

Solar Neutrinos: Questions and Hypotheses

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The “solar neutrino puzzle” has been a challenge for almost 20 years, posing broad and fundamental questions about astrophysics and neutrino properties. This article sketches some of the ideas that have been put forward to solve the problem. These ideas can be grouped into two main classes: those involving changes in the standard solar model or in the basic nuclear reaction data, and those that attribute the puzzle to as yet unobserved properties of the neutrinos.

THREE FIELDS—ASTROPHYSICS, PARTICLE PHYSICS, AND cosmology—have provided the motivation for detecting and measuring the fluxes of solar neutrinos. The astrophysical impetus came earliest: a desire to observe directly the nuclear reactions that had been postulated to explain the fundamental processes of stellar energy generation. The first neutrino detection experiment produced a large surprise—a very much smaller neutrino rate than expected (1). Critical reexaminations followed but have to date failed to yield an unobjectionable explanation. The theory of stellar evolution has developed into a detailed, quantitative formalism that is firmly based on physical laws and phenomenological data (2). The sun provides the only accessible testing ground and neutrinos are the only means of probing the dynamics of the solar interior. The gap between theory and experiment has implications for all stellar evolutionary theory and has become known grandly as “the solar neutrino puzzle.”

There is the possibility that the solar interior functions as pictured, but that something happens to the neutrinos in their transit from the sun to Earth; this, in turn, suggests that the means may be at hand to determine neutrino properties that would escape detection by any wholly terrestrial experiment. One of the mechanisms suggested (3), neutrino oscillations (oscillations among different neutrino species), depends on neutrinos having nonzero rest mass and is of particular interest in connection with “grand unified theories” (4) intended to unify the electroweak with the strong interaction. The concept of a neutrino mass fits naturally into these formalisms, but at this time there are many classes of predictions. Three families of leptons are now established, each with its own neutrino species. The oscillation hypothesis challenges the separateness of these neutrino species and, instead, proposes an exchange mechanism. The observation of the oscillation phenomenon would immediately establish the occurrence of mixing between the neutrinos and would constrain the important parameters: (i) the degree of the mixing and (ii) the mass square difference, Δm^2 , between neutrinos of different families. Such an observation would quantitatively establish a constraint for present speculations and promise a quantitative check on a unified formalism.

The determination of Δm^2 would not be the same as a mass determination, but it would provide bounds and remove any

question of a general principle that forbids neutrino masses. Neutrinos of finite mass have been proposed as one of the possible sources of the hypothetical “dark matter” (5). Whether there is enough nonvisible matter to close the universe has been a central question for many years. It has been conjectured that dark matter supplies the additional gravitational attraction required to explain motion at galactic perimeters and among galactic clusters.

However, before our understanding of solar neutrinos can be applied to these large conceptual questions, a first, critical step remains: actual measurements, as complete as possible, of the flux and kind of solar neutrinos arriving on Earth. In this article we review briefly the problems posed to solar models by the present experimental results, outline the ideas about solar mechanisms and neutrino properties put forward as solutions to the puzzle, and state the kinds of measurements that would permit a choice among them. In the accompanying article (6), we describe proposed detectors, consider how they will provide the desired data, and discuss whether those data will point toward a decision.

Energy Production in the Sun: Neutrino Sources

As early as 1903 Rutherford and Soddy (7) recognized that the sun’s energy production, until then quite inexplicable, could be accounted for by “subatomic processes.” The specific nature of the process responsible, namely, the transformation of four hydrogen atoms into helium, was first suggested by Eddington (8) and Perrin (9) soon after Aston’s mass spectrometric demonstration (10) in 1920 that the mass of four hydrogen atoms exceeds the mass of a helium atom. In 1939 Bethe (11) formulated in detail the nuclear reaction cycles involved in this transformation. Since then, these thermonuclear reaction cycles have been almost universally accepted as the source of solar energy and have formed the basis of the so-called standard model of the sun (2, 12).

