Comet Source: Close to Neptune

A computer simulation of orbital dynamics points to a close-in source for some comets; the source may even be observable, and astronomers are already beginning a search for it

COMETS have been considered outsiders, visitors from an enveloping cloud of inactive comets on the far fringes of the solar system. But new computer simulations of how comets can be drawn into the inner solar system eliminate a far-distant, spherical cloud of comets as the source for a major class of comets. The only practicable source in these simulations for the comets that now follow small, quick orbits near the planets is a flat disk of comets lying just outside the orbit of Neptune. That is a distance of little more than 30 times the distance between the sun and Earth (30 astronomical units) compared to the tens of thousands of astronomical units to the distant comet cloud.

Martin Duncan of Lick Observatory and Thomas Quinn and Scott Tremaine of the University of Toronto ran their computer model of the solar system to see how some of the peculiar characteristics of the short-period comets, those having orbital periods of 200 years or less, might have originated. Could the cloud of comets, called the Oort cloud, supply objects that behaved like these? It clearly is the source of the 589 known long-period comets, the ones rarely seen from Earth as they loop by the sun on circuits that require 200 to millions of years to complete. Everything about their motion is consistent with their having been jostled out of the Oort cloud by a passing star and into an orbit that passes near the sun.

But the short-period comets do not behave like typical Oort cloud comets gone astray. Most strikingly, the orbits of the short-period comets tend to lie within about 30 degrees of the plane of Earth's orbit, called the ecliptic. And only 4 of the 121 known short-period comets have their orbits tilted more than 90 degrees so that they are orbiting in the direction opposite to that of Earth.

In contrast, half the comets in the Oort cloud must have such retrograde orbits. They did not always. In the beginning, the dust and gas that went into forming the solar system collapsed from a cloud into a disk, just the way the trillions of particles orbiting Saturn form a ring. After forming by agglomeration in the vicinity of Uranus and Neptune, the future long-period comets could still not escape the confines of the disk as these two planets slung them into far-ranging orbits. Only the gravitational influence of the whole galaxy and random encounters with passing stars could finally disperse them into the randomly inclined orbits of a cloud.

Duncan and his colleagues first checked to see if, as reported 15 years ago, the four giant planets could select from comets falling in from the Oort cloud only those having low-inclination orbits typical of short-period comets. Using a mathematical model that included the sun, the four giant planets, and 5000 comets falling into the vicinity of massive Jupiter from the Oort cloud, they ran a simulation on their own souped-up Sun-3 microcomputer for a total of several months of computer time. They speeded up the calculation of millions of years of orbital evolution by increasing the mass of the giant planets by a factor of up to 40 in the simulation.

"There's a fairly good chance that with a concerted effort something can be found."

To their surprise, these modelers found that the planets are not selective at all when they deflect comets into new, smaller orbits. The comets from the Oort cloud that achieved periods of less than 200 years formed a cloud of their own, with no preference for orbiting in a disk near the ecliptic. In addition, three-quarters of them had periods greater than 15 years; only 21 of the 121 observed short-period comets have periods greater than 15 years. Starting the simulation with the Oort cloud comets coming no closer to the sun than Uranus and Neptune did not improve the situation.

Ruling out the Oort cloud, the modelers next tried a belt of low-inclination comets near Neptune's orbit. The idea dates back to a suggestion by Gerard Kuiper in 1951 that it would be only natural to find some debris from the formation of the solar system beyond Neptune. Comet-sized objects would not have formed planets there, but neither the giant planets nor passing stars could easily dislodge them or even smear their disk into a cloud. In fact, an absence of comets there would imply an oddly abrupt outer edge to the original solar system disk.

When the simulation was run with a disk of comets orbiting between a distance of 50 and 20 to 30 astronomical units, or well inside the orbit of Neptune, the comparison with reality was impressive. The mean orbital distances of both simulated and observed short-period comets cluster around 3 astronomical units with a lesser tendency to be near 5 astronomical units, the orbital distance of gravitationally influential Jupiter. That is also where the preponderance of maximum orbital distances lies for both sets of comets. Comets in each set also have a tendency to be passing the ecliptic when they are closest to the sun.

Most crucially, about half of the simulated comets, which started out with inclinations of 0 degrees to 18 degrees, retained inclinations of less than 30 degrees. More than 80% of observed short-period comets are confined to such a disk. About 8% of the simulated comets had their orbits tilted so much as to be in retrograde motion, compared with 3% of observed comets and 50% of Oort cloud comets. As is the case with observed comets, simulated retrograde comets tended to have periods longer than 15 years.

"You can't get good agreement," says Tremaine, "if you start off with inclinations far out of the ecliptic. It works if and only if the source is in the plane of the ecliptic." That moves the hypothesis of a comet belt lying beyond Neptune from being "very plausible to being the only plausible hypothesis." The Oort cloud as a significant source for short-period comets "is now ruled out," he says. Others would prefer retaining an Oort cloud source for at least some short-period comets, such as Halley's, that have retrograde orbits.

There are some hurdles yet for the "Kuiper belt." One is finding a means of bringing comets from the belt into Neptune-crossing orbits, which is where they were at the beginning of the simulation. Duncan and his colleagues suggest that the gravitational pull of exceptionally large comets within the belt might stir it up enough from within. Additionally, Neptune might be pulling in a trickle of comets whose closest approaches just bring them within Neptune's influence.

