Remembering a "Magical Genius"

Ramanujan was born 100 years ago and grew up poor and uneducated, but his work continues to draw and inspire mathematicians

If ever there were an exemplar of inborn mathematical ability it would be Srinivasa Ramanujan, a poor, uneducated Indian, born 100 years ago, who was one of the greatest and most unusual mathematical geniuses who ever lived. Although he died young—at age 32—Ramanujan left behind a collection of results that are only now beginning to be appreciated.

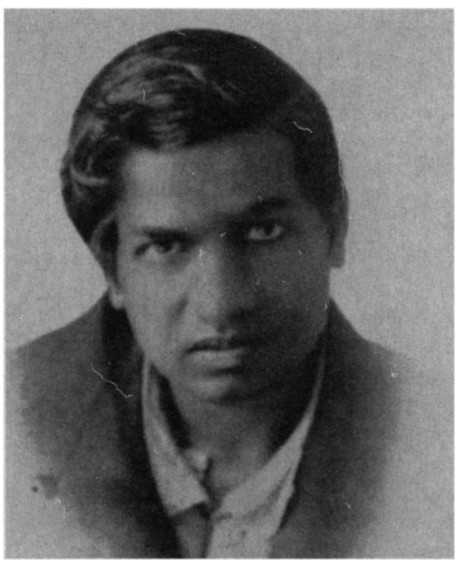
Ramanujan's story is one of the great romantic tales of mathematics, made all the more haunting because of the mystery surrounding the man. No one, no matter how much they try, has ever been able to understand the workings of Ramanujan's mind, how he came to think of his results, or the source of this incredible outpouring of mathematics.

On 1 to 5 June, mathematicians gathered at the University of Illinois at Urbana-Champaign to celebrate the centenary of Ramanujan's birth and to try to place his work in the context of modern mathematics. It was no surprise that Ramanujan's work is meaningful to mathematicians today—he touched on some very fundamental problems in number theory and analysis. But it was somewhat unexpected that his results are so relevant to problems he could have known nothing about—string theory in physics, for example, and fast algorithms in computer science. It also was surprising to see how many people were touched throughout their professional lives by Ramanujan—everyone from theoretical physicist Freeman Dyson of the Institute for Advanced Study in Princeton to number theorist Paul Erdös of the Hungarian Academy of Sciences.

Ramanujan, wrote mathematician G. H. Hardy of Cambridge University, was "a man whose career seems full of paradoxes and contradictions, who defies almost all the canons by which we are accustomed to judge one another, and about whom all of us will probably agree in one judgment only, that he was in some sense a very great mathematician."

He also was what the late Mark Kac called a "magical genius." Kac, says George Andrews of Pennsylvania State University, classified geniuses as "ordinary" or "magical." An ordinary genius is one of whom you might say, "Oh yes, I would have thought of that if I were 100 times smarter." A magical genius, on the other hand, is one who would lead you to say, "I have no idea where those results came from."

Ramanujan was born in 1887 in the town of Erode, in southern India, and grew up in the nearby town of Kumbakonam, where his father was an accountant for a cloth merchant. Although his family was of the middle class, he was actually very poor. Ramanujan, his brother, and his parents lived in a one-room adobe home. His entire mathematical education seems to have been gleaned from only two books, and these were books that mathematicians would not even give to students today because they are not particularly good. In fact, says Robert Rankin of Glasgow University, the books were not even good in Ramanujan's day.



Srinivasa Ramanujan. *A mathematical genius whose thought processes continue to baffle researchers.*

When he was 12, Ramanujan borrowed the first of these math books, S. L. Linney's *Plane Trigonometry*, from an older student and read it straight through. The book, according to Ramanujan scholar and mathematician Bruce Berndt of the University of Illinois at Champaign, contained more advanced math than its title would indicate. It included, for example, logarithms of complex numbers, infinite products, and infinite series.

At age 15, Ramanujan went to a government library and borrowed the second book—one by G. S. Carr called *Synopsis of Elementary Results in Pure Mathematics*. It was an unusual book and one that gave Ramanujan his unorthodox ideas of how to present mathematical results. Carr was a tutor at Cambridge University in England, and his book was essentially a list of results that he went through with his students. As many as 6000 mathematical theorems were stated, but almost never proved. Carr did give an extensive list of references, but these "would have been useless to Ramanujan because he had no access to a library that contained them," says Richard Askey of the University of Wisconsin in Madison. The lack of proof apparently did not bother Ramanujan. He simply worked through the book, presumably supplying the proofs on his own.

Ramanujan completed high school and tried twice to obtain a college education. But he failed both times because he was so obsessed by mathematics that he simply could not bring himself to spend any time on other subjects. In 1909, when he was 22, he married 9-year-old Srimathi Janki and took a clerical position in the Madras Port Trust Office to support her and his mother, who lived with the young couple. While he worked as a clerk, Ramanujan continued to pour out math results, using excess wrapping paper from the office to scribble down his formulas. He was so obsessed with his mathematics, in fact, that he did not want to stop even to eat. His wife, who is still alive, told Berndt that she and Ramanujan's mother used to feed Ramanujan at mealtimes so that he would be free to continue writing while he ate.