The principal energy-producing nuclear reactions (Table 1) form a series in which protons are eventually combined into helium nuclei with the emission of neutrinos (ν) and γ rays. The initiating and rate-determining reaction is the proton-to-deuteron process that proceeds via the beta-decay interaction. The PPI set dominates; PPII competes with the closing reactions of PPI; the rare branch PPIII utilizes the ${}^7\text{Be}$ produced as an intermediate product in PPII. We will refer to the neutrinos from the various reactions (ν_1 to ν_4) as pp, pep, ${}^7\text{Be}$, and ${}^8\text{B}$ neutrinos, respectively. There are additional nuclear compounding processes that “burn” still heavier nuclei and that are thought to play only minor roles in the sun’s energy production. The CNO cycle, in which ${}^{12}\text{C}$ has successive nucleons added until excited ${}^{16}\text{O}$ is reached and a ${}^4\text{He}$ is split off to complete the catalytic cycle, gives rise to two neutrino branches, ν_5 and ν_6 , the

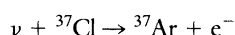
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^{13}N and ^{15}O neutrinos. We omit discussions of still rarer processes. As shown in Fig. 1, the basic energy-generating reactions take place deep in the sun; in the “standard model” nine-tenths of the pp reactions take place within a central sphere whose radius is one-fifth the solar radius, and the PPII and PPIII occur within still smaller spheres.

For a direct “view” of these basic stellar processes, use must be made of those emitted particles that can penetrate the solar mass with minimal interaction, and only neutrinos approach this property; as we shall discuss later, even they may not emerge untouched. It is this promise of a direct window onto the interior dynamics of the sun that has been a central motivation of solar neutrino experiments.

The ^{37}Cl Experiment: A Surprise

The first and, to date, only operating solar neutrino detector (1) is a radiochemical device based on the rate of production of radioactive ^{37}Ar by the neutrino-induced reaction



The basic apparatus consists of a tank containing 615 metric tons of perchloroethylene (C_2Cl_4). The production rate calculated on the basis of the standard solar model is about one atom of ^{37}Ar per day. It is a triumph for the techniques of nuclear chemistry that the few atoms so produced could be recovered out of the target material with almost 100% efficiency and reliably counted by means of the characteristic radioactive decay properties of ^{37}Ar . Shielding from cosmic rays was achieved by placing the experiment in a deep mine (Homestake Mine, South Dakota). This difficult but credible experiment, operational in 1967, accumulated data until 1985. As soon as preliminary results began to appear in 1968, attention was immediately focused on what was to become a formidable problem for the whole theory of stellar evolution. Far from agreeing with the calculations of the standard solar model, the neutrino flux as measured by the ^{37}Cl detector turned out to be much smaller; the present determination is about one-third of the calculated rate and might possibly be compatible with a zero solar neutrino flux.

The “Standard Model” of the Sun

We can give only the briefest sketch of this model and of the ideas put forward to alter it, none of which are free of strong objections. The “standard solar model” (2, 12) pictures a slowly evolving, quasi-static sun. Hydrostatic equilibrium is maintained between outward pressure and gravitational attraction; the energy produced by nuclear processes is conducted from the central to the outer region by radiative diffusion; with the exception of neutrinos, the nuclear reaction products remain very nearly in place. The evolution, during 4.5 billion years, from a primordial gas of hydrogen, helium, and a sprinkling of heavier elements to the present size and luminosity is computed from empirically determined reaction rates. By and large, it is a successful program; if we adjust the primordial composition, a star of the observed external properties is achieved, with values of the initial composition quite in line with cosmological compositions. Regrettably, it fails the neutrino test—the only direct test of the nuclear dynamics.

But just which aspects of the solar model are being tested? Because of the threshold energy, 0.81 MeV, and the form of the excitation spectrum of ^{37}Ar , the main part of the main neutrino branch, ν_1 , cannot contribute at all; the chlorine detector is sensitive principally (74%) (13) to ν_4 , the ^8B neutrinos; the ^7Be neutrinos (ν_3) contribute about 16%; the remaining 10% comes from other

Table 1. Energy-producing nuclear reactions in the sun.