Another problem is the need for a belt that is massive enough to supply the comets but not so massive that it unduly perturbs the rest of the solar system. Perhaps the most sensitive indicator of an unseen perturber would be the orbital motion of Halley's Comet, which spends most of its 76-year orbit in the vicinity of Neptune's orbit. Donald Yeomans of the Jet Propulsion Laboratory has calculated that a comet belt at 40 astronomical units having a total mass equal to that of Earth, a modest size for the hypothesized belt, would have twisted Halley's orbit a few thousandths of a degree from the position predicted assuming there were no belt. No such perturbation was detected this time around, notes Yeomans.

A close-in belt of 1 Earth mass is "most inconsistent with what Halley's orbit has been over the past several centuries," says Yeomans. "If you want to move the belt, say out to 100 astronomical units, that constraint goes away."

These problems would become academic if someone caught a glimpse of the belt itself. One member of the belt may already be known. Chiron is an oddball member of the solar system, as black as a comet nucleus, as big as a large asteroid, and orbiting between Saturn and Uranus in an unstable orbit. If there are 100,000 comets inside the orbit of Uranus, as the simulations suggest, and if they have a reasonable range of sizes, "the existence of an object as bright as Chiron inside the orbit of Uranus is not surprising," writes Duncan. Only one in a hundred thousand members of the belt need be as large as Chiron for there to be at least 10,000 belt comets of magnitude 22, which could be detectable.

As Tremaine points out, with that many objects near the ecliptic, the element of chance is removed. An observer can concentrate on detecting faint objects in one small area without worrying whether he picked the right spot. "There's a fairly good chance that with a concerted effort something can be found," says Tremaine. One search is already under way, others are being considered. The Hubble Space Telescope, scheduled for launch in 1989, would greatly simplify the search. ■ RICHARD A. KERR

ADDITIONAL READING

M. Duncan, T. Quinn, S. Tremaine, "The origin of short-period comets," *Astrophys. J. Letts.*, in press.

Fermat's Theorem Proved?

For the first time in memory, the mathematics community is optimistic that its most famous open problem—Fermat's Last Theorem—may finally have been proved. Experts are poring over a proof recently completed by Yoichi Miyaoka of the Tokyo Metropolitan University, currently at the Max Planck Institute for Mathematics in Bonn, West Germany. Although no one will be completely confident until all the details have been thoroughly checked, those involved feel that Miyaoka's proof has the best chance yet of settling the centuries-old problem. Until recently, serious mathematicians have shied away from working directly on Fermat's Last Theorem—it was considered quixotic to be working on a 350-year-old problem.

Fermat's Last Theorem asserts that the equation $x^n + y^n = z^n$ has no positive integer solutions x,y,z if the exponent n is greater than 2. French mathematician Pierre Fermat stated this "theorem" around 1637 in the margin of a book, with the tantalizing remark that the margin was too narrow to include the proof. Mathematicians have been trying to widen that margin ever since. Fermat himself did actually write down a proof of the theorem for the exponent $n = 4$, and Leonhard Euler contributed a proof for $n = 3$. In the 1840s, Ernst Eduard Kummer set up a mathematical theory that enabled him to prove the theorem for a large number of exponents. By last year improvements on Kummer's work, and high-speed computers, had enabled mathematicians to prove Fermat's Last Theorem for all exponents up to 150,000. Furthermore, it has been shown that counterexamples would have to be extremely large, with x, y, and z each having hundreds of thousands of digits.

Mathematicians are optimistic for Miyaoka's proof, in part because it is based on new ideas taken from a previously untried source: differential geometry, a branch of mathematics best known as the setting for General Relativity. In the 1970s, S. Arakelov and other mathematicians in Russia began the task of making arithmetical analogues of results in differential geometry. Their ideas reached a milestone in 1983, when Gerd Faltings, now at Princeton University, developed them into a proof of another important problem in number theory known as the Mordell conjecture. (The Mordell conjecture is much younger than Fermat's Last Theorem; it was formulated in the 1920s.) One consequence of Faltings's result is that the Fermat equation has only a finite number—presumably zero—of different solutions x,y,z for any given exponent n. (In the theory, two solutions are considered the "same" if one is merely a multiple of the other, such as 3, 4, 5 and 6, 8, 10 for $n = 2$.) Faltings's coup led mathematicians to think that many old problems in number theory and algebraic geometry might be accessible to the new methods.

Miyaoka's work involves the arithmetical analogue of one of the deepest results in differential geometry: a fundamental inequality involving certain topological invariants of surfaces. (A simple example of a topological invariant is the number of "holes" or "handles" on a surface, such as the hole in a doughnut and a handle in a coffee cup.) This inequality was proved true in 1974—by Miyaoka, who is a recognized expert in differential geometry. (This is one reason why mathematicians are taking his work on Fermat's Last Theorem seriously.) The connection of the inequality with Fermat's Last Theorem was made about a year ago by A. N. Parshin, a Russian mathematician, who proved that if the arithmetical analogue of the inequality is true, then Fermat's Last Theorem is also true. In other words, Fermat's Last Theorem would be a simple corollary to a much deeper theory. Miyaoka has now apparently put on the crowning touch by proving the arithmetical analogue of his own inequality, thus completing Parshin's argument.

Tempering their optimism, mathematicians also express caution at this point. Experts in the new theory are only now starting to check Miyaoka's proof, and it may take weeks or even months for the theorem to be accepted with confidence. In a sense the proof is like a carefully thought-out, very complicated computer program that has not been run very often—the logic looks good, but there may still be bugs. However, even if mistakes are found to invalidate Miyaoka's proof, the basic approach is considered promising. ■ BARRY A. CIPRA

Barry Cipra is a mathematician and writer based in Northfield, Minnesota.