though these experiments are still in their early stages. Willadsen does not expect to encounter insurmountable difficulties with them, however.

In addition, the overall efficiency of the cloning techniques needs to be improved. For example, the procedures now work better with oocytes that have matured, and embryos that have been produced, in live animals, but for practical reasons the researchers would prefer to perform these operations in vitro.

The success with cloning sheep and cattle embryos contrasts with the—at best—equivocal results with the cloning of mouse embryos. Early in this decade, Karl Illmensee, who was then at the University of Geneva, Switzerland, and Peter Hoppe of the Jackson Laboratory reported that they had cloned mice by transplanting the nuclei of embryonic cells into enucleated eggs. Other investigators have not been able to reproduce the results, however.

First points to two differences between the experiments with mouse embryos and those with sheep and cattle embryos that may help to explain the difficulties in achieving the cloning of mice. Illmensee and Hoppe used fertilized eggs for their recipient cells, whereas unfertilized oocytes are the recipients in the sheep and cattle experiments. "An oocyte will treat a new nucleus like that of a sperm," First says. However, a fertilized egg apparently no longer has that capacity.

Moreover, the nuclei that are transplanted into the oocytes must either be undifferentiated or just slightly differentiated if they are to be reactivated like the nuclei of recently fertilized eggs. Illmensee and Hoppe obtained the nuclei they transplanted from the cells of the inner cell mass of the mouse blastocyst, a fairly late stage of development.

Work by a number of investigators has indicated that mouse cells begin differentiating very early in development, even at the two-cell stage of the embryo. In contrast, First says, the cells of cattle embryos do not undergo irreversible differentiation until the 16- or 32-cell stage of the embryo, which is the latest stage used for cloning. The ability to clone the embryonic cells and obtain the birth of apparently normal lambs and calves points to the developmental potential of the embryonic cells. ■ JEAN L. MARX

ADDITIONAL READING

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Solutions to Euler Equation

Even the great ones can be wrong—but it may take 200 years for anyone to be sure. In 1778, the Swiss mathematician Leonhard Euler proposed that the equation $x^4 + y^4 + z^4 = t^4$ has no positive integer solutions for x, y, z, and t. Last summer, Noam Elkies, a young mathematician at Harvard University, showed that Euler was way off the mark: not only does the equation have solutions, it has infinitely many of them.

Euler's equation is a variant of a famous open problem in number theory known as Fermat's Last Theorem. Fermat's "theorem"—which is so called only because Format claimed to have a proof of it—states that the equation $x^n + y^n = z^n$ has no positive integer solutions if the exponent n is larger than 2. Fermat's assertion had been proved for many individual exponents—Euler proved it for $n = 3$—but a general proof has so far eluded the best efforts of mathematicians.

Euler's variant, generally, was that it requires the sum of three cubes to equal a cube, four fourth powers to equal a fourth power, and so on. In 1966, two mathematicians and a computer showed that Euler was wrong as far as fifth powers are concerned. By having a computer look at all combinations of small numbers, Leon Lander and Thomas Parkin, then with Aerospace Corporation, Los Angeles, California, discovered the counterexample $27^5 + 84^5 + 110^5 + 133^5 = 144^5$. However, similar searches with the equation $x^4 + y^4 + z^4 = t^4$ yielded no results. This is understandable in light of Elkies' solution: the smallest number involved in a counterexample has eight digits.

Elkies' proof relies on the fact that a related equation, $x^4 + y + z^4 = u^2$, *does* have positive integer solutions, in fact whole "families" of solutions. Elkies' approach was to ask if any of these families included a solution in which u itself was a square. (If $x^4 + y^4 + z^4 = u^2$ and $u = t^2$, then $x^4 + y^4 + z^4 = t^4$.) This approach transfers the crux of the problem into a field that mathematicians know quite well: the theory of elliptic curves. (Curiously, Fermat's Last Theorem has also been related recently to the theory of elliptic curves but in a much deeper fashion than Euler's variant.) Using techniques that had already been developed for dealing with elliptic curves, Elkies reduced the number of possibilities down to a single curve. The rest was left up to a computer, which "solved" the elliptic curve and hence lead to the infinity of solutions to Euler's equation. (More recently, Elkies says, Roger Frye of Thinking Machines Corporation, Cambridge, Massachusetts, found the smallest counterexample by means of an exhaustive search, which used approximately 100 hours on a Connection Machine. Frye's numbers are $x = 95{,}800$, $y = 217{,}519$, $z = 414{,}560$, and $t = 422{,}481$.)

In one way Elkies was lucky. Don Zagier, who holds a chair in number theory at the University of Maryland and is an expert on elliptic curves, happened to be working on the same problem at roughly the same time. According to Larry Washington, also of the University of Maryland, Zagier was close to a solution, but "Elkies put it on the computer quicker." Their analyses that reduce the problem to a single elliptic curve, Washington says, are "word-for-word, lemma-for-lemma identical."

Zagier did the analysis in the fall of 1986, but left the computer check for later while he traveled for most of 1987. Elkies, who already had a reputation for his brilliance and quickness of mind—as a high school student he once received a perfect score in the International Olympiad mathematical competition—tackled the problem in the spring of 1987, unaware of Zagier's efforts. He also set the problem aside, while he worked on his Ph.D. dissertation, and completed the solution last August. Zagier was stunned when a colleague casually mentioned at a conference that Elkies had worked on and solved the problem. "Don now carries a lap-top computer when he travels," Washington says.

The importance of Elkies' result lies more in its historical interest—resolving a 200-year-old question is always nice—than in its mathematical significance. Because the connection with elliptic curves is tied to the exponent 4, it is unlikely that the technique used by Elkies and Zagier will generalize to larger exponents in Euler's variant on Fermat's Last Theorem. Mathematicians suspect that Euler's variant is false for *all* higher powers, but so far, except for $n = 4$ and 5, nothing is known. ■ BARRY A. CIPRA

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