Mathematics Education: A Predictor of Scientific Competitiveness

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As WALL STREET TRACKS THE HEALTH OF AMERICAN BUSIness by monitoring indicators of economic productivity, so must scientists, parents, and taxpayers heed leading indicators of scientific productivity. Since mathematics is the foundation discipline for science, the state of mathematics education is a crucial predictor of future national strength in science and technology. Evidence suggests that our mathematics classrooms, like our smokestack industries, no longer provide adequate support for modern society. They deliver neither the mathematical foundation required for scientific research nor the quantitative literacy necessary for a democratic society.

International Comparisons

Because of its widespread utility in industrial, military, and scientific applications, mathematics is a crucial indicator of future economic competitiveness. The evidence is overwhelming, however, that the mathematics yield of U.S. schools—the sum total of mathematics learned by all students—is substantially less than that of other industrialized nations. Current levels of achievement in the United States are unacceptably low. Our mathematics curriculum is not what it ought to be, nor is it even close to what it could be. By looking downward through the grades, we can foresee the poor quality of mathematical understanding of future generations of scientists:

- Non–U.S. citizens who take the Graduate Record Examination in mathematics average 100 points higher than U.S. students. The performance gap is twice as high in mathematics as in any other field—the next highest being in physics, the most mathematical of the sciences (*1*).
- The mathematics achievement of the top 5% of twelfth grade students is lower in the United States than in other industrialized nations. The average twelfth grade mathematics student in Japan outperforms 95% of comparable U.S. twelfth graders (*2*).
- U.S. eighth graders, who are about average in rote computation, are well below international norms in solving problems that require higher order thinking skills (*2*). Indeed, as the "back-to-basics" movement has flourished in the last 15 years, the ability of U.S. students to think (rather than just to memorize) has declined accordingly.
- For fifth graders, the highest average mathematics achievement in typical U.S. schools (in Chicago and Minneapolis) is below the lowest average scores from similar schools in China (Beijing) and Japan (Sendai). Only one of the top 100 students in the fifth grade in these recent international studies was an American (*3*).
- Even in kindergarten and first grade, differences emerge. Japanese children enter school ahead of U.S. children in mathematical skills. Only 15 of the top 100 first graders in a U.S.–China–Japan study were American.

The unanimity of these studies, from different countries and different investigators, underscores their significance.

Contrary to popular myth, the United States is not among the world leaders in the percentage of its youth who receive advanced education in mathematics. In the eighth grade, virtually all students take mathematics in all industrialized countries. At the twelfth grade level, most countries (including the United States) enroll about 12 to 15% of 18-year-olds in advanced mathematics courses, although in some countries (such as Hungary) the number is as high as 50%.

These studies also show that there is no consistent correlation internationally between student achievement and time spent in mathematics instruction. Except in elementary school, where U.S. emphasis on mathematics is unusually light, many countries devote less classroom time to mathematics than we do. Similarly, average class size from country to country seems to be quite unrelated to achievement.

Since the cultural diversity of American society is so much greater than that of most other countries, many believe that lower U.S. scores are due to the greater challenge of achieving excellence in a diverse society. Yet even in culturally homogeneous Minneapolis-area schools, average performance is well below comparable schools in China and Japan.

Analysis of the data involved in these studies dispels many simplistic explanations for poor U.S. performance. Lower U.S. scores are not due simply to averages taken over a higher percentage of our population, nor are they due to less contact time in schools, or to the broadening effects of a heterogeneous population. As there are no simple causes, there are no simple solutions to the problem of poor performance.

Demographic Realities

Declining performance in mathematics is matched by declining numbers of graduates. The number of 22-year-olds in the United States will decline by nearly 30% between now and the end of this century, just as retirements of post–World War II teachers peak and the second baby boom population wave that is working its way through our schools will produce a 30% increase in the school-age population. To find and sufficiently educate many teachers who are internationally competitive will require an extraordinary redirection of energy by the American political and educational systems.

As the student population rebounds from its current low levels, it will do so with a very different demographic profile than ever before in American history. By the year 2000, one in every three Americans will be nonwhite. Of those under 18, the proportion of nonwhites

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will be nearly 40%, almost three times what it was just after World War II. Moreover, because black and Hispanic birthrates are above the national average, the percentage of children who are nonwhite will continue to grow.

In addition, according to Census Bureau estimates, 60% of children born in the 1980s will, before reaching the age of 18, live in a home with only one parent. More than one child in four will come from a family that lives in poverty; nearly one in five will come from a home in which English is not spoken; and one in three will come home from school to an empty house, with no adult there to encourage attention to homework. Hodgkinson summarized these bleak statistics in one graphic image: "Every day in America, 40 teenage girls give birth to their third child" (*4*, p. 5).

Among the many subjects taught in school, mathematics is probably the most universal, depending least on a student's background and culture. School mathematics should, therefore, transcend the cultural diversity of our nation. In fact, it does just the opposite. In the United States, mathematics is primarily part of white upper-class male culture, readily available only to those who have the nourishment, solitude, and luxury to spend time in concentrated thought. Except for shopkeeper arithmetic taught in the elementary school curriculum, few parts of mathematics are embedded in the family or cultural traditions of people from the many large "developing countries" that make up the American mosaic.

