between 40°N and 52°N 4.7%, and between 53°N and 64°N 6.2%. Although Dobson stations farther north are too sparse to yield an accurate trend, "the limited data suggest that there has been a decrease comparable in magnitude to that observed between 53° and 64° N." When the ozone trend between two consecutive 11-year solar cycles was determined, the pattern was the same—a smaller than average decrease in summer than in winter and within winter a larger decrease at higher, colder latitudes. Model predictions of wintertime losses are about one-third of these observed losses.

Reactions to the newly recognized losses run the gamut from heightened concern to outright consternation, Watson's near-concession of CFC involvement falling in the middle ground. Ralph Cicerone of the National Center for Atmospheric Research (NCAR) in Boulder, a panel working group member, says "I'm struck by the whole thing, but instead of being able to say clearly that CFCs are involved because the losses are so large, the changes are at the level of natural variability."

Panel member Sherwood Rowland of the University of California at Irvine, a cooriginator of the CFC hypothesis, is more confident about the possible conclusions. "My own view is that we're seeing appreciable losses in Antarctica and losses in the Northern Hemisphere," he said. "It seems quite likely that heterogeneous [ice-mediated] reactions could happen in the Arctic. I would be surprised if we don't find heterogeneous reactions of some significance in the Northern Hemisphere, especially in the Arctic."

A consensus must in all likelihood await direct chemical evidence that CFCs lie behind the loss of ozone. John Gille of NCAR, another panel member, noted that in the case of the Antarctic hole, there is now a smoking gun-high levels of chlorine monoxide, the ozone-destroying form of the CFCs' chlorine, as well as a wealth of other observations directly implicate CFCs. "This report says for the first time," in the case of the rest of the globe, "here is the corpse," observed Gille. Many researchers will be waiting to see if the same weapon was used in both crimes. A ground-based international expedition to the Arctic this spring may soon shed some light on the question. And a NASA-sponsored airborne study may be launched next year.

The panel had some suggestions. First, it complained that no validated data were yet available from the SBUV-2 instrument launched in 1984. Second, it noted that data interpretation must "be given higher priority and involve a broader cross section of the scientific community than in previous years."

The implications of the ozone decrease for the health of humans as well as flora and fauna were beyond the purview of the panel's report. It has been estimated that a decrease of 2.5% in ozone, such as that seen over Northern Hemisphere mid-latitudes, would lead to about a 10% increase in the rate of human skin cancer. That presumes decades of exposure that are required for the induction of skin cancer. It also presumes that the ozone reduction is spread evenly throughout the year, which it was not. Humans as well as the flora and fauna fortuitously tend to minimize their exposure in the season of greatest loss. What the future holds may be a bit clearer next month when NASA makes its periodic full report to Congress on the state of the upper atmosphere. **BRICHARD A. KERR**

Calculus: Crisis Looms in Mathematics' Future

Researchers and educators are debating how calculus should best be taught to increasingly recalcitrant students

ATHEMATICS, a discipline not normally known for its heated debates, is warming up to the subject of calculus reform. The teaching of calculus—when to do it, and even why to do it—has become a major issue in mathematics. The National Science Foundation has earmarked more than \$1 million for an initiative in calculus reform. And at colleges across the country, experimental teaching programs are springing up.

"The more computer power we have, the less the students know what they're doing."

Calculus reform is considered urgent because of the course's commanding position in the early undergraduate experience of students hoping to go on in science, engineering, business, and other fields. Robert White, president of the National Academy of Engineering, calls calculus "a critical waystation for the technical manpower that this country needs." Yet at some institutions as many as 50% of the students enrolling calculus either fail or withdraw from the course.

Many students today enter college with a weak background in the prerequisites to calculus: algebra, geometry, and trigonometry. Worse, in the opinion of college professors, many of these students have already had a watered-down taste of calculus in high school. The upshot is that many of these students retake calculus in college and do poorly; they think they know more than they really do, and the course does not "grab" them because they have already seen the highlights. The shortage of teachers capable of teaching calculus in high school adds another dimension to the problem.

In response, the Mathematics Association of America and the National Council of Teachers of Mathematics have sent a joint letter to high schools nationwide recommending that calculus be taught only to students who have a full 4 years of preparation in mathematics, and that it be taught with the expectation that students will not repeat the course in college.

While there is widespread agreement as to the problems that beset calculus instruction at the college level—unwieldy textbooks that have stuck to an outmoded emphasis on rote and repetition, unmanageable class sizes in many institutions, and unmotivated faculty who see teaching as a distraction from research—there is a correspondingly widespread disagreement as to the solutions.