Cycle and abundance	Reaction	$E(\nu)$ (MeV)
PPI 88%	$p + p \rightarrow d + e^+ + \nu_1$ (99.75%)	0 to 0.42
	$p + e^- + p \rightarrow d + \nu_2$ (0.25%)	
	$d + p \rightarrow ^3\text{He} + \gamma$	1.44
	$^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p$	
PPII 12%	$^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma$	0.86 (90%), 0.38 (10%)
	$e^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu_3$	
	$^7\text{Li} + p \rightarrow 2\ ^4\text{He}$	
PPIII 0.01%	$^7\text{Be} + p \rightarrow ^8\text{B} + \gamma$	0 to 14.1
	$^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu_4$	
	$^8\text{Be} \rightarrow 2\ ^4\text{He}$	
CNO	$^{12}\text{C} + p \rightarrow ^{13}\text{N} + \gamma$	0 to 1.20
	$^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_5$	
	$^{13}\text{C} + p \rightarrow ^{14}\text{N} + \gamma$	
	$^{14}\text{N} + p \rightarrow ^{15}\text{O} + \gamma$	0 to 1.73
	$^{15}\text{O} \rightarrow ^{15}\text{N} + e^+ + \nu_6$	
	$^{15}\text{N} + p \rightarrow ^{12}\text{C} + ^4\text{He}$	

nuclear reactions (those based on the CNO cycle and the pep reaction or ν_2). Because the solar fusion reactions require that the fusing nuclei tunnel through Coulomb barriers with energies orders of magnitude higher than the average thermal kinetic energy, the neutrino flux is enormously sensitive to temperature. However, since the pp reaction contributes most of the sun’s energy, the ν_1 flux, $\phi(\nu_1)$, is essentially determined by the observed luminosity and therefore model parameters are constrained such that $\phi(\nu_1)$ is nearly model-independent. On the other hand, $\phi(\nu_3)$ and $\phi(\nu_4)$ are not so constrained and both vary with high powers of the central temperature. It appears (14) that, for a wide variety of models, $\delta\phi(\nu_4)/\phi(\nu_4) \approx 2\delta\phi(\nu_3)/\phi(\nu_3)$. Model calculations by Iben (15) implicitly demonstrate these points.

Suggested Changes in the Solar Physics

Many of the proposed changes in the solar model have been aimed at achieving modest decreases in the central temperature, so as to lower the rates of the nuclear reactions giving rise to the neutrino flux. The effect on the ^8B flux would be greatest; on the pp flux, very little; and on the ^7Be neutrino flux, intermediate.

A number of interesting ideas have been proposed and examined to account for the ^{37}Cl result. A discussion of these ingenious and intricate suggestions and of the arguments against them may be found in a review by Rood (16). Here we limit ourselves to a fragmentary listing of some of the principal classes that typify this effort:

1) Lower primordial abundances of elements of atomic number $Z > 2$ would decrease the opacity to radiation, thus reducing the temperature gradient and hence the central temperature. Observational evidence on element abundances in the universe speaks against such a model, however, unless the sun is a very special star.

2) Large-scale mixing of solar strata, either convective or turbulent, would feed more hydrogen into the central region depleted by the fusion reactions. This would increase the importance of the PPI chain relative to the PPII and PPIII chains. It appears, however, to be difficult to construct a realistic model in which the large-scale mixing required to decrease the neutrino flux follows naturally from the physical dynamics. Furthermore, such models applied to low-mass red giants apparently lead to calculated element abundance anomalies.

3) Attempts have been made to account for a lower central temperature by postulating sources of outward pressure in addition

to that produced by the thermonuclear reactions, such as the centrifugal force due to rapid rotation of the interior, the “field pressure” of a large magnetic field, or a combination of the two. Failure to observe the solar oblateness that would result from rapid rotation and the inability to construct models with the requisite long-term stability of strong magnetic fields contradict these ideas.

4) The idea of a nonsteady sun in which the present epoch differs from the long-term average has been explored; possible effects, both of periodic variations and of a one-time change, have been considered. At least some of the models incorporating these ideas are contradicted by the earth’s climatic history.

