The 1985 Nobel Prize in Physics

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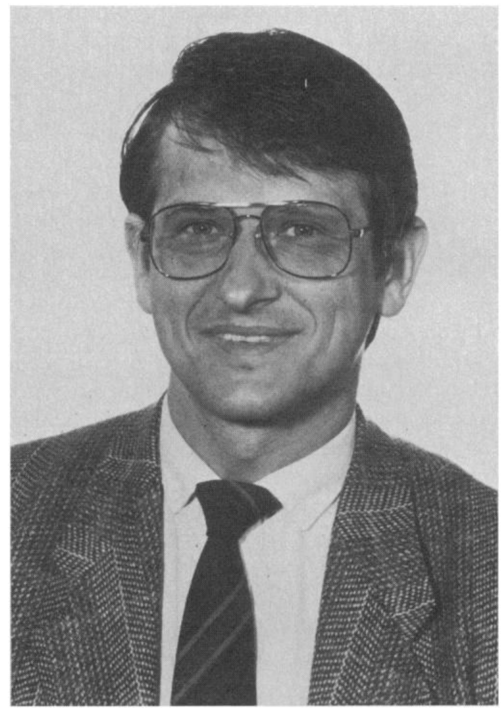
THE 1985 NOBEL PRIZE IN PHYSICS HAS BEEN AWARDED TO Klaus von Klitzing for the discovery of the quantized Hall effect, a remarkable phenomenon that occurs in certain semiconductor devices at low temperatures in very strong magnetic fields. The phenomenon was observed in 1980 in experiments by von Klitzing at the High-Field Magnet Laboratory of the Max Planck Institute, Grenoble, France, and was described in a paper by him and two collaborators, G. Dorda of Siemens Research Laboratory and M. Pepper of Cambridge University, who developed the samples used in the measurements (*1*).

The announcement of the quantized Hall effect came as a great surprise to condensed-matter scientists, and the subsequent effort to explain the effect has led to a major revision of our understanding of electronic conduction in strong magnetic fields. Moreover, the store of surprises was not exhausted by von Klitzing's original work. In 1982, during the course of an experiment designed to study the low-temperature magnetotransport properties of semiconductor systems with lower carrier density and higher mobilities, D. C. Tsui, H. L. Störmer, and A. C. Gossard, of AT&T Bell Laboratories, discovered another remarkable phenomenon, generally denoted the fractional quantized Hall effect (*2*), in contrast to the "integral" effect originally observed by von Klitzing. Although there is a great similarity between the two effects, the new discovery has led to a revision of theoretical concepts that may be even more radical than the first.

In addition to its impact on basic physical concepts, the quantized Hall effect has a direct application. As was noted in the original work of von Klitzing *et al.*, the effect can be used to construct a laboratory standard of electrical resistance that is much more accurate than the standard resistors currently available. Moreover, if the quantized Hall effect is combined with a new calibration of an absolute resistance standard, one should be able to obtain an improved measurement of the fundamental dimensionless constant of quantum electrodynamics, the fine-structure constant α.



Klaus von Klitzing

The Quantized Hall Effect

The quantized Hall effect is observed in artificial structures known as two-dimensional electron systems. The conduction electrons in these systems are trapped in a very thin layer, such that the electronic motion perpendicular to the layer is frozen into its lowest quantum mechanical state and plays no role in the conductivity of the device. In von Klitzing's original experiment, the system used was a silicon field effect transistor (MOSFET) of very high quality, similar in construction to the standard transistor on integrated circuit chips. The electrons here are trapped in a so-called inversion layer near the surface of a silicon crystal that is covered with a film of insulating silicon oxide, on top of which is deposited a metal "gate electrode," used to control the density of conduction electrons in the inversion layer. In many recent experiments the electrons have been trapped at the interface between two different semiconductors, such as a heterojunction between gallium arsenide and the alloy gallium-aluminum arsenide.

To study the Hall effect, a strong magnetic field B is applied perpendicular to the plane of the sample, while a small current I_x is made to flow within the plane. At the same time, one measures the voltage drop V_x parallel to the current and the "Hall voltage" V_y across the sample in the direction perpendicular to the current. The Hall resistance R_H is defined as the ratio V_y/I_x.

In most electronic systems the Hall resistance increases steadily with the strength of the applied magnetic field. The dramatic discovery of von Klitzing was the existence, in the two-dimensional systems at sufficiently low temperatures, of a series of plateaus in the Hall resistance where the value of R_H remained constant over a range of values of B and which satisfied the following simple formula with extremely high accuracy:

$$\frac{1}{R_H} = i\frac{e^2}{h} \tag{1}$$

where h is Planck's constant, e is electronic charge, and i is an integer that varies from one plateau to another (Fig. 1).

