length. The book deals with each of these subjects in a complete and thoughtful manner.

A similar advance has taken place with respect to electrical units in the last 20 years. The basic SI electrical unit is the ampere, and it is defined in terms of the force between two parallel current-carrying wires. It is technically difficult to realize this definition of the ampere, and for a long time it was the practice to use a standard cell for realization of the volt and a standard ohm for realization of the unit for electrical resistance. There was always a concern that these standards were not constant in time and that the standards used in geographically separated laboratories were not the same. The discovery of the Josephson effect, which describes the tunneling current between two closely proximate superconductors, has provided a unit for voltage that depends on a measurement of frequency and is much more reproducible than the standard cells used earlier. The recently discovered quantized Hall effect may provide an equally reliable standard for resistance. The author describes the recent advances pertaining to each of these units.

A subject closely related to the determination of fundamental constants is the use of precision measurements to test basic theories. The greatest effort has been devoted to measurements of the properties of elementary particles and atomic systems as tests of quantum electrodynamics. The improvements in standards and the more precise tests of theory drive one another; the tests of theories provide motivation for improvements in techniques for measurements. The book provides an excellent summary of this aspect of measurement science.

An equally important aspect concerns null experiments such as the Michelson-Morley experiment that look for deviations from the accepted theoretical framework. Examples are the validity of the inverse square force law for the Coulomb and gravitational forces, the isotropy of inertial mass, the equality of inertial and gravitational mass, the mass of the photon, the equality of the charge of the electron and the proton, the continuous creation of matter, and the constancy of the velocity of light as a function of frequency. The book provides a critical summary of the work on each of these subjects.

It is not straightforward to determine the best values for the constants from measurements since the measurements are not independent and the measured quantities depend on the several constants in a different manner. The fact that the available measurements of a given quantity may differ more than one would expect statistically from the stated uncertainties makes it advantageous to reject some measurements in a group of measurements. The book provides a summary of the methods used to obtain the best set of constants.

An essential component of a measurement is the determination of the uncertainty of the result. The last chapter provides a thoughtful and instructive discussion of this subject. Simple use of statistics is not sufficient. A good determination of the uncertainty requires insight and imagination. The history of the determination of the constants is filled with cases in which workers found that earlier work had systematic uncertainties such that new results differed by much more than would be expected if the stated uncertainty was the statistical uncertainty.

Petley gives a clear description of the manner in which most of the constants have been determined, with particular emphasis on recent developments and prospects for the future. The reviewer recommends this book to anyone interested in the subject.

FRANCIS M. PIPKIN
Department of Physics,
Harvard University,
Cambridge, MA 02138

How to Solve It

Mathematical Problem Solving. ALAN H. SCHOENFELD. Academic Press, Orlando, FL, 1985. xvi, 409 pp., illus. $58; paper, $29.95.

A mathematics professor is asked to construct a line parallel to the base of a triangle of altitude A so as to divide the area in half. He has not done any geometry for years, and he makes three false starts before successfully constructing a line of length $A/\sqrt{2}$. The false starts waste little time, however. Two undergraduates who have received high grades in math courses are given the same problem (to solve together). They use up their allotted time on a "wild-goose chase" and fail to solve the problem even though they are at least as familiar with geometry as the professor is.

What has the professor learned that accounts for this sort of difference? Can students be taught something that will improve their problem solving in unfamiliar domains? Recent literature explains such "expert-novice" differences in terms of what Schoenfeld calls resources, such as specific knowledge of geometry and speed of accessing such knowledge. Resources, however, cannot be taught so that they transfer across domains (J. Baron, in *Thinking and Learning Skills*, vol. 2, S. Chipman, J. Segal, and R. Glaser, Eds., Erlbaum, 1985). Schoenfeld argues convincingly that resources are not the whole story. He proposes three other types of difference between those who are "mathematically sophisticated" and those who are not: heuristics, control, and belief systems.

How to Solve It, G. Polya's 1945 book on heuristics, was Schoenfeld's original inspiration. Heuristics are weak methods whose usefulness, although limited, cuts across domains: do you know a related problem you have solved before? could you solve a simpler problem? would it help to consider a special case? Schoenfeld argues that such heuristics are actually families of heuristics. If students are to be taught useful heuristics, they must be taught a hundred rather than ten. For example, there are different ways to construct "special cases," such as by substituting numbers or by assuming an additional constraint. Students must be taught when to apply each specific heuristic. (It remains to be shown, however, that sudents can in fact make better use of a specific heuristic than a general one, when both apply.) Schoenfeld also argues that heuristics will not transfer if only practice is provided. Students must also be specifically instructed in what they are doing and why it is useful. (This conclusion is consistent with other research.)

"Control" refers to the allocation of time and effort to approaches. Good control involves asking oneself (or otherwise knowing) what goal one is trying to achieve with a certain approach, how likely one is to succeed, and whether other approaches might also be appropriate. Students often undertake a difficult calculation or construction without knowing how they will use the result or whether some alternative approach might be easier.

"Beliefs" concern a student's working assumptions. For example, many students attempt to solve geometry constructions empirically, by asking whether their drawing meets the required conditions. They fail to ask whether the construction would work for other figures of the same type, and they fail to use deductive mathematical argument (of the sort used in proofs) for the purpose of discovery. They tend to think of deduction as something done only after one knows the truth, to satisfy the teacher or to play the game, rather than to discover the truth or to check a conjecture. Schoenfeld admits that there are other relevant beliefs. I would suggest that beliefs about the efficacy of thinking in general, and one's own thinking in particular, are important in mathematics and elsewhere.

The first half of the book presents this theory and illustrates it with problem-solving protocols. The rest of the book presents extensive data (and further discussion), mostly from two successful attempts to improve the problem-solving performance of college students by teaching them heuristics and the like.