5) The nuclear data underlying the calculation of the rates of the relevant nuclear reactions have been carefully scrutinized and remeasured (17); although there have been a few ups and downs, and the calculated neutrino fluxes have undergone significant changes, this avenue has not provided a solution to the puzzle.

Of course, the mechanics, nuclear physics, and chemistry of the chlorine experiment itself have been scrutinized and tested over many years; it is now generally believed to be completely credible.

Neutrino Properties—Possible Alterations in Transit

It has been suggested that all could be well with the astrophysics, the nuclear data, and the solar mechanics, but that some hitherto unobserved neutrino properties might be responsible for the discrepancy. One suggestion is that neutrinos might decay over the long (by terrestrial standards) sun-Earth flight paths (18).

If the neutrino had a very small (not visible in laboratory studies) magnetic or electric moment, solar magnetic fields could flip the neutrino spin, and thus its helicity, into a state incapable of interacting to produce an electron in the $^{37}\text{Cl} \rightarrow ^{37}\text{Ar}$ reaction (19). Such a reversal of helicity would not only eliminate the charged-current interaction in which the neutrino is transformed into an electron, but also the neutral-current or weak scattering interaction, since both are coupled to the original helicity. Interaction with charged particles can occur via the magnetic interaction, but this is appreciably weaker than the neutral-current interaction (20). An additional possibility is that the neutrino could be flipped by the magnetic field into the antineutrino of another family (19); in this antineutrino state it would interact via the neutral current.

Another suggestion, neutrino oscillations (3), has been explored in some terrestrial experiments. It is now well established that there are three families of leptons (electron, e ; muon, μ ; tau, τ), each with its own neutrino; the weak interaction permits the emission or absorption of a lepton together with its own neutrino only: (e , ν_e), (μ , ν_μ), (τ , ν_τ). The oscillation hypothesis supposes that a neutrino emitted in these interactions is a particular mixture of two or more mass eigenstates; for a given energy the momenta of these mass components differ and so phase oscillation occurs over the travel path. The oscillation can then occur between the emitted mix and the other (orthogonal) combinations of the mass components; thus a ν_e will oscillate among ν_e , ν_μ , ν_τ states. Since only the ν_e can cause the electron (β) interaction required for the transformation of ^{37}Cl to ^{37}Ar , oscillation would change part of the neutrino flux into sterile forms. If the degree of mixing is maximal, on average a ν_e would spread into all three components equally and would thus effectively cut the active flux to one-third.

Terrestrial experiments, which have so far not observed the oscillation phenomenon, have provided some limits on the degree of mixing and on Δm^2 . Analysis of such experiments has usually been made on the simplifying assumption that just two of the neutrino species mix—which we take as ν_e and ν_X (ν_X for ν_μ , ν_τ , or perhaps

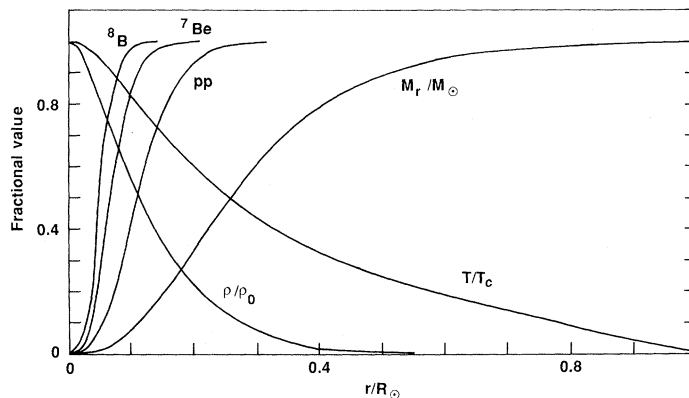


Fig. 1. Some of the important solar variables and neutrino fluxes plotted as a function of the fractional distance r/R_\odot from the center, where R_\odot is the radius of the sun. The temperature T and density ρ are given relative to their central values, $T_c = 15.5 \times 10^6$ K and $\rho_0 = 156$ g/cm³, respectively. The fraction of the pp, ^7Be , and ^8B neutrino fluxes produced and of the mass residing inside the indicated radial dimension are shown on the same plot. Here M_r is the mass within radius r and M_\odot is the solar mass. All the values have been calculated on the standard model and are taken from table 7 of (12).