For the ranges of magnetic field and carrier concentration where the quantized Hall effect is observed, the voltage V_x parallel to the current is observed to vanish, in the limit of zero temperature, so that the current flows without dissipation. Under these conditions the Hall resistance is independent of the precise shape or size of the sample. The universality of R_H on the quantized Hall plateaus is in

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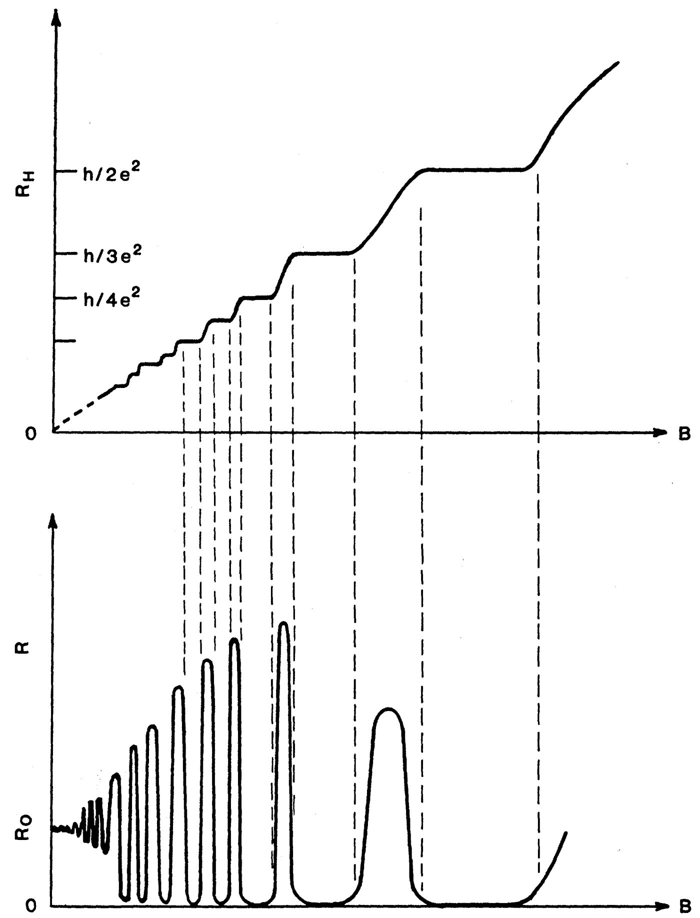

Fig. 1. Schematic plot of the Hall resistance $R_H = V_y/I_x$ and the ordinary resistance $R = V_x/I_x$ as a function of the perpendicular magnetic field B for a two-dimensional electron system at very low temperatures. Plateaus in R_H and vanishing of R are manifestations of the quantized Hall effect. The point R_0 is the resistance in zero magnetic field, which depends on the sample involved, while the universal quantity h/e^2 has the value 25,812.80 ohms.

strong contrast to the ordinary electrical resistance of a wire, which is sensitive to the size of the sample and its temperature, chemical purity, and density of structural defects. Indeed, for a standard calibrating resistor, one must take great care to avoid shape changes of any kind.

In the original measurements of von Klitzing, the accuracy of Eq. 1 was confirmed at a level of a few parts per million, with the principal error arising from the uncertainty of the calibrating resistor. More recent experiments have pushed this accuracy to 1 part in 6 million. The ratio of the values of R_H on different Hall steps and between silicon and GaAs samples has been measured with still greater accuracy, and found to agree with the ratio of integers at a level of 3 parts in 10^8 (*3*). It is almost without precedent for any physical measurement of a solid material except in superconductivity to be reproducible at this level of accuracy.

In samples that exhibit the fractional quantized Hall effect, one sees, in addition to the integer plateaus, steps in the Hall resistance where i is replaced by a simple rational fraction, p/q. There have been unmistakable observations of fractional plateaus at values of p/q that include 1/3, 2/3, 4/3, 5/3, 2/5, and 3/5, and there is at least some indication or indirect evidence for Hall plateaus at a variety of other fractions with odd denominators. There is no convincing evidence for a Hall plateau at a fraction with an even denominator, although there have been published reports of resistance anomalies that could be associated with such plateaus.

The fractional steps are observed only in samples of the highest quality, and generally require lower temperatures and stronger magnetic fields than the integer quantized Hall effect. The stringent conditions necessary for observation of the fractional steps have, in fact, been a motivating force to stretch the limits of semiconductor materials technology and to extend the experimental capability for measurements at very strong fields and low temperatures.

Historical Setting

Certain aspects of the integral quantized Hall effect had, in fact, been predicted in 1975, 5 years before von Klitzing's experiment, by the Japanese theorists T. Ando, Y. Matsumoto, and Y. Uemura (*4*). In an extensive theoretical investigation of the conducting properties of two-dimensional electron systems in strong magnetic fields, they noted that, under some special conditions, the Hall resistance would be given by Eq. 1. They were also aware that the current flow should be perpendicular to the direction of the voltage drop under these conditions, so that the current would flow without dissipation. However, the theoretical methods used by these authors were approximate ones and there was no expectation that Eq. 1 would hold with any great precision. Moreover, it was not expected that the Hall resistance would be constant over such broad intervals of magnetic field strength as was eventually found experimentally.

Several experimental studies of the Hall effect and of electrical conduction in MOSFET devices at low temperatures and strong magnetic fields were performed during the 1970's, and the results obtained were consistent with the approximate theory (*5, 6*). In general, however, there was no attempt to measure the Hall resistance with a high degree of accuracy, and the results of von Klitzing's 1980 experiment came as a surprise to workers in the field.

