematics at the University of Pennsylvania, and Doron Zeilberger at Drexel University, also in Philadelphia, have created an ingenious method for rendering ingenuity unnecessary in dealing with a class of (formerly fun) mathematical problems known as combinatorial identities. "I feel like the Grinch who stole Christmas," Wilf says gleefully.

Combinatorics, loosely speaking, is the art of counting complicated collections of objects, such as the number of ways a rowdy group of kids can choose up sides to play baseball or your chances of drawing to an inside straight. Beyond its recreational value, the subject has practical applications, especially in computer science. Algorithm designers, for example—the people who make computer programs work efficiently—frequently need to know the number of steps or amount of memory required by a potential algorithm. So they turn to combinatorial arguments, which, for mathematicians, means a line of reasoning, not a dispute between rival researchers.

Once it was an intellectual challenge to make a combinatorial argument. To do so you'd find a secluded spot and, with a pad and a sharp pencil, you'd prove a few combinatorial identities. An identity is an equation relating one combinatorial expression to another—equating a sum of binomial coefficients, for instance, to a power of 2. One side of the equation is usually rather complicated while the other side is relatively simple. The mathematician who could find and prove such an equation would succeed in simplifying an otherwise awkward calculation.

Now, here's where the fun used to come in. A standard proof of a combinatorial identity uses a series of cleverly chosen algebraic manipulations and rearrangements. The best proofs are a kind of mathematical poetry.

Wilf and Zeilberger's approach turns the poetry into a password. They use a computer algebra system to produce a "certificate of proof" from which an ordinary paper-and-pencil proof can readily be generated with no further insight needed. The certificate doesn't look like a proof at all. It looks like, and is, nothing more than an algebraic expression: a ratio of the two polynomials in two or more variables.

The secret? An algorithm developed in 1978 by William Gosper of Symbolics, Inc., in Palo Alto, California, and a clever idea added by Wilf and Zeilberger. Gosper's algorithm is incorporated as a single command in the computer algebra system Macsyma, and, like some kind of exquisitely trained hunting dog, it homes right in on the proof certificate.

Maybe you don't trust this techy, uninspired way of doing mathematics. Maybe you suspect these really aren't proofs, that the algorithm might accidentally certify an incorrect identity. But Grinch Wilf notes that even for anyone who doubts the certificate, there is an easy way to check it, in much the same way that a computer-generated factorization of a large number can be checked by simply multiplying the factors back together.

Gosper's algorithm occasionally meets an identity which it is unable to handle. But the Grinch's partner has the answer. For those identities Zeilberger has a general, more elaborate "machine" that is guaranteed to work in all cases (as long as the identity is in so-called "closed form"). Zeilberger actually developed the general method first. Wilf noticed that Zeilberger's proofs all "had much the same music to them," and helped find the simplified approach. "I had the advantage of not understanding his monster machine," Wilf explains.

One application of computer-generated proof certificates, Wilf notes, is to spare researchers the embarrassment of misstating identities or overlooking an obvious simplification. There are other spin-offs as well, which will likely provide deeper insights into the structure of combinatorial identities. In the process of proving one identity, the method actually proves a "companion" identity as well, and each of these has an associated "dual" identity. By repeatedly taking special cases, companions, and duals, Wilf and Zeilberger can grow an entire tree of identities out of a single proof certificate. They have already obtained several new identities this way.

So in trying to take the fun out of mathematics, the Grinch may have only put yet more presents under the tree. ■ BARRY A. CIPRA

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