another, as yet unknown); for two neutrinos the degree of mixing can be characterized by a parameter that varies between 0 and 1, generally written $\sin \theta$; θ is called the mixing angle. The fraction, P_e , of ν_e that would survive transformation in transit through a length L in the vacuum of space is that which is appropriate to an interference phenomenon:

$$P_e = 1 - \frac{1}{2} \sin^2 2\theta \left[1 - \cos \left(2\pi \frac{\Delta m^2 c^4 L}{4\pi E_\nu \hbar c} \right) \right]$$

where E_ν is the neutrino energy, c is the speed of light, and \hbar is Planck’s constant divided by 2π . The cosine with its L dependence can be traced back to the difference between the two phases

$$\frac{2\pi}{\lambda_1} - \frac{2\pi}{\lambda_2} = \frac{p_1 - p_2}{\hbar} = \frac{(m_2^2 - m_1^2)c^4}{E_\nu \hbar c} = \frac{\Delta m^2 c^4}{E_\nu \hbar c}$$

and the degree of mixing is reflected in the appearance of the $\sin^2 2\theta$ factor. In experiments with terrestrial neutrino sources one tries to observe the variability of P_e with L ; a significant change in P_e implies a corresponding change in

$$\frac{\Delta m^2 c^4}{4\pi E_\nu \hbar c} \frac{L}{\hbar c} \approx \frac{1}{2.5 \times 10^2} \frac{\Delta m^2 c^4}{E_\nu} L$$

where $\Delta m^2 c^4$ is in electron volts squared, E_ν is in megaelectron volts, and L is in centimeters. As a result of painstaking work Δm^2 is now known not to lie in the region above a few times 10^{-2} eV² unless the mixing is very small. At present the best limits come from reactor experiments (with $L \approx 5 \times 10^3$ cm, $E \approx 1$ MeV) (21). Future accelerator and cosmic-ray experiments may be sensitive to Δm^2 as low as 10^{-4} eV² (22). The solar neutrino experiments have the twofold advantage of a very long beam line, $L \approx 1.5 \times 10^{13}$ cm, and low energies, from a fraction of a megaelectron volt to ~ 10 MeV; these imply a sensitivity down to $\Delta m^2 \approx 10^{-12}$ eV². The simple estimates above are based on large values for the mixing parameter; the limits that can be derived are actually correlated pairs of the parameters Δm^2 and θ . For parameter values away from the extremes, averaging over E (because of the continuum spectra) and L (because of the eccentricity of Earth’s orbit and the finite solar source size) effectively reduces the cosine term to zero and yields $P_e \approx 1 - 1/2 \sin^2 2\theta$; the minimal value is one-half, as is appropriate for two-neutrino mixing; with three-neutrino mixing, a similar treatment would result in a minimal value of one-third.

For many years it has been thought that an experiment based on ^{71}Ga as a detector primarily ($\sim 60\%$) sensitive to pp neutrinos would give decisive information (23): Since the predicted flux of pp neutrinos is nearly independent of astrophysical model variations, a low result from a measurement of this flux, of order one-third of the predicted value, would point to an explanation based on neutrino transformation or disappearance properties. On the other hand, a result corresponding to the expected pp neutrino flux plus some fraction of the other components would be clear evidence for the need to revise the standard solar model.