Fortunately for Ramanujan, both the chairman and the manager of the office where he worked were engineers who recognized that he seemed to have extraordinary mathematical talent. The chairman, Sir Francis Spring, was English and the manager, S. N. Aigar, was educated in England.

Both urged Ramanujan to send some of his results to English mathematicians, who might be able to evaluate them.

So Ramanujan wrote to mathematicians H. F. Baker and E. W. Hobson of Cambridge University. Both returned his letters without comment. Then, on 16 June 1913, he wrote to G. H. Hardy of Cambridge University—a letter that was to change his life and the life of Hardy.

Hardy opened Ramanujan's letter, read it, and put it aside, not certain what to make of it. It was crammed with as many as 60 mathematical theorems and formulas, stated without any proofs. That evening, Hardy went with his colleague John E. Littlewood to the chess room at Trinity College of Cambridge University. Hardy showed Littlewood the strange letter he had received from Ramanujan and said that he could not decide whether Ramanujan was a crank or a genius. Two-and-a-half hours later, Hardy and Littlewood emerged from the chess room. Ramanujan, they decided, was a genius. Hardy declared that Ramanujan's results must be true, because "if they were not true, no one would have had the imagination to invent them."

So Hardy wrote to Ramanujan and invited him to come to England to study with him. Ramanujan accepted and arrived at Trinity College in April of 1914. For the next 3 years he "pitted his brains against the accumulated wisdom of Europe," Hardy said. And Ramanujan was successful. At Cambridge, he derived outstanding results in number theory in particular. Littlewood, in describing Ramanujan's work, wrote, "There is hardly a field of formulae, except that of classical number theory, that he has not enriched, and in which he has not revealed unsuspected possibilities. The beauty and singularity of his results is entirely uncanny."

But life in England was not easy for Ramanujan. He left his wife behind in India and had no one with him to be sure he ate or slept regularly. Because he cared more for mathematics than for eating and sleeping, Ramanujan reportedly would work for long stretches—24 or 36 hours—and then would collapse and sleep for 12 or more hours at a time. He was a vegetarian, which presented additional difficulties. "This was during World War I and it was difficult to get vegetarian food in England," says Rankin. "Ramanujan would have boxes of rice sent from India, and he would fry rice powder in ghee."

In May of 1917, Ramanujan came down with a mysterious illness that may well have been a vitamin B-12 deficiency, caused by his poor diet while in England. Ramanujan was so weakened and incapacitated by his illness that he returned in 1919 to India, where he died a year later.

Ramanujan's childhood home

A one-room adobe dwelling, it now houses a family of 11 and still has no running water.

Bruce Berndt

When he died, Ramanujan left behind three notebooks, which he wrote before coming to England and which are filled with as many as 4000 results, stated without proofs. He also left behind the papers he published in England, many of which were written in collaboration with Hardy. And he left behind results he discovered during the last year of his life. He mailed many of these results to Hardy, but the papers were never published. George Andrews of Pennsylvania State University discovered them about a decade ago in the library at Trinity College, where they were uncataloged. Andrews calls these papers Ramanujan's "lost notebook"—a phrase that annoys the Trinity College librarian, who points out that they are neither a notebook nor lost.

Berndt was told by Ramanujan's widow that during the last year of his life, when he was dying, Ramanujan filled a trunk with papers on which he scribbled his results. Berndt notes that, if this is so, many Ramanujan papers are still unaccounted for. Ramanujan's widow told Berndt that while Ramanujan's funeral was going on, Ramanujan's college mathematics teacher from the University of Madras came to his house and took all of his papers. "Many more papers existed after Ramanujan's death than came down to us," Berndt says. He suspects that at least some of these papers are hidden in dusty, uncataloged piles at the University of Madras library, but the librarian states that, as far as he understands, there are no Ramanujan papers there.

But even if the Ramanujan collection is incomplete, it has given researchers more than enough to work on. Berndt, as a labor of love, is going through the first three Ramanujan notebooks and attempting to prove the theorems.

"For 10 years," Berndt says, "I have done only this. My research has been directed by Ramanujan." The notebooks are spellbinding, according to Berndt. He feels, he says, somewhat the way mathematician George Polya did in 1925 when he came to England to visit Hardy. Polya asked Hardy if he could see Ramanujan's first notebook, and Hardy loaned it to him. "A day or so later, Polya returned the notebook in a state of panic," Berndt says. "He said that as long as he held onto the notebook, he would continue to try to prove the formulas in it. The notebooks were so fascinating that Polya was afraid that if he kept them, he would never again prove any result of his own."