Indicators of achievement and attitude support this general assessment. Students enrolled in advanced high school mathematics courses come disproportionately from white upper- and middle-class families. Differences in culture magnified by differential opportunities to learn imposed by years of multiple-tracked school produce widely different levels of performance.

By grade 8, U.S. students are typically grouped in four tracks: slow, average, accelerated, and early algebra. Assessment data show that each track completes the year with lower achievement than the next higher track had begun the year, thereby doubling the range of student performance while increasing its mean by less than one grade level. Moreover, those who benefit from placement in the top two tracks come disproportionately from upper middle-class families (*5*).

By grade 12, the proportion of Asian-American students who achieve high mathematics scores (above 650) on the Scholastic Aptitude Test (SAT) is twice the national average, while the proportion of Hispanic and black students is less than one-fourth the national average. Only about 200 black high school seniors receive mathematics scores above 650, about 2,000 above 550. Virtually none of the top 2,000 black freshmen (and fewer than 50 of the top 10,000) lists mathematics as an intended college major.

By grade 16, when prospective mathematics teachers graduate from college, fewer than 7% are non-Asian minorities, and most of these are black graduates of the traditionally black institutions in the South. Outside the South, fewer than 2% of bachelor's degrees in mathematics go to non-Asian minorities. Six or seven years later, the number of U.S. non-Asian minorities who receive a doctorate in the mathematical sciences can be counted on one's fingers (*6*).

The practice of tracking students in the United States introduces substantial variation in their opportunity to learn and magnifies the range of achievement from grade to grade. Moreover, socioeconomic factors relate closely with opportunity to learn, thereby introducing a multiplicative factor into the interaction of these other variables.

Since students who actually complete a mathematics major in college generally are in the top half of their class, with SAT mathematics scores above 500, there are virtually no prospects for developing an appropriate cadre of minority mathematics graduates from whom the next generation of teachers can be selected. Two demographic forces—increasing numbers of black and Hispanic youth in the classrooms and decreasing black and Hispanic graduates in mathematics—lead to a serious lack of classroom role models for those students who most desperately need not only quality but also motivation and incentive in the mathematics classroom.

Impact of Computers

The most visible force for change in mathematics education is the computer—a mathematics-speaking device that has totally transformed the way mathematics is used in science and daily lives. Computers (and calculators) change both what is feasible and what is important in the mathematics curriculum.

How many adults ever do long division, or even multidigit multiplication, by hand any more? Soon we might ask a similar question of scientists: how many engineers and scientists in 1990 will use paper and pencil to calculate derivatives, evaluate integrals, or compute Taylor series? Widely available computer packages, and even hand-held calculators, can carry out almost every mathematical technique taught through the sophomore year in college both in the purely symbolic form that mathematicians are fond of and in the graphical and numerical forms that scientists need.

Students will soon be sitting in school with small calculators that will do most of school mathematics faster and more accurately than their teachers. In addition, these new calculators have features that display advanced mathematics beyond what is normally taught in the school curriculum or even in the college preparation of schoolteachers. In the majority of classrooms with insecure teachers facing unmotivated students, these calculators will undermine whatever plausible arguments the teacher can muster for why students should learn dreary school calculations. Powerful mathematics-speaking calculators will change forever the rationale, the dynamics, and the incentive for much of what is emphasized in traditional high school and college mathematics (*7*).

As computers have changed the way we calculate, so they have changed the way we investigate. Simulation, computer graphics, and expert systems now constitute a third mode of scientific investigation, complementing empirical methods and theoretical models whose roots go back to Galileo and Newton. Computers make mathematics a partly empirical science, and science a partly mathematical investigation. To become well versed in the new symbiosis of mathematics and science, students will need to explore these new strategies of investigation throughout their school years.

Computers have mathematicized society. Major issues of public policy, such as the Strategic Defense Initiative, AIDS, and arms control, depend on mathematical issues such as algorithms, statistics, and game theory. Political perceptions depend on opinion polls; investment decisions rely on computer models; economic policy debates often amount to arguments about coefficients in econometric computer models. The "vulgar arithmetic" of the 19th-century public school is no longer adequate to ensure an enlightened citizenry in the 21st century.

New Directions

The mathematics curriculum in the United States forms the primary mathematical experiences of some 25 million school children, 10 million secondary school students, and 3 million college students. In higher education, mathematics courses account for 10% of all course credits and for nearly one-third of all science and engineering credits. *(Continued on page 302)*

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Poor performance, changing demography, and technological pressure require vigorous response if we are to provide all Americans with an education in mathematics appropriate to their needs. Americans more than any other people attribute success in mathematics to innate ability rather than to hard work (*8*). Students, parents, and teachers the world over, except in the United States, believe that everyone can learn mathematics if only they work hard at it. America must come to understand that achievement in mathematics is possible for all students, not only for those with a special talent.

