Quantum Chaos: Enigma Wrapped in a Mystery

Indisputably, chaos is a part of the world, from atoms to solar systems. But quantum mechanics, the fundamental theory of nature, seemingly has no place for chaos

This is the fourth in a series of stories on chaos in various areas of science. The next article will deal with chaos and fluid dynamics, including chaos in the weather.

WHAT IS QUANTUM CHAOS? Don't ask. "There is no agreed upon definition of quantum chaos," says Yale University physicist Rick Jensen. He adds, "There may be no such thing as quantum chaos." Daniel Kleppner of the Massachusetts Institute of Technology says, "The very term 'quantum chaos' itself gives people the shivers." Joseph Ford at Georgia Tech says quantum chaos challenges fundamental beliefs about how the world works. "I'm claiming that we're headed for a revolution," he says.

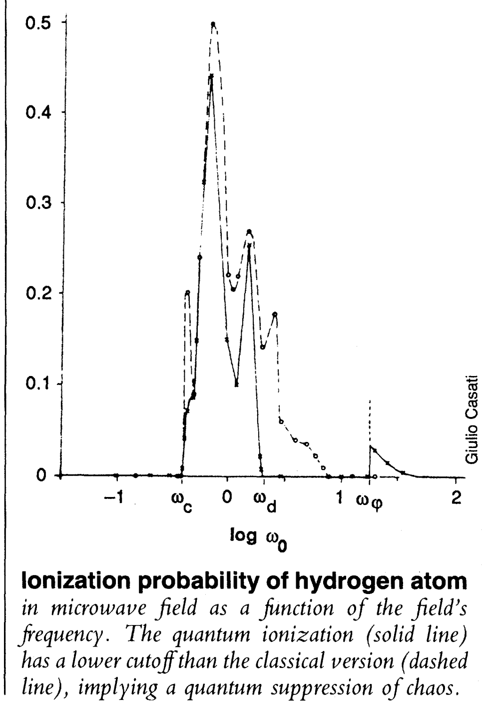
Chaos, a special type of disorder that pops up in Newtonian physics, may have no place in quantum physics. But the researchers looking into the question believe the existence or nonexistence of quantum chaos has fundamental implications about the underlying reality of the physical world. If chaos does exist on the quantum level, it raises confusing questions about the meaning of quantum mechanics; if it does not, the result could be even more profound.

To answer the question of what—if anything—quantum chaos is, physicists are exploring the border between classical and quantum physics. There, where the Newtonian equations of motion still provide good approximate answers but the fuzziness of quantum mechanics is starting to encroach, researchers hope to unearth clues about how classical chaos can be accounted for in quantum terms. Their tools include computer simulations of classical and quantum models, experimental measurements of the corresponding physical systems, and comparisons between them.

Roughly speaking, chaos is a type of randomness that appears in certain physical and biological systems and is intrinsic to the system rather than caused by outside noise or interference. A well-known example is the orbit of Pluto. Although the orbits of the other planets in the solar system are mostly regular, Pluto's path is complicated and impossible to predict over the long term. This chaos arises because Pluto is gravitationally kicked by other planets as it circles the sun, resulting in a path so sensitive to deviations that a change of a few feet in its position now could completely change its resulting location in the next several thousand years.

Over the past few years, scientists have found chaos in a number of physical systems, from something as large as the orbit of Pluto to something as small as a pair of atoms. It is generally accepted that chaos is a fact of life in our universe, so any physical theory must be able to account for it. "Quantum mechanics is supposed to be our universal theory of nature," Ford says. "It had better have chaos."

But there is a problem. The random behavior of chaos, which is well understood in classical Newtonian physics, does not seem to exist on the quantum level, where motion is described not in terms of position and velocity but rather as the evolution of a wave function. "Chaos is a classical feature associated with the trajectory of a particle," Kleppner says. "You do not have chaos in quantum mechanics."



Ionization probability of hydrogen atom *in microwave field as a function of the field's frequency. The quantum ionization (solid line) has a lower cutoff than the classical version (dashed line), implying a quantum suppression of chaos.*

Indeed, researchers have had a difficult time deciding what quantum chaos should be. The Schrödinger equation, which describes the evolution of a wave function, does not seem to allow chaos to emerge as in classical systems. One possible definition of quantum chaos would be randomness in a quantum system over and above the randomness inherent in the indeterminacy of the wave function, but so far no one has been able to see where such randomness would come from. Quantum mechanics posits a certain amount of randomness in the behavior of an electron, for instance, because the electron can only be described in terms of the probability it will be found in a certain state. But although the electron may not have a definite position, its wave function evolves in a nonrandom way.

