suffice to construct the specialized representations required for different tasks. The performance of a weight-adjustment algorithm generally depends on the prespecified architecture of the network within which it operates (for example, the number of layers, the number of units within each layer, and the overall pattern of connectivity). In all the simulations reported in the book, the network is crafted by the researcher to perform some specific task. As yet there seem to be no principles that constrain network architecture and no proposed learning techniques that might allow induction of modular subnets or other specialized architectures.

Although major hurdles loom ahead, it is clear that the PDP approach so forcefully articulated in this book will have a major impact on cognitive science. Researchers in cognitive psychology, artificial intelligence, parallel computation, neuroscience, linguistics, and other fields as well will find the work immensely stimulating. The greatest value of the book is that it clearly lays out a paradigm and applies it to concrete and interesting examples. The book opens a door that cognitive scientists can enter: some will stay and join the movement, others will steal a few ideas and leave, and others yet will learn why they prefer to stay outside. All will want to take a peek.

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Shapes in the Plane

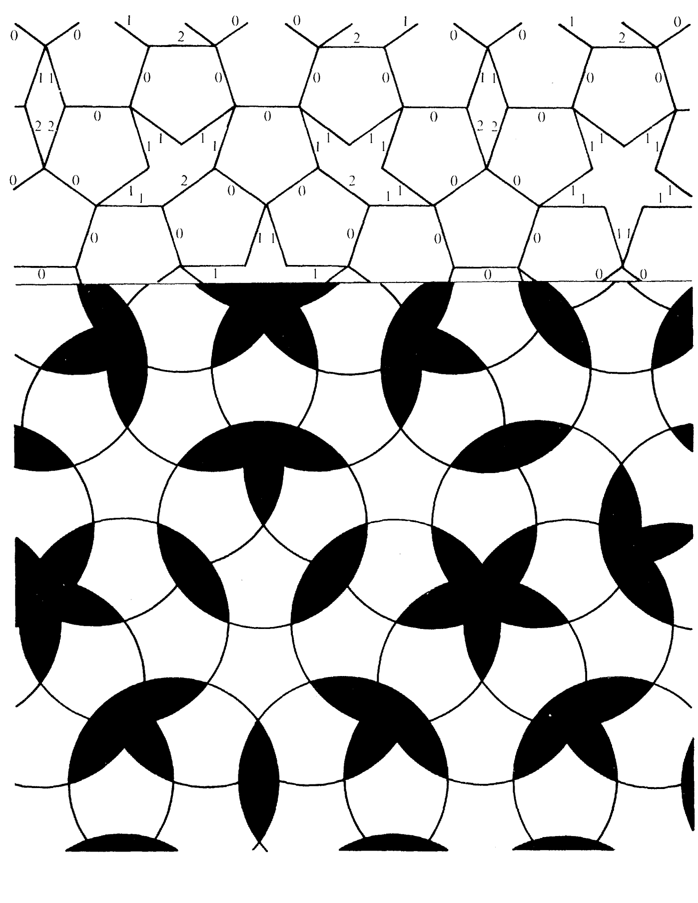
Tilings and Patterns. BRANKO GRÜNBAUM and G. C. SHEPHARD. Freeman, New York, 1986. xii, 700 pp., illus. $59.95.

Throughout history people have filled floors, stained-glass windows, and fabrics with shapes that occupied the plane without holes or overlaps. These shapes, or tiles, often endow plane surfaces with patterns of remarkable symmetry and beauty. Although tilings of the plane have been found in varied contexts and cultures, the systematic study of their types and properties is surprisingly new. Except for a modest beginning in the 17th century by Johannes Kepler, there were few studies of tilings until the 20th century, and not until the research of this book's authors did the study of tilings become a full-fledged subbranch of geometry.

To develop a theory of tilings, one must adhere to the rules concerning which shapes are allowed for tiles and the rules about how shapes can be placed next to one another. For example, if one starts with a supply of identical rectangular tiles, one may place the tiles so that the edges of the rectangles match, producing the familiar tiling of floorings, where four tiles meet at a point. But other tilings by rectangles can occur if we allow the tiles to adjoin in additional ways. Infinitely many types of tiling are possible when the tiles are not laid down edge to edge. Patterns, the other word sharing the book's title with tilings, involve symmetry considerations that arise when motifs of various kinds are systematically located in the plane. Although beautiful patterns have been created and examined by mathematicians in addition to craftsmen and artists for thousands of years, a coherent theory of patterns was not developed until that presented in this book.

What Grünbaum and Shephard have done, in a dazzling display of scholarship, erudition, and research, is collect in one volume a compendium of the accumulated knowledge about tilings and patterns developed by a wide range of individuals including artisans and craftsmen, mathematicians, crystallographers, and physicists. In doing so they were forced to take a fresh look at what was known, what was thought to be known, and what had yet to be investigated, and to provide a framework in which all of this information could be succinctly put down. The project, which was started about 10 years ago and has only now been brought to (partial) completion, is well worth the wait.