About a month ago, with the occasional help of other mathematicians, Berndt reached a milestone. "Everything I can make sense of in the 21 chapters of the second notebook is now proved," he says. The results, Berndt says, are important to mathematicians, but his proofs, unfortunately, do not reveal what led Ramanujan to these results in the first place. Too often, says Berndt, the proofs are verifications. Nonetheless, Berndt is continually amazed by what the notebooks contain. For example, in 1979, the French mathematician R. Apéry of the University of Caen solved a famous problem about a quantity known as $\zeta(3)$. The quantity is represented by an infinite series, and the question that nagged at mathematicians was whether the number the series converges to is rational or irrational. Apéry proved that $\zeta(3)$ is irrational, using "two beautiful new ideas," according to Berndt. But, Berndt continues, "one of the ideas had to do with a continued fraction representation of $\zeta(3)$ and it turns out that this continued fraction representation is a special case of a very general continued fraction in Ramanujan's notebook."

More and more often, said the conference participants, mathematicians are finding that their clever new ideas were discovered first by Ramanujan. In fact, William Gosper of Symbolics, Inc. in Palo Alto, California, called his talk "Ramanujan as Nemesis." "How can we all love this man if he is forever reaching out from the grave and snatching our neatest results?" Gosper asked. Gosper recently devised a new computer algorithm to calculate the number pi to 17.5 million digits. But over and over again, he found that his best ideas were already discovered by Ramanujan. "If Ramanujan were still alive, what I would do is show him my computer and hope to seduce and distract him," Gosper said.

This rediscovery of Ramanujan's work is particularly unexpected because, in his day, Ramanujan was thought to be a genius whose work had been misdirected because of his lack of a formal education in mathematics. "Hardy said that Ramanujan's work would have been greater if it weren't so strange," says Andrews. "We're coming to appreciate it more." For example, R. J. Baxter of the Australian National University found that some of Ramanujan's work was exactly what he needed to solve the hard hexagon model in statistical mechanics. Carlos Moreno of the City University of New York says that Ramanujan's work in the area of number theory known as modular forms is exactly what physicists need when they work on the 26-dimensional mathematical models of string theory.

Some researchers, however, say that they never had to rediscover Ramanujan because he has been a stimulus to them for years. For example, Freeman Dyson says, "many of the things I've been doing over the years, right up to the present, come out of Ramanujan's work." Dyson was introduced to Ramanujan early, when he was still in high school in England. He won a math prize and was awarded a copy of Ramanujan's collected works. "It was such beautiful stuff. I fell in love with it right away," Dyson says.

Since Ramanujan was so poorly educated, mathematicians throughout the years have wondered what he might have become if he had been brought up in different circumstances. There is no answer, of course, but it can at least be argued that, for Ramanujan, a formal education was almost beside the point. "It is nice to say that if Ramanujan had had more education, he would have done more," says Andrews. "But we can't run a controlled experiment on geniuses that occur uniquely in history. It is at least plausible to say that more education would have ruined Ramanujan."

Although Ramanujan was unique in mathematics, some researchers say they never give up hope that a new Ramanujan will appear. "Of course, we're always hoping," says Dyson. "That's one reason why I always read letters that come in from obscure places and are written in an illegible scrawl. I always hope that it might be from another Ramanujan." ■ GINA KOLATA

Mockingbird Song Aimed at Mates, Not Rivals

An opportunity to monitor closely the singing behavior of mockingbirds produced a result that was entirely unexpected

MOCKINGBIRDS can be pretty cheeky creatures. And noisy too. But to Randall Breitwisch and George Whitesides of the University of Miami, these two sometimes annoying habits of mockingbirds presented an opportunity. During the spring of 1985 the researchers were able to monitor closely the birds' singing behavior right on the university campus. And what they found was a big surprise. "The birds appeared to spend a great deal of their time directing their song into their territories rather than out of them," says Breitwisch. "And that is important, because it tells you something about the function of their song."

Bird song traditionally has been interpreted principally as a territorial warning from one male to another. "The idea of a 'keep-out signal' is very popular among biologists," says Breitwisch, "and very often it must operate that way. But not always." The discovery that male mockingbirds spend a lot of time singing into their territories rather than broadcasting out of them from the edges implies that in this case at least, it is probably not true. "This was an unexpected finding," acknowledges Gene Morton of the National Zoological Park in Washington. "The interesting question now is, how common is this?"

When Breitwisch, who is now at the University of Pennsylvania, first observed this pattern he looked into the literature and was surprised to find that no one else had even looked at the question of directionality of singing. Position of the singer within the territory, yes. But not direction of the singing. The problem most observers face in trying to get data on this is simply being able to be close enough to the birds to make reliable and repeated observations of the position of the beak during singing. The relatively habituated mockingbirds of the University of Miami campus offered that opportunity. "Finding out how general a phenomenon this is will therefore be difficult," says Breitwisch.

The northern mockingbird is, of course, famous for its extensive repertoire and its

Hal Harrison/Grant Heilman Photography

The northern mockingbird. *Its varied song is meant to impress its mate.*