"There were a number of attempts early on to define quantum chaos," Jensen says, "but there are not so many now." Instead, scientists have decided they need more information. "What almost everyone agrees on now is that the important question is how quantum systems behave when the corresponding classical system is chaotic." In short, researchers are looking at systems that can be described either by classical or quantum methods and are trying to determine what is happening at the quantum level when the classical system turns chaotic.

Not surprisingly, it turns out that quantum systems do interesting things when their classical counterparts are chaotic.

The system that has been examined most extensively is the ionization of hydrogen atoms by microwave radiation. A hydrogen atom—an electron orbiting a proton—will be ionized if the electron absorbs enough energy from the microwave radiation to escape the electrical attraction of the proton. The electron can absorb this energy either from a single photon, if the photon is energetic enough, or from a number of less energetic photons.

Although a hydrogen atom in a microwave field is a quantum system, it can be modeled quite accurately by a classical system in which the electron is a point particle rather than a quantum wave (see box). As the strength of the microwaves increases, the motion of the electron becomes increasingly complicated until it becomes chaotic. The electron gains energy from the radiation and eventually becomes energetic enough to escape the nucleus. The onset of chaos in the hydrogen atom leads to ionization.

These calculations, done in the early 1980s, were somewhat surprising because the classical ionization thresholds were very close to experimentally measured values, and ionization is an inherently quantum process. Since the classical model gives such accurate results, researchers had a good system in which to examine quantum behavior when the classical calculations say there is chaos.

Over the past few years, researchers from a number of institutions have worked this system over in great detail. Using both the classical and the quantum models of the system, they have done numerical calculations to discover the behavior predicted by each approach, and they have performed experiments to see how the system really does behave. The tentative conclusion is that although the quantum systems behave somewhat peculiarly when their classical counterparts are chaotic, there is no sign of real chaos at the quantum level, at least in the sense of random, unpredictable behavior.

What the quantum system does is "mimic" chaos. The probabilities of the electron wave function evolve in a complicated way that may look chaotic for long periods of time. Eventually, though, given enough time, the system will start to repeat itself, which is something classical chaotic systems never do.

The quantum behavior of the electron also lacks another of the hallmarks of chaos at the classical level: it is not irreversible. A classical chaotic system cannot retrace its tracks because in some sense it has lost the knowledge of where it has been. A quantum system, on the other hand, can go home again.

In 1986, Giulio Casati in Milan, Italy, Boris Chirikov in Novosibirsk, U.S.S.R., and colleagues compared the classical and quantum behavior of a hydrogen atom with principal quantum number $n_0 = 100$ exposed to a microwave field for a certain amount of time. (The principal quantum number n_0 describes the energy of the electron. The higher the number, the more energetic the electron, the farther away from the nucleus it will be, and the easier it will be to ionize.) The quantum and classical calculations gave almost the same predictions for the expected state of the hydrogen atom after exposure to the microwaves. The final state of the atom could be anywhere from $n_0 = 75$ to well above 100, with approximately equal probability of any outcome (see figure).

Then the researchers reversed the systems. The classical system remained spread out, but the quantum system returned to its starting point. Although the quantum system had mimicked the classical system and looked chaotic, it was not random and had not lost track of where it had been.

Comparison of the classical and quantum

Chaos in a Hydrogen Atom

The hydrogen atom, a single proton orbited by a single electron, is a natural subject for studies of quantum chaos. Because it is relatively simple both to understand theoretically and to observe and measure, researchers can make and test experimental predictions that would be impossible in more complicated systems. Much of the work done on quantum chaos has concerned hydrogen atoms, either illuminated by microwave radiation or put in a strong magnetic field.

Classically, one can think of a hydrogen atom as a miniature solar system, with the electron orbiting the proton just as a planet circles a sun. If the planet or the electron is undisturbed, the orbit is an ellipse. But if another influence is present—a second planet, for instance—the orbit becomes more complicated. And if that extra influence is strong enough, it can overcome the stability of the orbit and throw the planet into chaos. A chaotic orbit is so complex and so sensitive to any change in position that prediction of its orbit is impossible—an error of a few feet in measuring its position at one time would lead to a mistake of millions of miles in forecasting its location at some future time. The planet Pluto, tugged by the larger planets orbiting closer to the sun, has a chaotic orbit.