Many argue that computers and the new generation of sophisticated hand-held calculators will force a change in calculus instructions, whether change is wanted or not. The Hewlett-Packard HP 28S, a programmable calculator that incorporates symbolic manipulations, equation-solving algorithms, and graphing capabilities—in short, much of what students are currently taught to do—represents a threat to some and an opportunity to others.

"This identifies a new wave," says John Kenelly of Clemson University, who has worked with several of the pocket calculators. Kenelly and five colleagues are using the HP 28S this year in calculus and other courses at Clemson. Engineers do not work with paper and pencil anymore, Kenelly points out, adding "We have a colleague or two who think we're going to bring up a bunch of button-pushing dead-heads, but the bulk of the community is behind it."

A number of colleges-among them Colby College, Denison University, Harvey Mudd College, Oberlin College, Rollins College, St. Olaf College, and the University of Waterloo-are experimenting with calculus courses that use mainframe or minicomputer-based Computer Algebra Systems such as MACSYMA, Maple, or SMP. These systems are more powerful than their handheld cousins, and also more user-friendly. Their main drawback is one of access, which may limit their use to smaller colleges where only a few hundred students would need to be accommodated. Proponents believe these systems will allow teachers to focus more on the concepts of calculus rather than on computational techniques and can foster in students a more exploratory and experimental attitude toward mathematics.

Alvin White of Harvey Mudd College is dubious. "The more computer power we have, the less the students know what they're doing," he says. "The promise we were given by the calculator people is that we can spend more time on the underlying ideas. But in my experience, the time is spent on showing them more buttons to push."

Even within the pro-computer camp there is disagreement over the issue of programming: should students write their own programs or not? Most of the Computer Algebra Systems only require the user to learn some special commands, and at least one— Maple, developed at the University of Waterloo—is aimed at being "user friendly." Herbert Greenberg of the University of Denver, which has developed its own software called "Calctool," says that students should not be aware of programming, only of the mathematical applications. "We are teaching programming, and the students are not doing programming," he says.

James Baumgartner of Dartmouth College, New Hampshire, disagrees with this outlook. "If you choose the problems correctly, they're simple, they're short, and they teach the essence of programming." Dartmouth—whose former president, John Kemeny, created the programming language BASIC—emphasizes programming from the beginning. "What we're doing is building a base for further down the line," Baumgartner says.

Another promising approach to calculus reform is the introduction of writing as part of the mathematics curriculum. Proponents such as David Smith of Duke University stress the relationship between writing and reasoning. According to Smith, "failure to read and analyze instructions prevents students from getting started on a problem, and their ability to understand a solution process is related to their ability to explain in English what they have done." Robert Webber of Longwood College, for instance, requires students to write documentation of computer programs: explanations of what their programs are meant to do and how they work. He requires revision of any project that has inadequate documentation. Webber notes that about one quarter of the students receive a lower letter grade because



Education in a technical world: the demand for an understanding of calculus is increasing in an increasingly technical world, but the teaching of it is suffering.

of their writing. He tells them that "if they don't write good documentation as professionals, they'll lose more than just a letter grade."

Why teach calculus to students who will not be using it? some ask. Alternatives to calculus have been proposed, but generally found wanting. In particular, "discrete mathematics," which emphasizes the study of finite sets and combinations, has been suggested as more pertinent to students of computer science. Experiences at Dartmouth College and the University of Denver, however, suggest the opposite. Greenberg helped to develop a course at the University of Denver in "discrete structures and calculus." He later retitled it "Destruct Creatures." Students did not like the course. Greenberg says. Even the computer science majors said it had nothing to do with the computer science they were studying, and called the course "a complete waste of time."

Dartmouth College suffered a similar experience: their discrete mathematics course, which was designed for the freshman year, was dropped as a requirement by the Computer Science Department. Most colleges now offer discrete mathematics as a course subsequent to a full year of calculus.

Two major conferences on calculus reform have been held in the last 2 years, including one sponsored by the National Academy of Sciences and the National Academy of Engineering. Organizers of the conferences hope that the visibility they have attained will convince people that change is both necessary and possible. Lynn Steen of St. Olaf College points out that the American Institute of Physics is embarking on a similar reform, focusing on the development of a calculus-based physics course. Ronald Douglas of the State University of New York at Stony Brook sees desk-top publishing as an important ingredient, enabling individuals to create and disseminate new curricular materials that the conservative textbook publishing industry is reluctant to pick up.

Although funding levels have not been settled, the NSF initiative, which includes plans to fund several small projects as well as conferences and workshops, should also help. Timothy O'Meara, Provost of Notre Dame and himself a mathematician, observes an element of self-interest: if mathematics isn't better taught, there will be fewer mathematicians in the future. "The approach of the NSF to funnel money into research in the long run isn't good for research—the pipeline dries up."

BARRY A. CIPRA

Barry Cipra is a mathematician and writer based in Northfield, Minnesota.