Very recent work has shown that the oscillation phenomena are more complicated. Mikheev and Smirnov (24), working with a formalism put forward earlier by Wolfenstein (25) for the oscillation phenomenon in matter, have pointed out that the solar medium can have profound effects on neutrinos. Because the weak force produces an additional interaction between ν_e 's and electrons that is not shared by ν_μ 's or ν_τ 's, the ν_e 's are effectively placed in a shallow potential whose depth is proportional to the electron density, ρ_e . This potential is sufficient so that, in some region within the sun where ρ_e is in a particular range, it just compensates for Δm^2 ; in this region there is an effective degeneracy and effective maximal mixing occurs even when the original mixing is small. As a result, even quite small intrinsic mixing can be sufficient to explain the large reduction in ν_e flux consistent with the chlorine result. The properties of the solar medium make this compensation possible for a wide range of values for the mixing parameter and Δm^2 . Because the ν_e interaction with the solar medium is attractive, the compensation occurs only if the ν_e mass is lower than the ν_χ mass; although this ordering seems credible, there is not now an overwhelming theoretical basis that compels such a choice. Figure 2 illustrates the fraction, P_e , of ν_e that would survive transformation in the long passage through the dense solar medium. These calculations are performed with the simplifying assumption that only two neutrinos mix. It is striking that P_e can take on quite small values for some range of the parameters. P_e is not a monotonic function of the energy and mass parameters. Figure 3 shows the parameter range that could explain the result of the chlorine experiment; also shown are the effects on a low-energy solar neutrino experiment based on a ^{71}Ga detector. There is no longer any simple one-to-one relation; this can be traced back to the

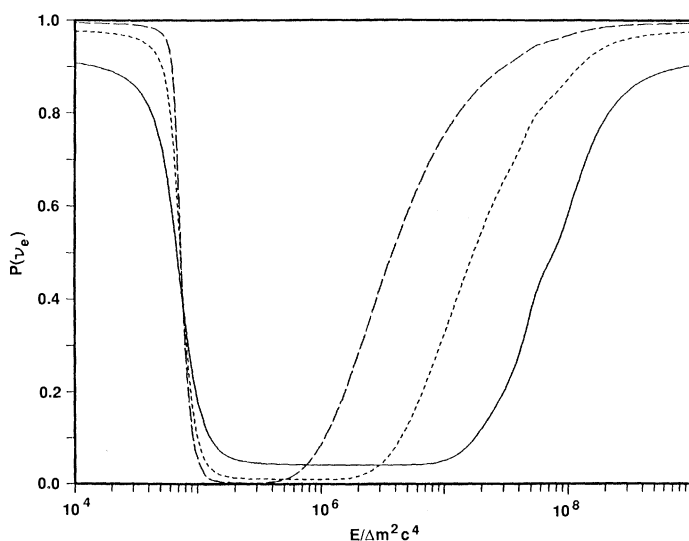
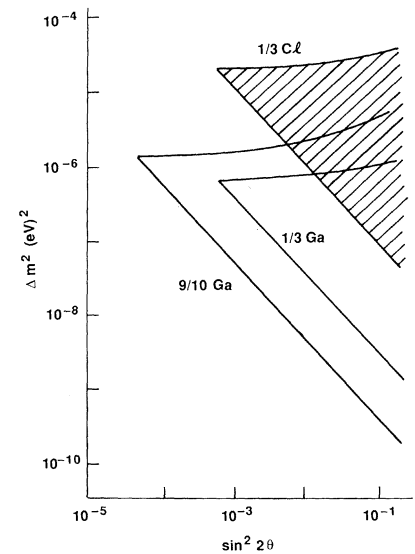


Fig. 2. The fraction, P_e , of electron neutrinos, ν_e 's, that start from the solar center and remain as ν_e 's, plotted as a function of the parameter combination $E/\Delta m^2 c^4$ (with E in megaelectron volts, $\Delta m^2 c^4$ in electron volts squared) for three values of the mixing parameter θ (27). The curves are for $\sin^2 2\theta = 0.4$ (—), 0.2 (---), and 0.1 (- - -).

Fig. 3. The Mikheev and Smirnov locus of solutions for the parameter sets (θ , Δm^2) that result in a reduction to 1/3 of the standard model neutrino flux for a ^{37}Cl experiment. The cross-hatched area corresponds to more severe reductions. Also shown are the loci that correspond to ^{71}Ga responses of 1/3 and 9/10 of the standard model expectation. The values plotted are taken from figure 2 of (24).



nonmonotonic nature of P_e . In the region of very small θ , for example, the ^{71}Ga response would be almost unaffected while the chlorine response is cut to one-third; for larger values of θ there is a finite range of Δm^2 where both the ^{37}Cl and ^{71}Ga are sizably diminished. A low result in a ^{71}Ga experiment will thus still be clear evidence for some neutrino transformation mechanism. However, a result near the standard model expectation would not exclude matter-induced oscillations as an explanation of the ^{37}Cl result. In a recent publication Bethe (26) expressed sentiments on behalf of the small- θ region and, therefore, made a definite choice of Δm^2 and a definite prediction that a ^{71}Ga experiment will show only little reduction (only the ^8B neutrino contribution being affected). We see no a priori reason for making this choice on the basis of present evidence.