In a hydrogen atom, researchers can imitate the effect of adding a second planet by exposing the atom to an electromagnetic force. In practice, researchers have either placed the atom in a constant magnetic field or exposed it to electromagnetic radiation in the form of microwaves. Either stimulus disturbs the orbit of the electron and can push it into chaotic motion if the stimulus is strong enough and the electron is distant enough from the nucleus.

Rick Jensen of Yale University likens the behavior of a hydrogen atom irradiated by microwaves to the motion of a child on a swing being pushed by someone from behind. Like an electron orbiting a proton, a swing oscillates in a regular cycle, and the microwave radiation striking the electron gives it a periodic push just as the person does to the swing. One difference is that the impulses from the microwaves are not necessarily in time with the oscillations of the electron—it is as if the person behind the swing is giving it a push every 2 seconds no matter where it is in its cycle. Sometimes he pushes in time with the swing, and the child will go higher than before, and sometimes he will push out of sync, and slow the swing down. In the same way, an electron placed in microwave radiation will gain and lose energy as it moves in and out of resonance with the field, but will generally maintain a constant average energy because the effects even out.

If the pushes are strong enough, however, everything changes. In Jensen's analogy, this point comes when one pushes hard enough that the swing goes so high it misses the next push entirely and comes back just in time to catch the second one. In technical terms, the electron comes into resonance with the second harmonic of the microwave field. Now the electron no longer ends up giving back, on average, whatever energy it absorbs from the radiation. Instead, it starts to gradually gain energy from the pushes, moving slowly and erratically from the second harmonic to the third, the fourth, and so on up the ladder. "Once you can push hard enough for that one step, you can go from the second step, to the third step, and so on," Jensen explains.

The result is chaos—the movement of the electron becomes unpredictable. Its average energy increases regularly, and the electron moves to higher and higher energy levels. Eventually, the electron has so much energy that the pull of the proton can no longer hold it, and the electron is torn away. The hydrogen atom is ionized.

This picture of the microwave ionization of a hydrogen atom is purely classical. Quantum mechanics sees the electron not as having a well-defined position like a planet orbiting a sun, but instead as being spread out over a certain area of space. Instead of being a particle, it is a rather nebulous "wave packet." And unlike a planet, which can orbit at any distance from its sun, an electron can orbit a nucleus at only certain energy levels. In the quantum picture, before seeing the microwaves, the electron wave packet is localized—the electron has a definite energy. Microwave radiation at a high enough energy will delocalize the wave packet—the electron will become "spread out" over several energy levels, with a certain probability of being found in each one upon observation. This delocalization of the wave packet corresponds to the chaos in the classical motion of the electron. ■ **R.P.**

models of the hydrogen atom has revealed another difference: For certain frequencies of the microwave radiation, classical analysis predicts a much higher ionization probability than does quantum analysis. This "quantum suppression of chaos" has recently been observed experimentally by Peter Koch at the State University of New York at Stony Brook. Physicists are still arguing about what causes it, but it seems to involve resonances between some of the different quantum states of the hydrogen atom.

A second simple quantum system whose classical counterpart shows chaotic behavior is a hydrogen atom in a strong magnetic field. Classical chaos appears when the electron is far enough from the proton that the magnetic force and the attraction between proton and electron are of the same magnitude. Then instead of moving in a regular orbit about the proton, the electron wanders chaotically around.

Although in quantum terms it makes no sense to speak of the path of the electron, the quantum model does show its own behavioral change when the magnetic field is strong enough to throw the classical system into chaos. As the magnetic field crosses the classical chaos threshold, the spectrum of the energy levels in the hydrogen atom undergoes a dramatic shift. (The energy levels are the possible energies that the orbiting electron can have, and the state of the electron is described by the various probabilities that it will be found in each of these levels.)

"The statistics of the energy levels are one of the aspects of quantum chaos," Kleppner says. When the magnetic field is in the regular, nonchaotic region, the spacings of the energy levels are arranged in a Poisson-like distribution—a distribution one would expect if the spacings were random. When the magnetic field is increased into the chaotic regime, the spacings become much more regular. Curiously, classical chaos manifests itself here as an increased order in the energy level spectrum.

This change in the energy level spectrum for a hydrogen atom in a magnetic field or the spreading out of the wave function of the hydrogen atom in a microwave field are what Jensen calls "symptoms of quantum chaos." Instead of trying to define what quantum chaos is, it is more profitable, he says, to determine the characteristics of a quantum system that correspond to chaos in the corresponding classical system. In this view, quantum systems mimic chaos, but they do not have the randomness or irreversibility that mark classical chaotic systems.