Currently, in the United States, more than two-thirds of the bachelor's degrees and more than 80% of the doctorates in mathematics are held by one-third of the population—Asians and white males. To meet the nation's needs for scientifically trained personnel, we must make mathematics attractive to everyone. Equality of opportunity will not be possible unless we make a national commitment to dramatic improvement in the respect, expectations, and standards of school mathematics.

The need for such a commitment is beginning to register not only among beleaguered teachers, but also at the highest levels of national policy. "Many of our schoolteachers still treat mathematics as a minefield to tiptoe through," commented presidential science advisor William R. Graham in a recent address (*9*, p. 247). "Worse yet, we've inherited a woefully limited set of expectations of what schools and students can accomplish" (*9*, p. 247).

Graham urged research scientists to help create new mathematics curricula that are interesting, sophisticated, innovative, and well integrated with the applications of mathematics. He cited the newly created Mathematical Sciences Education Board at the National Research Council as "exactly the right kind of mechanism to work on this problem" (*9*, p. 248). He also stated that educational policy in the United States is made by more than 15,000 independent local school boards. Influencing these boards to change their priorities for mathematics (and science) education is one of the most important tasks facing the American scientific community. Here is an agenda worth starting on:

Only teachers who like mathematics should teach mathematics. Fear of new mathematics is common among teachers, especially in elementary school. Moving toward a system of elementary specialists in mathematics is an important long-term goal. In the short term, creative juggling of teaching duties could at least ensure that all children have the benefit of the best mathematics instruction available in their districts.

Only tests that measure higher order thinking skills should be used to assess mathematics. Teachers will teach whatever is required to enable their students to do well on assessment tests, and textbooks will be written to match the test objectives. Too often today's assessments dictate a curriculum filled with rote calculation and mimicry mathematics.

The chief objective of school mathematics should be to build student confidence. Retaining natural curiosity, promoting confidence in clear reasoning, and building favorable attitudes are far more important than specific techniques for solving school book problems. Confidence is a prerequisite to learning; once lost, it is nearly impossible to restore.

Good teaching must be rewarded both professionally and financially. Low salaries and nonprofessional working conditions for teachers discourage anyone with better options from entering the profession. In mathematics, there are always better options. To attract and retain able young teachers, the profession of teaching mathematics must be made as attractive as competing professions.

Mathematics teaching must be based on both contemporary mathematics and modern pedagogy. Both the nature of mathematics and our knowledge of how to teach it have changed significantly in recent years. Teachers, no less than doctors or airline pilots, need to be at the leading edge of their profession.

School mathematics should use computers and calculators. Computers now compute, so students should learn to think. More important, students need to learn at every grade level when to use their heads and when to use their machines.

Mathematics in the schools should be linked to science in the schools. To achieve this goal science teachers must actually use mathematics, mathematics teachers must use science, and mathematics and science teachers must discuss coordination of their teaching.

Changes such as these will require immense effort at local levels led by scientists and mathematicians who know the motivation and importance of the necessary changes. Teachers and educators alone cannot bring about the necessary changes; only with support from the leadership of the scientific community will it be possible to mobilize the national will to address this pressing problem. For this reason the National Science Foundation is working to guarantee the participation of the university research community in precollege education (*10*).

With the creation of the Mathematical Sciences Education Board the mathematics community has launched a unique national enterprise to harness the energies of mathematicians and mathematics educators in the common endeavor of improving mathematics education. This effort needs to be joined by scientists, educators, and policy-makers at all levels. A good place to begin is by reading *The Underachieving Curriculum* (*2*), the best contemporary analysis of the status of mathematics in U.S. schools. Armed with data from that report, and cautioned by its analysis of "deceptive explanations," scientists and mathematicians can begin the long process to improve mathematics education in the United States.

REFERENCES

1. K. M. Wilson, *The GRE Subject Test Performance of U.S. and Non–U.S. Examinees 1982–1984: A Comparative Analysis* (Educational Testing Service, Princeton, NJ, February 1987).
2. C. C. McKnight *et al.*, *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective* (Stipes, Champaign, IL, 1987).
3. H. W. Stevenson, S.-y. Lee, J. W. Stigler, *Science* **231**, 693 (1986).
4. H. L. Hodgkinson, *All One System: Demographics of Education* (Institute for Educational Leadership, Washington, DC, 1985).
5. F. J. Crosswhite *et al.*, *Second International Mathematics Study: Detailed Report for the United States* (Stipes, Champaign, IL, 1986).
6. S. L. Coyle, *Summary Report 1984: Doctorate Recipients from United States Universities* (National Academy Press, Washington, DC, 1986).
7. P. Zorn, in *Toward a Lean and Lively Calculus*, R. Douglas, Ed. (Mathematical Association of America, Washington, DC, 1986), pp. 237–246.
8. S.-y. Lee, V. Ichikawa, H. Stevenson, "Beliefs and Achievement in Mathematics and Reading" (University of Michigan, Ann Arbor, 1987).
9. W. R. Graham, *Not. Am. Math. Soc.* **34**, 245 (1987).
10. E. Bloch, testimony before the Senate Subcommittee on HUD—Independent Agencies (12 March 1987).