Bases for Future Experiments

What key experiments would allow clear decisions among these many hypotheses? The important elements that provide the critical basis can be grouped into five categories:

- 1) The mechanisms that alter the solar physics have the greatest effect on the flux of ^8B neutrinos, have an appreciably lesser effect on the ^7Be neutrino flux, and leave the primary pp rate almost unchanged. Stated in terms of the solar neutrino spectrum, if solar physics and solar models are changed to decrease the predicted intensity of the highest energy neutrinos by a factor of 3 or more compared to the standard model, then the flux of the lowest-energy neutrinos will necessarily be almost unchanged independent of astrophysical details, whereas the intermediate energy (0.81 MeV) ^7Be neutrino flux will be changed from the standard model, but by much less than the high energy ^8B flux.

- 2) The hypotheses that focus on the properties of neutrino propagation have different critical characteristics. Thus, neutrino decay would lower the flux of low-energy neutrinos to an even greater extent than the factor indicated for the ^8B flux by the chlorine-detector response since the effective lifetime is, via the Lorentz factor, proportional to the energy.

- 3) The spin-flip effect of large solar magnetic fields on neutrinos that have electric or magnetic moments would be energy-independent.

- 4) Neutrino oscillation theories are somewhat more complex since they enjoy a richer choice of parameters. For those values of the oscillation parameters, Δm^2 and θ , that explain the chlorine

result and for which there is no effective influence of the solar medium, all solar neutrino fluxes would have much the same fate; a reduction of the ^8B flux would imply a reduction of the ^7Be and pp fluxes by much the same factor.

5) For Δm^2 and θ values that imply solar-medium effects, the parameter choices permit (i) a ^8B reduction, with ^7Be and pp moderately or slightly affected or (ii) a ^8B reduction with even more severe reduction of ^7Be and pp. These reductions, of course, imply the transformation into another neutrino species; demonstration of the presence of this transformed neutrino, ν_χ , would serve as unmistakable proof of neutrino oscillations. An energy spectrum of the surviving ν_e 's that is characteristically different from that produced by the solar reactions would be implied; again, if shown experimentally, it would serve as an unmistakable proof.

More generally, the demonstration that there is a sizeable diminution of the ν_e flux originating from the pp reaction would unmistakably point to some neutrino propagation property, whether decay, spin-flip, or oscillation.

However, one should not base one's expectations on just one experiment. In a situation where at present there is only a single experimental datum resulting from an extraordinarily complex set of phenomena, there is a need for more experiments that can shed light on the energies, directions, and kinds of neutrinos arriving on Earth, even if no single result can give all the answers. Proposed experiments of this kind are discussed in the following article (6).

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$$\phi(\nu_3)/\phi(\nu_1) \propto \frac{X(^4\text{He})}{X(\text{p})} T_c^{6.7}$$

$$\phi(\nu_4)/\phi(\nu_1) \propto \frac{2X(^4\text{He})}{1+X(\text{p})} T_c^{20.3}$$
 where $X(^4\text{He})$ and $X(\text{p})$ are the present-day concentrations by weight of ^4He and ^1H and T_c is the central temperature. The variation in $X(^4\text{He})$ and $X(\text{p})$ as a function of input parameters adds to the central temperature dependence so that

$$\phi(\nu_3)/\phi(\nu_1) \propto T_c^{10.9}$$

$$\phi(\nu_4)/\phi(\nu_1) \propto T_c^{22.9}$$

$$\phi(\nu_4) \propto [\phi(\nu_3)]^{2.1}$$
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