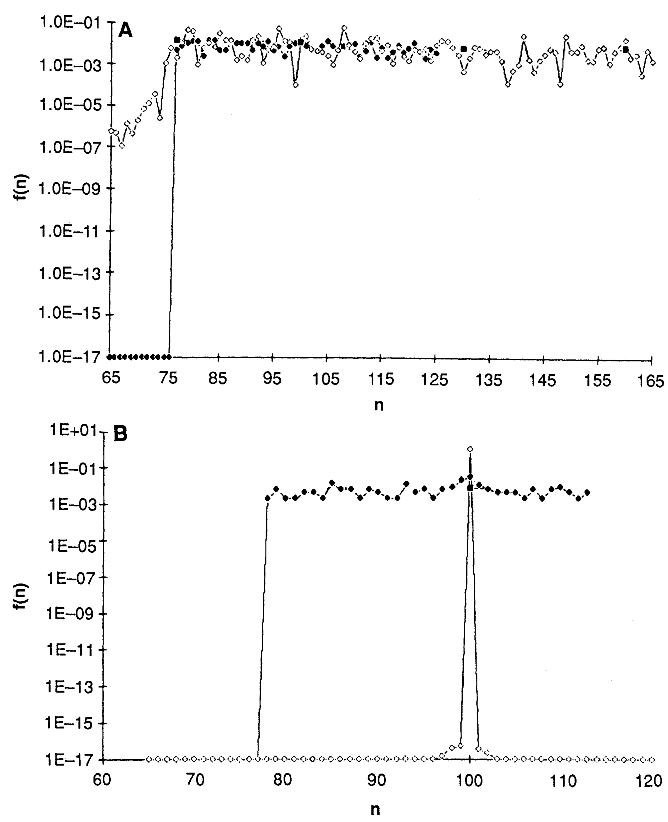
Ford believes this approach ignores a major problem: There should be something chaotic on the quantum level, he says, but "if one examines all the evidence to date, there is no chaos." He defines quantum chaos as "randomness in quantum mechanics over and above that which already appears in the wave function." Because classical systems have the capacity for chaos, the correspondence principle seems to imply chaos should be available to quantum systems as well. (The correspondence principle states, roughly, that quantum mechanics makes the same predictions that classical physics does for systems in which classical physics is applicable.) "I can prove for at least one time-dependent system that the correspondence principle fails," Ford says. He says he believes the correspondence principle fails much more generally and predicts chaos should not exist in any finite, bounded classical systems, but he cannot yet prove it.

Ronald Fox, a colleague of Ford's at Georgia Tech, believes he has a true example of quantum chaos. His example involves a large number of two-level systems—objects that can be in one of two states, such as an atom with an electron that can be in either of two energy levels. When these two-level systems are put in a box and illuminated with electromagnetic radiation, they will move from one state to the other as they absorb or emit photons. Fox calculates the expectation value that all the systems will be in one state. This expectation value changes chaotically over time, Fox says. "I think it's bona fide quantum chaos," he says.

Fox's work does not contradict Ford's belief because the system Fox has offered is not bounded. But even if this is a real example of quantum chaos, it does not avoid Ford's paradox: How can chaos exist in classical systems when their quantum counterparts have no random behavior?

Both Ford and Jensen believe the existence of quantum randomness is important for quantum statistical mechanics. Classical chaos helps to provide the randomness that lies at the heart of classical statistical mechanics, and presumably some version of chaos is needed for the quantum version. "How do you justify quantum statistical mechanics when it depends on the assumption there is additional randomness?" Ford asks. No one seems to have looked at this problem very closely yet.

Jensen says his gut feeling is that although, strictly speaking, quantum chaos does not exist, there is quantum behavior that is chaotic for all practical purposes. "Quantum mechanics involves linear dynamics in Hilbert space, and Hilbert space is awfully big [that is, it has many dimensions]," he says. "Quantum systems can mimic classical chaos for a very long time." A quantum system with 50 particles, he says, could mimic chaos for the age of the universe. That still does not explain where the randomness necessary for quantum statistical mechanics would come from, Jensen admits. It is a deep question that is only now being asked. ■ ROBERT POOL



Giulio Casati

No quantum chaos here: *The probability distribution of energy states in a classical (solid boxes) and quantum (open boxes) hydrogen atom after being exposed to a microwave field for a fixed time (A) and then reversing velocities for the same amount of time (B). The quantum atom returned to its starting point, the 100th energy level, thus exhibiting reversibility, while the classical atom remained diffuse.*