In one study, both experimental and control groups were shown how to solve the same problems. The experimental group was given explicit instruction in the heuristics used in the solutions. Subjects in this group transferred at least some of these heuristics to new problems. (To prevent wild-goose chases in the transfer test, subjects in both groups were reminded to reassess their progress.)

In the second study, the experimental group was given an intensive course in problem solving while the control group had a course in structured programming. Again, the experimental group improved in the use of the heuristics taught and in subjective measures of control such as whether they planned their solution or just plunged into it. Analysis of protocols showed objective evidence of improved control: experimental subjects were indeed less likely to "plunge in" without evaluation. Subjects in this group also came to classify problems more in the way experts did, by methods of solution rather than superficial features. Although this result will appeal to cognitive psychologists because it employs one of their official paradigms, it adds less to our understanding than other findings.

Also presented is a framework for the analysis of problem-solving protocols that explicitly incorporates judgments of the appropriateness of control. Such a prescriptive approach to analysis ensures the relevance of the research to instructional questions. The analytic framework could, for example, serve as the basis for a tutorial approach to the teaching of problem-solving.

Schoenfeld is rightly critical of classroom practices (which he documents) that encourage rote memorization of geometric constructions, an empirical attitude toward discovery, two-minute exercises to the exclusion of real problems, "step by step" procedures, and preparation for standardized examinations. His analysis should enrich discussions of curriculum reform.

This book is worthwhile and engaging reading for anyone who teaches mathematics or related subjects at the high school or college level. It should also be read by those concerned with curriculum and policy and by scholars. It is a fine demonstration that worthwhile scholarship is possible in a prescriptive domain. It should inspire similar work in other disciplines and on other aspects of mathematics learning, for example, on understanding as distinct from problem solving, as well as efforts to discover the common features of good thinking across domains.

JONATHAN BARON
*Department of Psychology,
University of Pennsylvania,
Philadelphia, PA 19104*

New Looks at Faraday

Faraday Rediscovered. Essays on the Life and Work of Michael Faraday, 1791–1867. DAVID GOODING and FRANK A. J. L. JAMES, Eds. Stockton, New York, 1985. xiv, 258 pp., illus. $70.

Michael Faraday came from a family of modest circumstances and Sandemanian faith. As a bookbinder's apprentice interested in science, he read science books that came his way and attended Humphry Davy's popular lectures at London's Royal Institution, which was founded at the turn of the century as a private institution for promoting practical scientific research and popular scientific lectures. Entering the Royal Institution as Davy's servant, Faraday eventually became the most prominent member of its staff and one of England's most famous scientists. In 1831 he discovered electromagnetic induction and in 1845 the "Faraday effect." Grounded in his experimental researches, his ideas prompted the development of field theory, greatly influencing the mathematical physics of William Thomson and James Clerk Maxwell.

Research on Faraday can draw on an enormous manuscript record of correspondence and laboratory notebooks as well as his experimental apparatus itself. Evidently, Faraday scholarship could expand into an "industry" to rival those devoted to Newton and Darwin. Though not quite an industry in itself, this excellent volume, which brings together essays by some dozen scholars, could claim to have "rediscovered" Faraday in three basic ways: as experimentalist, as member of the Royal Institution, and as Sandemanian.

Faraday has always been recognized as a consummate experimental scientist, but recent research has extensively explored his considerable theoretical insights. Though not avoiding theoretical issues, this book above all seeks to comprehend the expertise and importance of Faraday the experimenter. The chapter by David Gooding, the leading Faraday scholar these days, goes beyond description of experiments in their final form to study Faraday's development of experiments from rough beginning to polished completion. "Experiments, like experimentalists, have biographies," Gooding explains. In identifying the typical stages in the life of a Faraday experiment, Gooding can rely on Faraday's *Diary*, which "allows us to see how a scientist worked day by day, sometimes hour by hour." Ryan Tweney's chapter on induction emphasizes Faraday's strategies of experimentation, noting his searches for both confirmatory and disconfirmatory results. Tweney sees the latter as especially significant in Faraday's methodology. Frank James's chapter on Faraday's "optical mode of investigation" highlights Faraday's use of light as an experimental tool, for example in his research on magnetism and on the structure of matter. L. Pearce Williams, whose massive 1965 biography of Faraday was a milestone in the study of 19th-century science, contrasts Faraday's experimental caution with the mathematical boldness of his French contemporary André-Marie Ampère. Concentrating on experimentation, Faraday forced Ampère to retreat from his bold theorizing of the early 1820's to less hypothetical research later applauded by Maxwell.

Two chapters attempt to revise Morris Berman's 1978 discussion of Faraday and the Royal Institution, which questioned the value of each to the other. Sophie Forgan's essay on "the institutional context" of Faraday's career maintains that Faraday's research *was* appreciated by the managers of the Institution and, indeed, that the Institution was probably the best place Faraday could have been for pursuing his own research interests. Moreover, denying that the Institution declined after 1844 under Faraday's leadership, Forgan declares that it entered the last third of the 19th century stronger than ever. Gooding's chapter, too, concerns the institutional context of Faraday's experimental work, as his experiments were begun and worked out privately in the Institution's basement and then brought upstairs as polished experiments for presentation (and general acceptance) in Faraday's public lectures. Since Faraday also lived at the Institution, there was an extraordinary dovetailing of an outstanding establishment and an eminent career.

The extremely conservative Christianity of Faraday's Sandemanianism has long been known but has been little investigated with respect to his science. In an imaginative use of Sandemanian sources, Geoffrey Cantor has taken on the task in his contribution on "reading the book of nature." For Faraday,