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18 May 1988; accepted 20 September 1988

Technical Comments

Toward a Universal Law of Generalization

When sentient organisms make decisions about how to react to novel stimuli or situations, they may do so in accordance with consequences associated with previously learned stimuli or situations. Thus, the probability that a novel stimulus is located perceptually in a consequential region is important in determining the subsequent behavior of the organism. A generalization experiment gives the probability that a response learned to stimulus S_i will be made to stimulus S_j. Shepard (*1*) proposed the basis for a law of generalization that involves two central ideas: first, that the probability that a response learned to stimulus S_i will be made to stimulus S_j is approximately a negative exponential function of the distance between the stimuli in a space of a certain dimensionality; second, that the metric used to define this distance will be Euclidean when the psychological dimensions are correlated and city block when they are not. Some of the examples used by Shepard (*1*) involved similarity, rather than generalization, and he noted that both probabilities arise from the same basic process. As presented, Shepard's theory applies only to experiments in which generalization is tested immediately after a single learning trial with a novel stimulus. The need to invoke this limitation results from the fact that, with highly similar stimuli or with delayed test stimuli, the relation between similarity and distance has been found to be Gaussian in form, and the distance metric appears to be Euclidean for cases in which the theory would predict city block. The work of Nosofsky (*2*) exemplifies this kind of result. Shepard conjectured that perceptual "noise" contributed in some way to these departures from theory but, since he did not have a formal model for dealing with these cases, he treated them as exceptions.

This self-imposed limitation on Shepard's theory can be removed by treating the mental representations of learned and novel stimuli (in generalization experiments) or pairs of stimuli (when estimating similarity) as momentary values from multivariate normal distributions (*3*). In other words, one can formulate Shepard's theory in stochastic terms. If d is the distance between the momentary psychological magnitudes and $g(d) = \exp(-d^{\alpha})$ is a measure of similarity, then $E(g)$ (*4*) is the expected value of the similarity measure. This model was evaluated in two dimensions when the perceptual dimensions are uncorrelated and, consistent with Shepard's theory, g is a negative exponential function ($\alpha = 1$) of city block distance, d. The evaluation revealed that the relation between $E(g)$ and the distance between the means of the distributions of psychological magnitudes (δ) is best described as a modified Gaussian function of Euclidean δ. This result is consistent with Nosofsky's results (*2*) and with several exceptions to Shepard's theory (*1*). The theory as originally discussed by Shepard regarding generalization and similarity was applicable when perceptual "noise" was absent; this extension allows for the possibility of a certain kind of noise (multivariate normal) and consequently extends Shepard's theory to include pairs of perceptually confusable objects.

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4. $$E(g) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty} \exp(-d^{\alpha}) \times \frac{\exp\{-0.5(\mathbf{z}-\boldsymbol{\mu})'\,\mathbf{V}^{-1}\,(\mathbf{z}-\boldsymbol{\mu})\}}{(2\pi)^{n/2}\,|\mathbf{V}|^{1/2}}\,dz_1dz_2\ldots dz_n$$
where $\mathbf{V}$ is the variance-covariance matrix of the difference between psychological values, $\mathbf{z}$; $\boldsymbol{\mu}$ is a vector of differences between the means of the momentary psychological values, $\boldsymbol{\mu}_x$ and $\boldsymbol{\mu}_y$.

25 March 1988; accepted 27 April 1988

Response: I welcome Ennis's demonstration that my recent theory of generalization (*1*) can be reconciled with Nosofsky's (*2*) impressive body of data on human performance in identification tasks. Results obtained by Ennis, by Nosofsky, and by me now appear consistent with the following theoretical characterization. In Nosofsky's identification experiments, subjects are primarily uncertain about the precise locations of individual stimuli in "psychological space." In the generalization experiments I considered, subjects are primarily uncertain about the location, size, and shape of the region of psychological space corresponding to the set of stimuli having the same important consequence as a given training stimulus.

Some of my own earlier results (*3*) had suggested, and Ennis has now more fully and rigorously demonstrated (*4*), how my theory of generalization can be extended to accommodate uncertainty about the locations of stimuli. The generalization theory then yields response probabilities that fall off in an approximately Gaussian manner with Euclidean distance from the training stimulus—the result empirically obtained by Nosofsky (*2*). When the uncertainty is primarily about the disposition of the consequential region, however, probability of response falls off, as I originally deduced (*1*), in close approximation to an exponential decay function of distances of either the Euclidian or "city block" varieties—depending on whether the subject assumes that the possible extensions of the consequential region along the underlying dimensions of the space are correlated or uncorrelated.

Clearly, my theory of generalization need not entail an exponential decay of response probability with distance under every condition. The exponential generalization function is a candidate for a "universal" psychological law in the sense only that the effects of any consequences associated with the *first* encounter with a novel stimulus may decrease exponentially with distance for all sentient organisms—wherever they may have evolved.

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12 October 1988; accepted 13 October 1988