To present this material Grünbaum and Shephard needed to develop a standardized terminology. They have chosen terminology that is clear yet flexible and thus well suited to the rich range of phenomena encountered. Since this book begins with the simplest of concepts, it can be started and read with enjoyment by a high-school student. However, the reader needs staying power. As the book progresses, relatively elementary concepts and definitions are developed, but in the building-block style typical of a mathematical theory. Furthermore, the authors have felt free to use elementary ideas from topology, group theory, and number theory, defining the necessary terminology as they go along. Thus, although the treatment is elementary in the sense that all concepts and ideas that are not well known by persons with modest mathematics backgrounds are fully, clearly, and carefully explained, one cannot hope to understand the statements in the middle of the book with-



"A modification by Penrose of his set P1 of aperiodic prototiles. Each edge is replaced by a circular arc whose center is the 'point' of a pentacle or half-pentacle. A small portion of the original P1 tiling is reproduced at the top of the diagram to show the relationship between the tilings. Three of the prototiles have been colored black and three are white. It is conjectured that these 'curvilinear' tiles are also aperiodic." [From *Tilings and Patterns*] out retaining earlier concepts. Furthermore, although the ideas presented are related to a variety of applicable situations, the material is not developed with a view to demonstrating its "usefulness." The discussion, for example, of aperiodic tiles—sets of prototiles that admit tilings of the plane but for which there is no possible periodic tiling—is pursued for its own interest. Yet this beautiful and exciting new field of aperiodic tilings is setting crystallography and physics topsy turvy by providing a framework for the controversial and exciting developments in the area of what have come to be called quasicrystals. These are a form of matter that appears to violate the so-called crystallographic restriction, which states that the only types of rotational symmetry that can be present in physical lattice systems are two-fold, three-fold, four-fold, and six-fold. In particular, the high level of icosahedral symmetry, seemingly confirmable for these new materials from x-ray crystallographic techniques, cannot be explained by classical methods.

Laypeople often think mathematics is a subject with no problems for which answers are not known. It is exciting to see throughout this book a large number of unsolved, easy-to-understand problems. (Sample: Is there a collection of tiles of only one shape that is aperiodic? The so-called Penrose tiles are a set of two shapes that tile the plane aperiodically.) In addition, the format of individual chapters is felicitous. Each chapter begins with an introduction giving a general account of the material to be developed, the technical material follows, and the chapter closes with informative and entertaining remarks and notes. These notes document the historical development of the material, indicate areas where the ideas have hope of application, and suggest connections with other disciplines and other areas within mathematics. The book is stuffed with visually beautiful and mathematically fascinating examples of tilings. One wishes that the economics of publishing had allowed the examples of colored tilings (both of an historical nature from such sources as the Alhambra and in the theoretical discussion of color symmetry) to be shown in color. Persons with a wide range of interests, from chemists and physicists to artists, mathematicians, and crystallographers, will find many things to enjoy in this book. It will serve as a standard reference for this material for many years to come.

As a headline in *Science* indicated (30 May 1986, p. 1087), there is "a math image problem." Mathematics has a reputation for being highly symbolic, highly structured, and impenetrable. Grünbaum and Shephard's superb book shows that it is possible to write a self-contained book using elementary concepts that combine to create a rich fabric of exciting and useful ideas. But without the precision of a technical vocabulary and the compression and increase in clarity allowed by well-chosen notation, mathematics could not do the jobs it sets for itself. If nonmathematicians and laypersons are willing to meet mathematicians halfway by providing the work and staying power to understand and probe the beautiful and useful ideas present in so many portions of mathematics, then with the help of such books as *Tilings and Patterns*, this image problem can be made to disappear.

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Symmetries

Symmetry through the Eyes of a Chemist. ISTVÁN HARGITTAI and MAGDOLNA HARGITTAI. VCH, New York, 1986. xii, 458 pp., illus. $95.

Hermann Weyl's 1952 monograph, *Symmetry*, is by now a classic. Richly illustrated with examples from art and nature and using little mathematics, it introduced the reader to the concept of symmetry, its foundations in group theory, and its applications in science. *Symmetry through the Eyes of a Chemist* is written in the same spirit. The early chapters are reminiscent of Weyl, with some examples and illustrations in common. István and Magdolna Hargittai are Hungarian, and there is a nice mixture of illustrations of art and architecture from Eastern Europe with more familiar Western European and American themes. After establishing the basis of the study of symmetry in group theory, the authors describe how symmetry concepts are used in a number of areas in chemistry: molecular structure, spectroscopy, chemical reactions, and crystallography. The range is broad, and the applications of symmetry to chemistry are interwoven with an array of examples from art, architecture, music, literature, and nature.

There are today several excellent books that introduce the principles of symmetry and group theory and their applications to chemical problems (for example, Harris and Bertolucci's *Symmetry and Spectroscopy* and Orchin and Jaffe's *Symmetry, Orbitals, and Spectra*). While covering much of the same material, the Hargittais' book has a different flavor. The development is less formal: for example, in the chapter on group theory, results are given without proof. The intention is to convey an appreciation for the ways in which symmetry concepts are incorporated in chemistry rather than to present a comprehensive development. Some topics

Two- and three-fold rotational symmetries, displayed by sculptures of a pair of fish in Washington, D.C. [*left*], and a trio in Prague [*right*]. [Photographs by István and Magdolna Hargittai; from *Symmetry through the Eyes of a Chemist*]