Current Theoretical Picture

The 1980 paper by von Klitzing *et al.* (*1*) emphasized not only the experimental precision of the effect and its potential importance as a standard of electrical resistance, but also the fact that existing theories were not adequate to account for this precision. Since then, a fairly complete understanding of both the integral and fractional quantized Hall effect has been developed. Here, however, I can present only a rough outline of the current theoretical picture.

We may begin by considering a highly idealized model in which a collection of noninteracting electrons move in a two-dimensional system with an electrostatic potential that is constant, except for a small gradient due to a weak, uniform applied electric field. In such a system the Hall resistance is simply given by the classical formula

$$R_H = \frac{B}{ne} \qquad (2)$$

where n is the density of electrons (per square meter) in the sample. If the two-dimensional system is connected to an external reservoir of electrons and the magnetic field B is allowed to vary, then the density of electrons in the layer will vary with B in such a way as to minimize the combined energy of the layer and reservoir, so Eq. 2 may be nontrivial.

It has long been known that the allowed states of a two-dimensional electron in an applied magnetic field are quantized into a series of discrete energy levels known as Landau levels and that the maximum density of electrons that can fit into a single Landau level is given by the quantity Be/h. Because the total energy of the layer has a sharp local minimum when the density is chosen to just fill an integral number of Landau levels, the ratio nh/Be will tend to be stabilized at each integer value for some range of magnetic field strengths. Combined with Eq. 2, this gives a heuristic explanation of the integer quantized Hall effect (*1*).

In actuality, the electrostatic potential in a two-dimensional electron system is never constant on a microscopic scale. In particular, there are always random variations in the potential arising from impurities in the material, and the classical Hall formula, Eq. 2, is no longer accurate under these circumstances. The impurity potential also broadens the Landau levels into energy bands, and if the potential is strong enough, then the density of electrons in a filled band is no longer given precisely by Be/h. It appears, moreover, that most of the electron states in the Landau band are physically localized in space, and these states cannot participate in the conduction of electricity by the system. The current is carried entirely by a few spatially extended states whose energies are near the center of each Landau band. Remarkably, the total Hall conductance of these extended states is precisely the same as the Hall conductance of the entire Landau level in the absence of impurities. Thus, as long as the Fermi energy, which separates the occupied electron states from the empty states at higher energy, is not located in a region where the states are spatially extended, there will be an integral number of occupied bands of extended states, and the Hall resistance remains quantized according to Eq. 1. We may say, then, that the nonconducting localized states act as an internal reservoir for the conducting electrons, and an external reservoir is therefore not necessary to observe plateaus in the Hall resistance. It is also possible to explain why the electrical current flows without dissipation under the conditions of the quantized Hall effect.

A quantum-mechanical derivation of the insensitivity of the Hall resistance to the perturbing potential of the impurities emerged gradually in a series of papers by various theorists (*7*), subsequent to the publication of von Klitzing's experiments. General arguments have been developed that exploit such principles as the "gauge-invariance" of quantum mechanics and the topological properties of the phase of a wave function in certain geometries as a function of magnetic flux. As may be imagined, the development of these ideas has led to an increased understanding of some of the hidden consequences of quantum mechanics, and there may well be applications of this understanding to elementary particle theory or to other branches of physics outside the quantized Hall effect.

A model of noninteracting electrons, with or without the random potential of impurities, can never give rise to the fractional quantized Hall effect. For this, it is necessary to take into account the effects of the electron-electron interaction in a fundamental way. The theoretical explanation of the fractional quantized Hall effect, developed by R. B. Laughlin in 1983, together with extensions by other theorists, required the construction of a new type of quantum-mechanical wave function in which the electrons form a highly correlated liquid that is stable and incompressible when the ratio nh/Be is an appropriate rational fraction (*8, 9*). According to these theories, the stable states exist and thus the quantized Hall effect can be observed for many fractions with odd denominators but not for all such fractions. The stable states have a number of exotic properties, including elementary excitations whose charge is a fraction ($1/q$) of the electronic charge e.

Although current theories can explain the general features of both the integral and fractional quantized Hall effects, the subject remains a very active one for experimental and theoretical study. In particular, there are several quantitative measurements, and even qualitative features, that remain poorly understood (*10*).

Biographical Notes

Klaus von Klitzing was 36 years old at the time of his discovery of the quantized Hall effect. He had published his first scientific papers 9 years earlier, in 1971, together with Professor G. Landwehr at the University of Würzburg, on the electrical properties of tellurium in strong magnetic fields (*11*). His research contributions during the 1970's included studies of silicon inversion layers in strong magnetic fields (*6*) and under conditions of uniaxial stress (*12*).

Since 1980, von Klitzing has played a leading role in the exploitation of the quantized Hall effect as a standard of electrical resistance and has investigated many other aspects of the behavior of two-dimensional electron systems (*13*). He is currently a director at the Max Planck Institute for Solid-State Research in Stuttgart.

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